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Truck driver scheduling with combined planning of rest periods, breaks and vehicle refueling

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Abstract

Fuel is one main cost driver in the road haulage sector. An analysis of diesel price variations across different European countries showed that a significant potential for cutting fuel expenditure can be found in international long-haul freight transportation. Here, truck drivers are often on the road for several consecutive days or even weeks. During their trips, they must comply with the rules on driving hours and rest periods which in the European Union are governed by Regulation (EC) No 561/2006. In the literature, refueling problems have attracted limited attention so far. In the present study, we show why a joint consideration of drivers' rest periods and breaks and refueling is important and how the choice of time windows, the planning of driver activities, and the determination of refueling stops and quantities can be done accordingly. For a given sequence of customer locations and gas stations with different fuel prices along the route chosen to serve these customers we propose a mixed integer linear programming (MILP) model and describe the corresponding solution process. In this multicriteria optimization problem with the goals to minimize lateness, traveling time and fuel expenditures, we consider multiple soft time windows at customer locations. We extend the MILP model developed by Bernhardt et al. (2016) by integrating refueling decisions. Additionally, a preprocessing heuristic is described which reduces the number of gas stations to be considered along the route and thus the solution space and the computational effort. Numerical experiments were conducted for instances derived from real data that include vehicle routes for one week and information on gas stations along the vehicle routes. Different parameter settings for the preprocessing heuristic were analyzed.

Keywords: road transportation, refueling, fuel cost, driver scheduling, rest periods, breaks, driving hours, Regulation (EC) No 561/2006, mixed integer linear programming models

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1 Introduction

Considering the competitiveness in the road haulage sector, cost levels are a key factor. According to the European Commission (2014), converging cost structures will more and more urge transport undertakings to improve their efficiency and quality of service. As depicted in Figure 1, fuel is a main cost driver and represents between 24 % and 38 % of the total costs in the EU member states.[1]

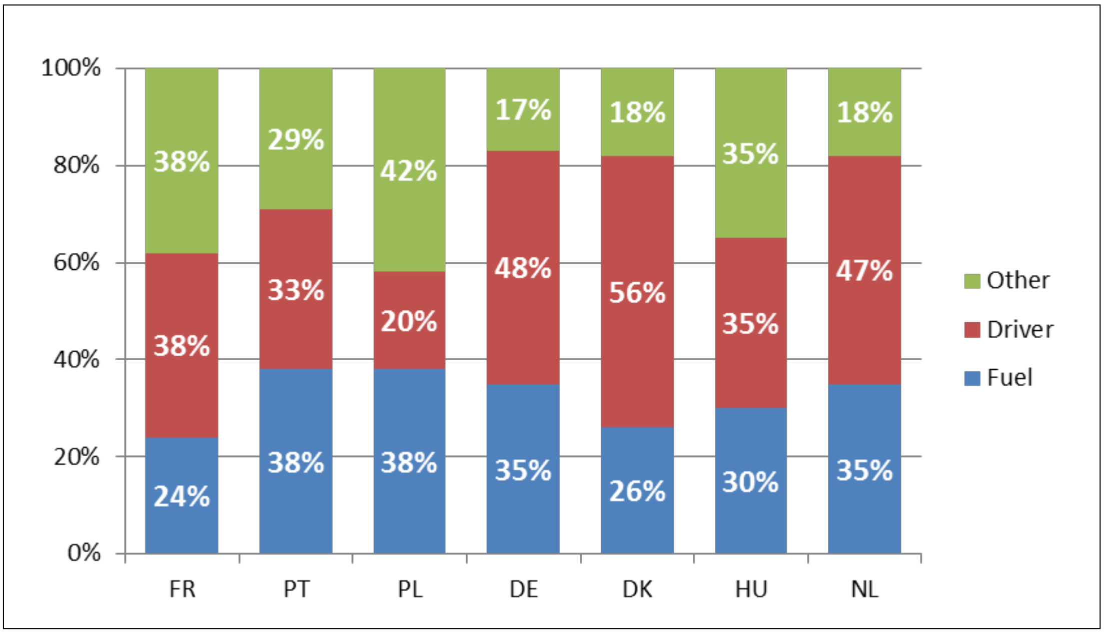

Figure 1: Cost breakdowns of hauliers from selected EU member states

Considering the 100 largest cities in Germany, price variances may amount up to 22 ct per liter diesel considering the cheapest and the most expensive price.[2] This seems to be a high cost saving potential but many haulage companies have contracts with fuel card operators that make diesel prices at gas stations dependent on list prices per country. In that case, price variations within one country become less important. A significant potential for cutting fuel expenditure can especially be found in international long-haul freight transportation. In the European Union, international transport operations account for almost one third of all road freight transport activities (European Commission (2014)). Diesel prices vary strongly across different European countries. Variations may amount to 30 ct per liter and more (see Figure 2). Price relations between countries are not constant over time as the comparison of Figures 2 and 3 shows. This means that even if countries with cheap diesel prices were chosen for refueling in the past and fixed contracts exist

[1] Source: Collection and Analysis of Data on the Structure of the Road Haulage Sector in the European Union, AECOM 2013 (retrieved from European Commission (2014))

[2] Source: clever-tanken.de (2017). The referenced prices include VAT.

causing traveled routes to remain the same, new refueling plans are necessary regularly to exploit the cost saving potential.

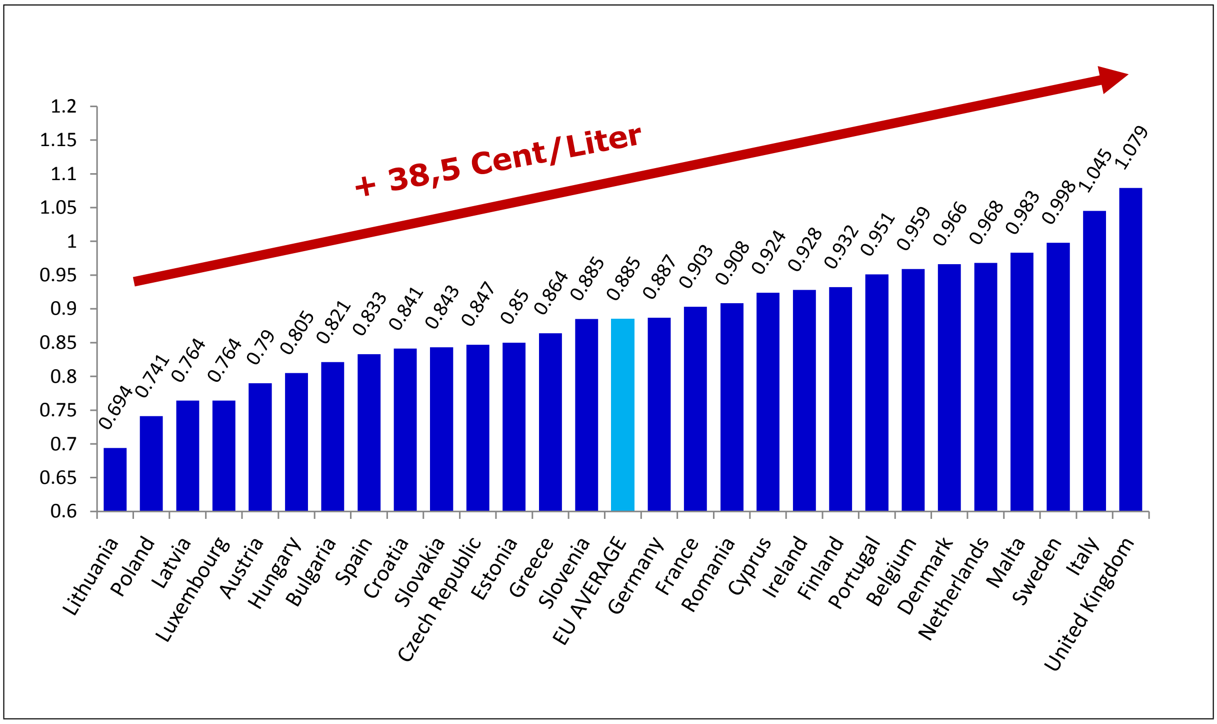

Figure 2: Diesel prices excluding VAT in Euro per liter on August 13, 2016[3]

Not only the choice of gas stations but also the refueling quantities have an impact on the refueling costs of a trip and determining good refueling strategies is a non-trivial task. Additional cost or non-cost factors can be taken into account and are considered in the literature (see Section 4).

One important factor is the time needed for detours to gas stations and for refueling and its impact on driver schedules.

Besides cost levels, quality of service is another important key factor (European Commission (2014)) for which in the haulage sector punctuality is a quantifiable distinctive feature for the customer. Lateness may cause contractual penalties and may lower customer satisfaction which has a strong impact on future requests and thus on the economic viability of a haulage company. The time needed for detours and refueling influences the transport duration and should be taken into account. In case a cheap refueling may bear the risk of a late arrival, the refueling plan should be reconsidered.

The cost breakdowns in Figure 1 depict that labor is, with fuel, one of the two main cost drivers in the road haulage sector. Opportunity costs arise when a driver has to wait several hours for a new customer time window or until the next day to load and/or unload the

[3] Source: Europe's Energy Portal (2016)

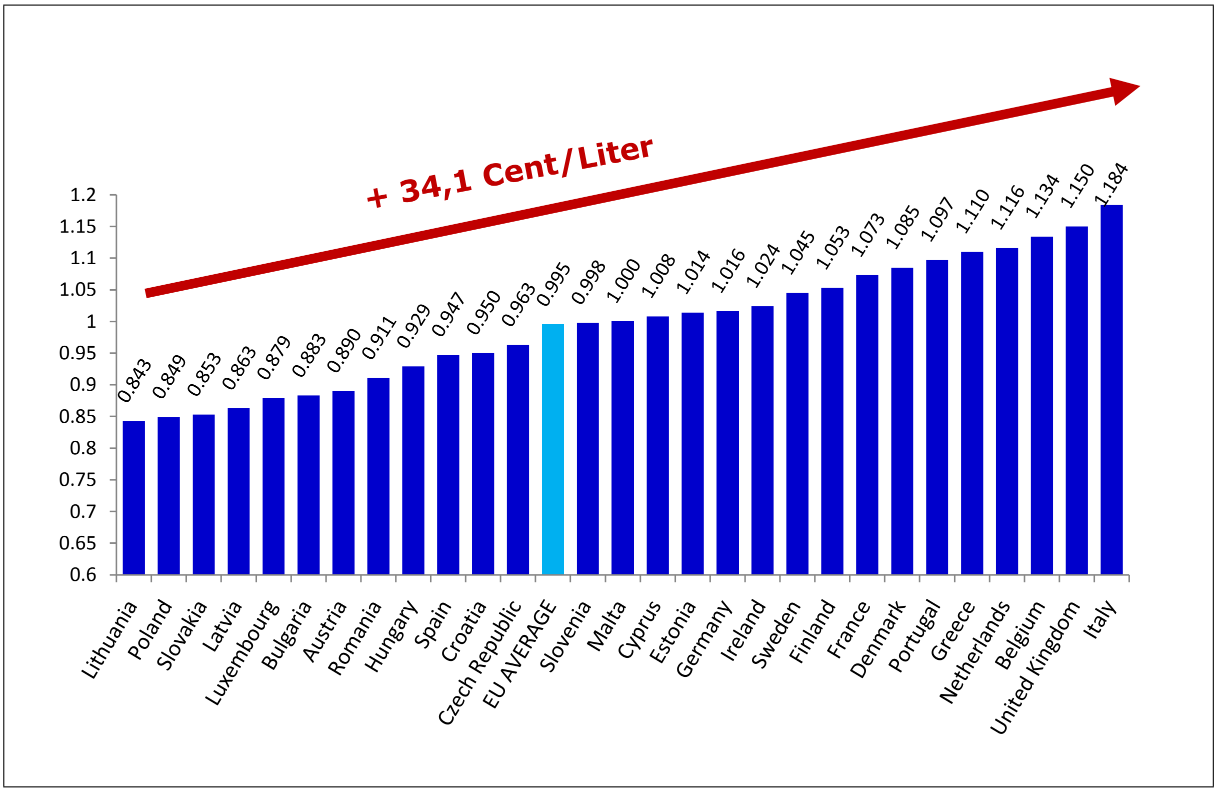

Figure 3: Diesel prices excluding VAT in Euro per liter on December 9, 2017 [4]

vehicle because he missed a time window or arrived after the opening hours. If he arrives much too early, this is disadvantageous as well for the same reason. Finally, one should not forget the annoyance for the driver if, for example, deviations from the original schedule disturb his or her plans for the weekend or a resting location with basic amenities.

When planning arrival times at customers for long-haul trips that require several days, Regulation (EC) No 561/2006 on driving times and rest periods of drivers in road transport is obligatory in all member countries of the EC. Set up to improve safety and working conditions of drivers in road transport, they have a high influence on the execution time of a transport request and disregarding them may be fined severely.

Despite the rules being rather complex in their application, as often many different possibilities to plan driver activities have to be evaluated, a dispatcher has to set up his plans ensuring that drivers are able to stick strictly to the regulation. Rest periods and breaks cannot be split arbitrarily or interrupted to serve customers or to refuel. Thus, it is not recommendable to consider resting activities in the form of a fixed proportion of the overall travel time in the schedule when planning arrival times as deviations would occur frequently. This would be disadvantageous especially if narrow time windows are involved.

[4] Source: Europe's Energy Portal (2017)

2 Motivation

Technology as on-board computers, digital tachographs and telematics equipment poses new challenges for transport companies but at the same time opens many new opportunities. Online availability of telematics data such as latest position data of drivers and time management data which reflect their exact status considering rest periods and breaks, gas station prices and locations are some of the data which could help dispatchers and drivers in their daily work. As there is plenty of distributed data to be evaluated decision support systems with advanced planning tools can be an important contribution to support decision-makers.

Fuel costs and driver rest periods and breaks are two important issues that transport companies have to take care of to be profitable. Moreover, dependencies between them suggest a joint analysis to identify good strategies.

Figure 5 shows an example of the interdependencies of planning refueling and choosing among customer time windows thereby considering daily rest periods and breaks. It is assumed that the driver crosses two country borders and thus has the possibility to refuel in three different countries with three different list prices.[5] Due to the remaining fuel quantity in the tank, a "corridor" has been determined in which the next refueling has to take place such that the vehicle does not run out of fuel. Without considering time windows, a refueling plan would recommend to stop in the area where the fuel price is 1.10 € and completely refill or at least refuel as much as needed to cross the area with a fuel price of 1.14 €. The latter recommendation depends on the development of future fuel prices in the following regions (countries) to be passed, namely if they are expected to be cheaper or not.

At the bottom of Figure 5 driver activities were planned without considering the duration for refueling. The start and the end times of the chosen customer time windows are marked with bold, black vertical lines. Loading and/or unloading at a customer location has to start within a time window but can be finished after the upper bound of that time window. The red vertical lines show the estimated arrival times of the driver at the customer locations. The symbols that are used to describe driver activities are explained in Figure 4. It can be seen that there is no lateness and at the first and last customer location the driver even has to wait 30 minutes until the start of the time window. At the second customer location, loading and/or unloading is planned to start at the end of the time window and thus any delays between the first and the second customer would cause lateness.

In this simple example, the estimated duration for refueling is assumed to be 20 minutes, the times for detours to gas stations are neglected. If we try to bring the two plans together, the first one only made for refueling, the other one for driver activities (without refueling) and time windows (see the continuous lines from top to bottom), we see that refueling for the optimal price of 1.10 € would cause lateness at customer 2. If we consider punctuality to be more important than fuel costs, refueling (at least) has to take place in the corridor

[5] For reasons of simplicity we assume that the gas station prices in a country are equal to the list price of this country.

with a diesel price of 1.17 € if the replanning of time windows, rest periods and breaks is not to be considered.[6]

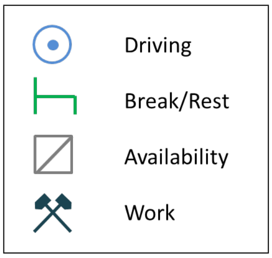

Figure 4: Symbols describing driver activities

Figure 6 shows the advantages of an alternative planning technique which simultaneously considers the choice of time windows and the determination of driver activities together with refueling. For the second customer a different time window has been chosen thus allowing the driver to be on time and also making it possible to refuel for the cheapest price of 1.10 €. Refueling is depicted in yellow.

Models and algorithms that plan refueling considering restrictions on drivers' working hours can be an important value added. Embedded in decision support and planning tools they can help dispatchers to plan vehicle movements, estimate arrival times at customer locations and reduce fuel expenditures. But the integration of both issues - the scheduling of drivers' rest periods and breaks and the refueling planning - into one planning process has not been addressed in literature so far. In the following, we will deal with the mathematical modeling and the integration of the two issues described above into one solution process.

During our research, we cooperated with a medium sized company operating truckload shipping services in Europe. While a significant part of transport requests result from fixed contracts, others are not known in advance. Several partner companies pass on requests and additionally, freight exchanges are used to supplement partial loads or to acquire additional requests for the return trip. Conversely, shipping orders are passed on to subcontractors. For our analysis of the developed solution methods, we concentrated on the international transport requests which represent a large proportion of the business activities of the freight company. Each request for transportation consists of a pickup and a corresponding delivery location and often, the locations are far apart of each other. Moreover, for the arrival at customer locations, opening hours have to be taken into consideration and it is common that loading and/or unloading at customer locations has to take place during so-called time windows, i.e. one or several time intervals proposed by the customer among which a choice has to be made. Especially for this group of requests, the joint consideration of driver rest periods and breaks and refueling is promising. As national borders are passed and fuel prices vary considerably across different countries in Europe, there is a high potential to cut fuel expenditures. Since travel times of several days are considered, the integration of

[6] Note that this is a simple example. In reality, if the driver schedule is executed, the complete plan of driver activities may have to be reconsidered after the first refueling. Because of detours breaks may have to be taken earlier and because of the additional time required daily rest periods may have to be rescheduled. This is also the case if gas station prices are neglected.

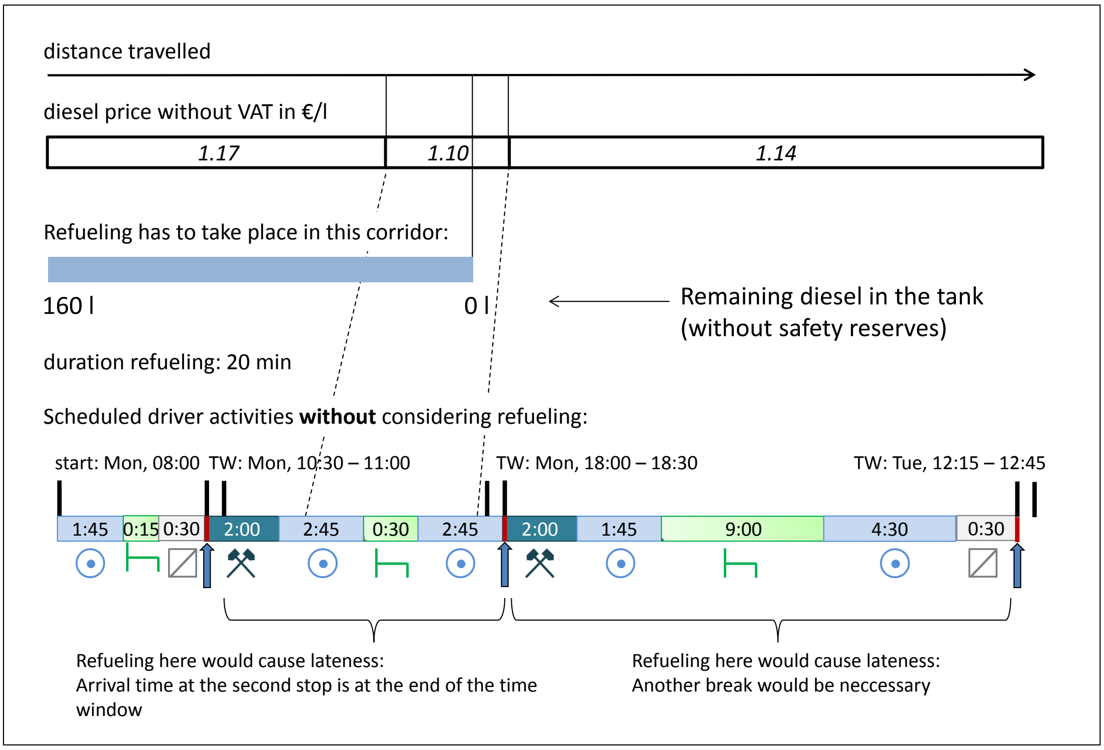

Figure 5: Planning of rest periods and breaks without considering refueling

rest periods and breaks into the planning of arrival times and the choice of time windows must not be neglected.

The planning tasks that are relevant for our consideration are distributed among dispatchers, drivers and a decision maker for refueling matters. An on-site process analysis gave detailed insights into the planning process. Dispatchers are responsible to acquire additional or to subcontract transport requests, to plan vehicle routes, choose among customer time windows and plan arrival times. Drivers plan their rest periods and breaks and regularly inform the dispatcher about their status. The dispatcher uses this information for the planning of time windows and to react on unforeseen events. The decision maker for refueling matters analyzes refueling data from the past and negotiates contracts with fuel card operators. If a gas station is favorable because of its geographical position near to often traveled routes, the decision maker may try to negotiate a special price and informs the dispatchers accordingly. Moreover, the tasks include the monitoring of diesel list prices of the countries and to give advice to the dispatchers in which countries refueling should preferably take place. The dispatchers inform the drivers about changes in the refueling strategy that are relevant for the current route. If refueling is necessary, drivers fill up completely if no cheaper gas stations are expected to be passed until the next refueling stop. Otherwise, a smaller amount is refueled.

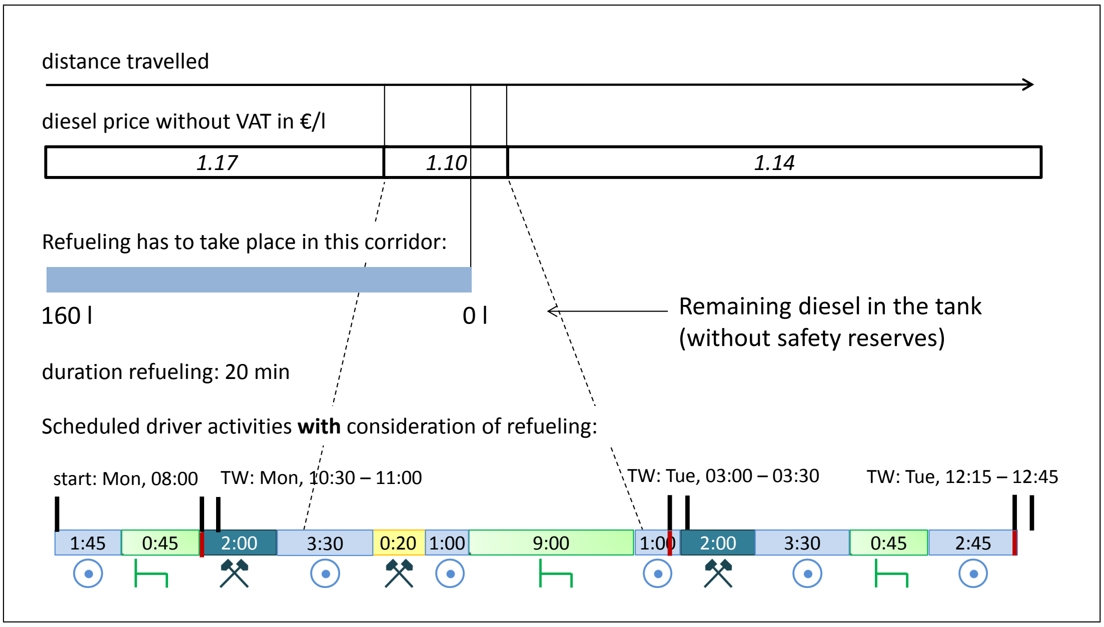

Figure 6: Joint planning of rest periods and breaks and refueling

3 Objective

A sequence of customer locations that are assigned to a vehicle in the current week and the route to be traveled are given. Geographical positions of gas stations along the route and the corresponding diesel prices are known. Driving durations and fuel consumptions between consecutive customer locations are additional input parameters as well as the current time and the driver status concerning rest periods and breaks. As we consider a planning horizon that comprises several days, we also take Regulation (EC) No 561/2006 on driving times and rest periods of drivers into consideration. For each customer location there are one or more time windows among which a choice has to be made. The time that is needed for loading, unloading and handling activities at each customer location is given as well.

The objective is to optimally choose customer time windows and gas stations, plan refueling amounts and schedule driver activities including refueling with the goals to maximize punctuality, i.e. to minimize lateness, and to minimize fuel costs. Inefficiencies that arise from the distributed decision making of drivers and dispatchers shall be minimized by considering these tasks simultaneously and determining a global optimum. Minimizing lateness and minimizing fuel costs are two conflicting goals. For example, choosing a very cheap gas station may cause a greater detour distance and travel duration. This may lead to lateness at following customer locations that could have been avoided by choosing a different gas station for refueling that is more expensive but causes less detour duration.

The MILP model that will be proposed builds on the MILP model with optional rules[7] for scheduling driver rest periods and breaks described in Bernhardt et al. (2016). Before we start to describe the model, we give a review of the literature dealing with vehicle refueling problems in Section 4. For a review of research on including regulations concerning rest periods and breaks in operational transportation planning considering Regulation (EC) No 561/2006 we refer to Bernhardt et al. (2016). Then, in Section 5, different graph structures for the refueling subproblem are analyzed and a graph structure together with the corresponding model is chosen to work with. In Section 6 it is shown how to merge the MILP model chosen in the previous step and the MILP model with optional rules developed by Bernhardt et al. (2016) to simultaneously plan refueling, customer time windows and driver activities in accordance with Regulation (EC) No 561/2006. The solution process to solve the resulting multicriteria optimization problem with the help of an optimization solver is described in Section 7. The creation of base instances for our numerical experiments is presented in Section 8. A heuristic preprocessing which was used to eliminate unattractive gas stations and thus to reduce the problem size is introduced in Section 9. In Section 10, the environment and the different parameter settings for the numerical experiments are described. The analysis is given in Section 11. Section 12 summarizes our findings and identifies directions for future research.

[7] In Bernhardt et al. (2016) two MILP models, one which considers the optional rules of Regulation (EC) No 561/2006 and one that ignores these rules, are developed. Both can be chosen as basis but in our mathematical experiments we concentrated on the one with optional rules.

4 Literature review

In the literature different refueling problems are analyzed. Besides studies for the road transportation sector, there are works that deal with refueling problems in railroad networks, the airline industry and maritime transportation. The positioning of fueling facilities is also a field of research.[8]

In the following review we consider studies that concentrate on road transport and vehicle refueling problems that include the identification of gas stations to be visited and the amounts of fuel to be purchased. For other modes of transportation we refer to Suzuki and Dai (2013) who propose corresponding literature. Routes may be given in advance or chosen together with the refueling strategy.

In the problem considered by Lin et al. (2007) the vehicle traverses a series of gas stations with different fuel prices while traveling along a fixed route. Detours to gas stations are ignored. At each gas station a decision has to be made on how much to refuel. The goal is to reach the destination with minimum total fuel costs. Lin et al. (2007) relate the problem to the inventory-capacitated lot-sizing problem and propose a linear-time greedy algorithm. The idea is at each gas station to fill just enough to reach the next cheaper gas station or to fill up the tank if no cheaper gas station is reachable even with a full tank.

Lin (2008b) deals with the problem of finding an optimal refueling policy in a transportation network with fixed start and target vertices. The other vertices are gas stations with different fuel prices and other locations such as cities, suppliers or customers that may be but do not have to be visited. The goal is to find the cheapest path in the transportation network along with the corresponding refueling quantities without running out of gas. The starting and ending fuel levels may be arbitrary between a minimum fuel level (reserve quantity to remain in the tank at all time) and the tank capacity. Lin (2008b) takes all possible integer fuel levels per stop into consideration and proposes a polynomial time dynamic programming algorithm to solve the problem depending on the number of vertices and the difference between the minimum fuel level and the tank capacity.

Khuller et al. (2007) consider several different refueling problems. One of them is the same problem addressed by Lin et al. (2007). Another one is the "gas station problem", which is similar to the problem addressed by Lin (2008b) with the difference that the ending fuel level is set to be equal to the minimum fuel level. Khuller et al. (2007) present a different dynamic programming recursion to solve the problem. For the all-pairs version, a faster algorithm is proposed. Both algorithms run in polynomial time. Khuller et al. (2007) also study the "tour gas station problem" where a set of cities has to be visited in arbitrary order in a minimum cost tour. There may be cities with gas stations but some cities may not have a gas station. Gas stations located outside of cities may also be visited for refueling. Khuller et al. (2007) at first concentrate on the uniform cost case where fuel prices are the same everywhere. As a generalization of the traveling salesman

[8] Corresponding literature is presented for example by Suzuki (2008).

problem (TSP) the problem is NP-hard. Under certain assumptions[9] the "uniform cost tour gas station problem" can be reduced to the TSP and can be solved with standard techniques. Building upon the Christofides heuristic for the TSP, the authors develop an approximation algorithm for the more general problem where for each city to be visited there is a gas station within a specified distance with a fuel consumption of less than a half of the tank capacity. This algorithm is used within the heuristic for the "tour gas station problem" with arbitrary fuel prices. In the "sequence gas station problem", a cheapest way from a source to a final destination has to be found in the transportation network, visiting a set of locations in a given order. This problem can be reduced to the original gas station problem. The technique used will be discussed later when we consider the different possible graph structures (see Section 5). At last, Khuller et al. (2007) consider the "single gas station problem" where the vehicle starts from the gas station and always has to return to it before it runs out of gas while visiting a number of cities.

Lin (2008a) considers a refueling problem that is similar to the gas station problem addressed by Lin (2008b). By analyzing the structure of optimal refueling policies he reduces the problem to the classical shortest path problem. For this purpose, a transition graph is derived from the original graph, modeling all extremal transitions between gas stations where the vehicle arrives with the lowest fuel level allowed and gas stations that are left with a full tank. A corresponding distance measure that represents the transition cost is introduced. Lin (2008a) presents an algorithm that is faster than the one proposed by Khuller et al. (2007) for the all-pairs version. In addition, on the basis of the all-pairs version, he gives a solution method for the single-pair case with given ending fuel level that may differ from the minimum fuel level.

Lin (2016) shows how to efficiently maintain and update routing and refueling information to be able to determine an optimal refueling strategy in quadratic time depending on the number of gas stations (n). With the help of shortest path trees and the usage of the transition network described in Lin (2008a), important routing information is determined in $O(n^3)$ time using quadratic space which also depends on n.

Suzuki (2008) refers to software products already in use by transport companies in the United States to plan vehicle refueling. He develops a mathematical programming model that mimics the behavior of standard fuel-optimizer products such as ProMiles, Expert Fuel, Fuel and Route, or Fuel Advice. The model can be used to optimally plan refueling stops at gas stations along with refueling quantities for a given route considering detour distances to and from gas stations. He stresses additional parameters that are taken into account by most fuel optimizer packages that are important in practice. These include, for example, the detour distances to gas stations or the availability of certain amenities to be able to eliminate unattractive gas stations that are far off the route or that do not have shower facilities. A minimum purchase quantity allows to control the frequency of refueling stops. A limitation to "network" gas stations (i.e. gas stations with purchase contracts) has also to be taken into account. Suzuki (2008) identifies the shortcoming with respect to other non-fuel cost elements that are interconnected with out of route miles to gas stations and the frequency of refueling stops such as vehicle deprecation cost, vehicle

[9] It is supposed that every city has a gas station and the largest distance between any two cities is less than or equal to the tank capacity. No additional gas stations are considered, i.e. the set of gas stations is equal to the set of cities to be visited.

maintenance and opportunity costs in standard fuel optimizer software. He also stresses the underestimation of cost elements such as fuel consumption rates on non-highway roads that are of special importance if highways are left and detours to visit cheap gas stations are accepted. In Suzuki (2008) a MILP formulation is introduced to include such cost components in order to minimize the total cost of operating a vehicle in a given route. Numerical results for randomly generated instances are presented that compare the fuel purchasing cost and the total vehicle operating cost for the solutions obtained with the standard fuel optimizer model with those of the extended version. As solution method, the simplex algorithm in conjunction with the branch-and-bound method is used.

Suzuki (2009) addresses a refueling problem that differs from the ones discussed so far. Usually, refueling strategies applied by standard-optimizer products deal with the questions which gas stations to choose and how much fuel to purchase. Suzuki (2009) mentions that transport companies are reluctant to introduce fuel optimizer products as they are afraid to suffer from limited actual cost savings because of low driver compliance rates and they even fear that drivers may move to other companies. He proposes a method that considers fuel price fluctuations over time and allows drivers to freely choose the gas stations they wish to visit. It is assumed that drivers take their daily rest period at a parking area of a truck stop where they also refuel. A corresponding refueling policy comprises the decisions on whether to refuel before or after taking a daily rest period at a truck stop chosen by the driver and on the refueling quantity. The latter may be equal to the minimum purchase quantity or the amount needed to fill the tank completely. Expected future prices at subsequent gas stations are taken into account by the stochastic dynamic programming model proposed. To predict future fuel prices at truck stops the OPIS (2017) database which provides fuel price information for truck stops in the U.S. and Canada is used. Computational results for randomly generated test instances are presented, comparing the costs for the case of random refueling behavior, those obtained when using the standard fuel optimizer model and those provided by the method proposed. Considering several scenarios, not only fuel costs but also driver compliance rates and driver replacement costs are taken into account. Although the lowest fuel cost is attained for the standard fuel optimizer model, under certain conditions the overall cost savings are higher for the proposed method.

Suzuki and Dai (2013) consider the vehicle refueling problem in combination with the route selection and propose a corresponding bicriteria MILP model. In contrast to Lin (2008a,b) and Khuller et al. (2007), the presented transportation network comprises vertices solely incorporated for the route selection subproblem. Gas stations are considered between each pair of those vertices in a similar way as it is done by Suzuki (2008, 2009) for the fixed path refueling problem. Additional constraints involve a limit on the maximum number of refueling stops and a limit on the maximum route duration. The duration for refueling is considered to be constant. Suzuki and Dai (2013) emphasize that it is important to consider both, fuel costs and vehicle miles and thus also integrate pollutants emission caused by increased fuel consumption into the decision process. The authors propose an optimization technique that involves the usage of a commerical optimization solver to construct the Pareto front. Different strategies are proposed to select the final solution according to the user preferences.

Suzuki (2014) outlines that there is no efficient algorithm in the literature that can solve the complex fixed-route vehicle-refueling problem to optimality taking into account the minimum refueling quantity as well as detour distances to gas stations. He suggests the usage of a preprocessing heuristic to eliminate gas stations that are guaranteed not to be chosen for refueling in the following solution process. For 16 instances based on real data provided by a fuel optimizer vendor, the variable-reduction technique removed between 46.9 % and 60.1 % of the gas stations. On average, this reduced the run time to about one-fourth of the original time needed to find an optimal solution. Suzuki (2014) also considers the solution quality of solutions determined by the heuristic used in the software of the fuel optimizer vendor. For the instances considered the difference between the optimal solution and the one determined by the heuristic method was 0.3 %. In some of the solutions produced by the software of the fuel optimizer vendor, less than the minimum purchase quantity was refueled at gas stations implying that the minimum purchase quantity is considered as a soft constraint.

The weight of the fuel in the tank as a variable part of the overall weight of the vehicle has an influence on the fuel consumption. Suzuki et al. (2014) aim at incorporating this weight as a factor for refueling decisions modifying the standard fuel optimizer model presented by Suzuki (2008, 2009) accordingly. Additionally, they consider the possibility to modify the minimum quantity of fuel to be left in the tank to not run out of fuel in case of unforeseen events depending on the gas station density that varies along the route. For the resulting nonlinear model, the authors propose a simple heuristic approach. To this end, they develop a relaxed MILP model based on the standard fuel optimizer model. By adding a penalty term in the objective function, the portion of the fuel tank that is never used is rewarded. The minimum fuel level is set per route segment. In their experiments, Suzuki et al. (2014) show the saving potential of their approach compared to the standard approach. They discover that in their experiments the overall fuel consumption is only reduced by up to 0.25 %, whereas the savings in the overall fuel costs amount up to 1.74 % compared to the standard approach presented in Suzuki (2008, 2009). This indicates that the reduction of the minimum fuel level for areas with a high gas station density is taken advantage of very extensively. At cheap gas stations, this allows to buy more fuel. Suzuki et al. (2014) also argue that the effectiveness of their approach may improve as the maximum tank capacity increases. Based on Suzuki (2008), the authors also consider the impact of their approach on non-fuel cost. Since in their approach the detour distance and the frequency of refueling stops is decreased, the sum of other (direct and opportunity) costs associated with detour distances and durations and the time needed for additional refueling stops is reduced as well.

Lin (2014) introduces two MILP models for vehicle refueling problems with route selection in a transportation network that is similar to the network considered by Lin (2008a,b). The models either minimize fuel cost or travel time giving an upper bound on the overall travel time[10] or the fuel cost, respectively, and thus only differ by the objective function and a single constraint that has to be chosen accordingly. First, the author proposes a formulation that restricts the solution space to only allow a simple path and then shows how to relax this condition.

Lin (2015) proves that the computational task to solve the MILP models presented in Lin (2014) is NP-complete even if fuel prices do not vary or the fuel consumption and the travel time are linearly dependent. For these two cases the author proposes two fully polynomial-time approximation schemes.

[10] Note that this is equivalent to having a customer time window at the target location.

Vehicle refueling in the context of vehicle routing problems with time windows is considered by Bousonville et al. (2011). In this study, the standard fuel optimizer model introduced by Suzuki (2008, 2009) is extended considering additional vertices representing customer locations with time windows. The resulting graph structure with detour distances to gas stations is obtained by mapping gas stations into the main path. Thus, the detour from the route to a gas station may differ from the detour from that gas station back to the route. The resulting model is used to integrate refueling decisions into the Solomon I1 heuristic (see Solomon (1987)) for solving the VRPTW. Test instances are constructed on the basis of the well-known Solomon benchmark instances. The numerical results show the impact of price variations on the tour length.

Suzuki (2012) considers vehicle refueling in combination with the time-constrained single-vehicle routing problem (traveling salesman problem with time windows, TSPTW). He proposes a two-stage solution technique. In the first stage, the TSPTW is solved using a variant of the simulated annealing technique. Not only the best feasible tour is kept but also the M best feasible tours. In the second stage, for each tour chosen from a subset of the M tours determined in stage one a MILP model is solved using the simplex algorithm. The chosen subset depends on a customizable parameter. Similar to Bousonville et al. (2011), the MILP model is an extension of the standard fuel optimizer model with additional time window constraints. Strategies for improving solution time are discussed. Numerical experiments for the proposed method are conducted for three real-word instances and a set of hypothetical instances (simulation experiments). The solution quality and the run time are compared to benchmark methods.

In the literature, no algorithms or models have been proposed so far that simultaneously plan vehicle refueling along with driver rest periods and breaks. The main contribution of this study is to present a MILP model to fill this gap.

5 Graph structures

In the literature dedicated to refueling problems, different graph structures are analyzed. Three of them are interesting for our objective and will be described in the following. Additionally, we will show their possible integration with the graph structure of the MILP models developed by Bernhardt et al. (2016)[11] for the scheduling of driver activities with multiple soft time windows considering European regulations on rest periods and breaks. Before we start discussing the different graph structures, we first describe the input of the problem we addressed in Bernhardt et al. (2016).

The starting point of the problem consists of an origin location, a sequence of customer locations which have to be visited by the vehicle for loading and/or unloading goods and a final destination. Regulation (EC) No 561/2006 on rest periods and breaks should be taken into account. To be able to plan in a rolling horizon manner and to reschedule activities in case of unforeseen events, the driver status at the beginning of the planning process provides information about when the next daily or weekly rest period or break is necessary. Moreover, it also gives information on the usage of the optional rules. Thus, the planning may start at the beginning of the week but it is also possible during the week in case that the driver has already started to serve customer requests. We do not consider customer locations with time windows that start after the presumed end of the next weekly rest period. This means that we do not schedule driver activities spreading over two weeks. Loading and/or unloading at customer locations has to start within time windows or opening hours (modeled as large time windows). Multiple time windows may be available and a choice has to be made. We allow lateness but we penalize it in the objective function to be able to give feedback to the dispatcher in case that there does not exist a solution without lateness. Driving durations between consecutive locations and estimated durations for loading, unloading and handling activities are known.

To integrate refueling decisions and to determine the corresponding input parameters, at first a decision about the problem definition and the degree of abstraction has to be made. For two of the graph structures described in the following it is assumed that the choice for the optimal route to serve the customers has been made in advance. In one of these graph structures the problem is reduced by neglecting detours to gas stations. The third graph structure bases on the idea that for the choice of a route prices and locations of gas stations already play an important role. It can be argued that when choosing the route independently gas stations along the route may be very expensive and detour distances may be large as this was not included into the decision. This can be overcome when integrating the choice of gas stations into the process of finding an optimal route between consecutive customers. Therefore, the graph structure of this subproblem represents a complete transportation network with vertices for customer locations and for gas stations and arcs between them.

[11] An integration is possible for both models, for the one that considers the optional rules and the one that ignores them. In the next section, we show the integration for the model with optional rules. Numerical experiments are only described for the MILP model resulting from this step. If optional rules are not considered, then constraints (6.14) and (6.15) must be dropped and the objective function (6.26) has to be modified. For reasons of simplicity, in the following, we talk about "the MILP model developed by Bernhardt et al. (2016)" referring to the model which considers the optional rules.

Independently of the graph structure that will be chosen, the goal is to select time windows and gas stations, determine refueling quantities and plan driver activities that comply with Regulation (EC) No 561/2006 in such a way that lateness, overall travel time and fuel expenditures are minimized. Whether or not we include routing decisions will be revealed at the end of this section.

We will now describe the three approaches in more detail and will explain their advantages and disadvantages.

Lin et al. (2007) consider refueling along a fixed route. No detours to gas stations are taken into account. Adapted to the problem of finding an optimal refueling policy between an origin and a destination where no refueling may be allowed at the origin and destination, the resulting linear graph for n locations (origin, destination and $n-2$ gas stations) looks like the upper graph depicted in Figure 7. The origin may be the start location for the vehicle at the beginning of the planning horizon or a customer vertex where loading and/or unloading takes place. The destination may be the subsequent customer location or the end location that should be reached until the end of the planning horizon. For a sequence of customer locations to be visited, the corresponding graph is shown below. The r different locations (origin, destination, customer locations and gas stations) are numbered from 0 to $r-1$. It is simply the concatenation of origin and destination pairs and the linear graph structure remains.

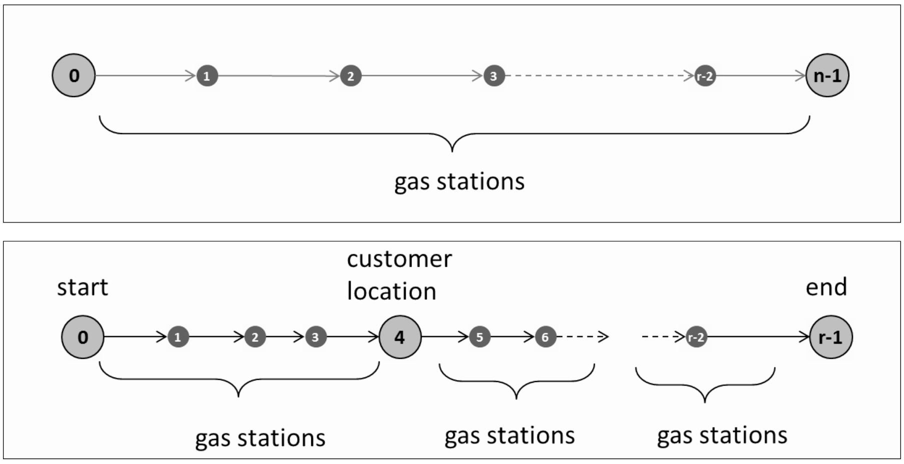

Figure 7: Linear graph

The disadvantage of this graph structure is the underlying assumption that gas stations are always located on the route or at least extremely nearby such that detours to reach gas stations and to return to the route may be ignored. Gas stations located along the motorways are usually more expensive than stations that are a little farther away even if fuel cards are used. In addition, neglecting gas stations requiring a detour may reduce the solution space too much. On the other hand, considering gas stations with a detour but ignoring the detour distance will lead to solutions that are suboptimal in practice. As a detour to a gas station consumes time and fuel we actually want to know whether a price difference is worth a detour. Detours not considered in the planning phase may jeopardize the driver schedule and thus a punctual arrival at customer locations. Furthermore, they may lead to higher fuel expenditures than originally expected.

In Lin (2008a,b) and Khuller et al. (2007) (in their gas station problem), refueling in a transportation network is considered. In the basic problem, only the origin and destination vertices are set in advance, other vertices that represent gas stations or other locations may or may not be visited and together with the origin and destination pair form a complete graph. The goal is to find the cheapest path from the origin to the destination. Extended by vertices for customer locations that have to be visited, the complete graph looks like the one depicted in Figure 8 in the upper part. To ensure that the sequence in which the customer locations have to be visited is kept, we need an extension. Khuller et al. (2007) call the underlying problem of finding the cheapest way starting from an origin to a final destination visiting a set of locations in a given order during the trip "the sequence gas station problem". For this task, the authors provide a corresponding graph structure. For each location in the sequence that is not equal to the origin or destination, a copy of the complete graph is made, i.e. if n is the number of these locations, $n-2$ copies are obtained. These graphs are joined by merging the equivalent to the i-th location (unequal to origin and destination) of the sequence from the i-th copy with the one from the $(i+1)$-th copy. If there is at least one customer location to be considered, the original graph is jointed with the first copy by merging the first customer location and its equivalent in the copy. Figure 8 demonstrates the case of one location that does not equal the origin or destination.[12] Thus, the "sequence gas station problem" can be reduced to the "gas station problem", as a solution to the original problem can be obtained by finding an optimal path from the origin to the destination in the new graph.

One major drawback of this representation is the huge number of additional indicator variables that have to be provided when describing the graph in a mathematical model. If n is the number of locations to be visited, including the origin and destination, and m is the number of gas stations, the whole graph has a number of arcs equal to $(n-1)(n+m)(n+m-1)$. If in our mathematical model each arc is connected with the decision on whether or not to use this arc in the solution, we obtain for the case of only 10 locations to be visited in a sequence including origin and destination and 20 gas stations 7830 binary decision variables. Additional decision variables that are relevant to model rest periods and breaks on arcs have to be considered for each arc. However, there are additional reasons against this graph structure. In practice, an optimal route between locations is chosen based on different criteria, not only depending on refueling costs. Relevant aspects are, for example, the driving duration, toll costs, suitability of the streets for trucks, durations of border controls and regional holidays with driving bans. Therefore, it is not a good idea to base the choice of routes between customer locations solely on the selection of gas stations. As long as other aspects to be taken into account cannot be formalized in a sufficient way and integrated in the routing decision, it is better to directly plan the routes between customer locations and try to stick to them as much as possible.

The mathematical models presented by Suzuki (2008, 2009) build upon the standard fuel optimizer model. The underlying idea of this model is that drivers follow the route previously determined (for example by a routing algorithm and then modified to also take

[12] Note that in Figure 8 a consecutive numbering of all locations, including gas stations, is chosen. The vertex with number 2 represents the first location of the sequence of locations that have to be visited.

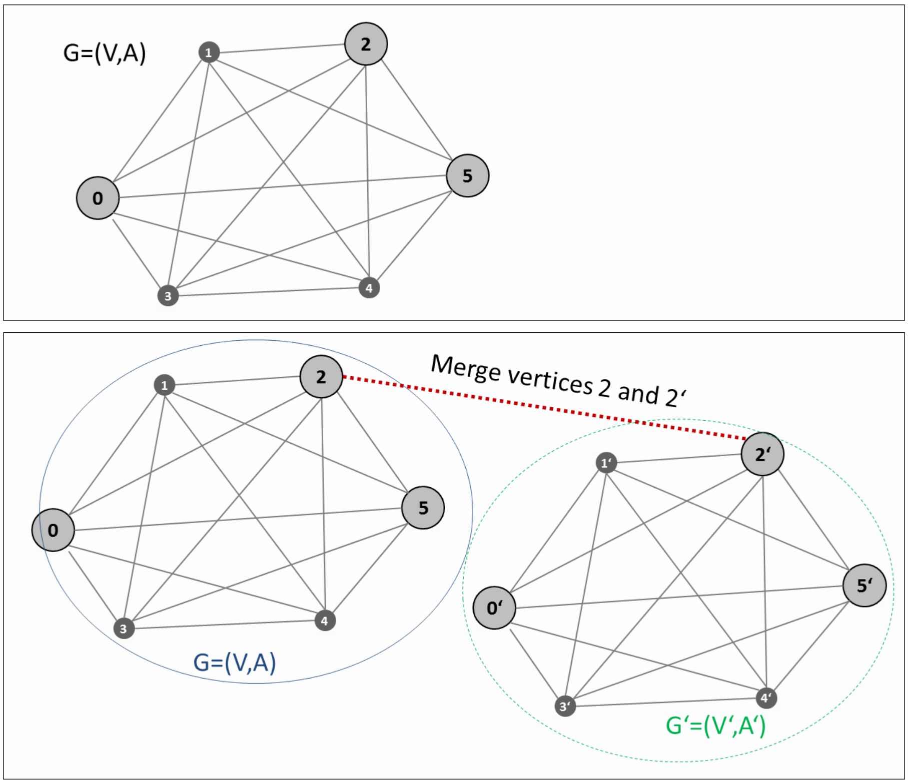

Figure 8: Subgraphs as copies of a complete graph merged at customer vertices

care of the aspects described above) and only leave it for refueling to return to the route afterward (see the upper graph in Figure 9). To simplify the problem, it is assumed that the route is left for refueling and entered after refueling at the same position and that the detour distances to and from the gas station are the same. In reality, this may not always be the case, especially if the motorway is left for refueling. However, this graph structure allows us to consider detours without considering complete subgraphs for refueling. The original route is kept and only extended by detours. If a decision variable indicates that a gas station is chosen for refueling, the consumption for the detour to the gas station is added to the consumption for the path between the preceding location (gas station, customer location, origin or destination) and this gas station. The path between this gas station and the next location is extended by the detour from the gas station back to the route. Due to the reasons described above and the drawbacks of the other graph structures we decided to use this base graph structure to model the combined problem of planning refueling, customer time windows and driver activities in accordance with Regulation (EC) No 561/2006.

The extended graph for a sequence of customer locations is depicted in Figure 9 at the bottom. Note that in contrast to Suzuki (2008) and similar to Bousonville et al. (2011) we decided to allow detour consumptions (and durations) to and from gas stations to differ.

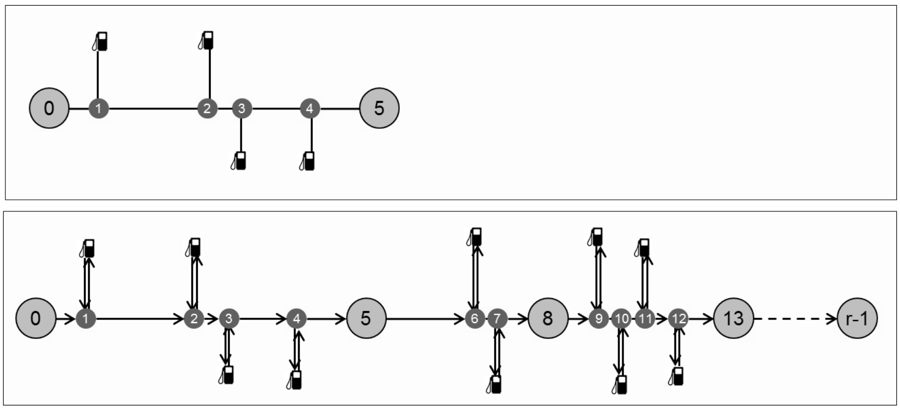

Figure 9: Linear graph supplemented by detour arcs

To allow for the consideration of fuel consumption that depends on route lengths and also on properties like road types and geographical data and to enable the possible usage of historical fuel consumption rates for route segments often traveled, we do not consider fixed fuel consumption rates as input parameters but a concrete fuel consumption for each path between locations.

In the following section we will extend the standard fuel optimizer model presented by Suzuki (2008) and show the necessary changes on the MILP model developed by Bernhardt et al. (2016) to be able to merge the two models.

6 Mathematical formulation

In the following, we show how to merge the two MILP models, the standard model for refueling decisions as described by Suzuki (2008, 2009) and the model for rest periods and breaks from Bernhardt et al. (2016) and which modifications have to be made. Figure 10 takes up again the graph structure from Figure 9.

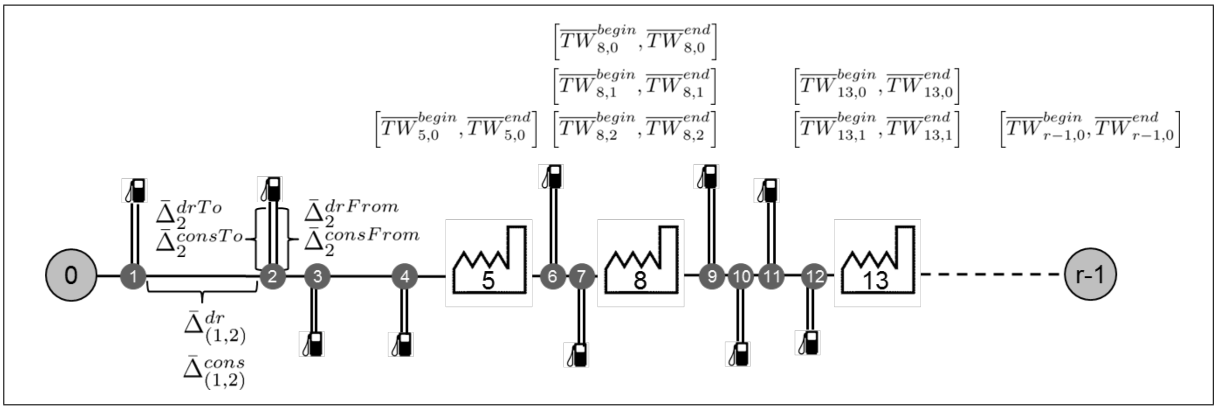

Figure 10: Parameters for driving durations and consumptions and time windows

The factory symbol emphasizes that the sequence of locations between the origin and destination consists of customer locations where loading and/or unloading takes place. A consecutive numbering is chosen for all locations, i.e. for origin, destination, customer locations and gas stations not differentiating between the kind of location. For the sorting of gas stations the point where the route has to be left to head for the corresponding gas station is the decisive criterion. Note that only gas stations that lie in a previously specified (linear or real) distance to the route are relevant and are listed with their detour durations ($\bar{\Delta}_i^{drTo}$ and $\bar{\Delta}_i^{drFrom}$ for the detour duration to and from the gas station i, respectively) and detour consumptions ($\bar{\Delta}_i^{consTo}$ and $\bar{\Delta}_i^{consFrom}$ for the detour consumption to and from the gas station i, respectively) as potential refueling points.

Observe that if a gas station can be visited between several consecutive customer locations, it is listed for each of those pairs. Thus, depending on the time windows, it can be decided between which pair(s) of customer locations the gas station shall be visited if it is chosen for refueling. We used a mapping table to memorize the assignment of customer locations and gas station IDs to location numbers.

The set of all locations is given by $S^{locations}$. To differentiate between location types, $S^{customers}$ denotes the set of all customer locations and $S^{stations}$ denotes the set of all gas stations. The origin and destination are mapped by the first vertex 0 and the last vertex $r-1$, respectively.

Driving durations and fuel consumptions between two consecutive locations i and $i+1$ without detours (in case one or both locations are gas stations) are given by $\bar{\Delta}^{dr}_{(i,i+1)}$ and $\bar{\Delta}^{cons}_{(i,i+1)}$, respectively.

Similar as in Bernhardt et al. (2016), each customer location has at least one time window, i.e. a time interval in which the loading and/or unloading of goods should start. We consider multiple customer time windows, as in reality, a dispatcher often has the possibility to choose among a set of time windows proposed by a customer. The start of the z-th time window interval at location i is given by $\overline{TW}_{iz}^{begin}$, the end by $\overline{TW}_{iz}^{end}$ ($z = 0, 1, ..., nTW_i - 1$).

In Section 6.1, we describe the modifications that have to be made for constraints adopted from the MILP model described by Bernhardt et al. (2016) to be able to integrate the time aspects of refueling decisions. Then, in Section 6.2 additional parameters, variables and constraints are introduced. A complete list of all parameters and variables of the whole MILP model is given in Appendix A.

6.1 Modifications for constraints from the model for rest periods and breaks

All constraints from Bernhardt et al. (2016) are adopted for the extended vertex set that now additionally consists of refueling vertices. Some modifications are necessary and will be described in the following.

In the MILP model of Bernhardt et al. (2016), the driving durations between two consecutive locations $\bar{\Delta}_{i,i+1}^{drive}$ were constant. As now gas stations are included in the list of locations, the durations contain a variable part if at least one of the locations is a gas station. Depending on whether a gas station i is chosen for refueling ($\alpha_i^{refuel} = 1$) or not ($\alpha_i^{refuel} = 0$), out of route driving durations have to be added. Note that $\bar{\Delta}_i^{drTo}$ and $\bar{\Delta}_i^{drFrom}$ are set to be zero if location i is no gas station. In the equality conditions (6.1) to be added to the MILP model the driving duration from gas station i to the point where the route is entered is added if i is a gas station chosen for refueling. The driving duration to gas station $i+1$ is added if $i+1$ is chosen for refueling.

$$\Delta_{(i,i+1)}^{dr} = \bar{\Delta}_{(i,i+1)}^{dr} + \bar{\Delta}_i^{drFrom}\ \alpha_i^{refuel} + \bar{\Delta}_{i+1}^{drTo}\ \alpha_{i+1}^{refuel} \qquad \forall\ i \in S^{locations} \tag{6.1}$$

As a vertex may represent a customer location or a gas station, there are two different kinds of working activities that may take place: working activities associated with loading and/or unloading or refueling. Both can be treated similarly as far as time aspects are considered. It is important to know the estimated duration of the working activity. A new variable Δ_i^{work} is introduced that represents the duration of the working activity at location i (see (6.2)). It is composed of the working time for refueling $\bar{\Delta}^{refuel}$ and the working time for loading and/or unloading $\bar{\Delta}_i^{service}$. The working time for refueling is set to be zero for all non gas station locations. For reasons of simplicity it is assumed to be constant for each gas station and it is only taken into account if the corresponding gas station i is chosen for refueling ($\alpha_i^{refuel} = 1$). The working time for loading and/or unloading is set to be zero for all non customer locations.

$$\Delta_i^{work} = \bar{\Delta}_i^{service} + \bar{\Delta}^{refuel}\ \alpha_i^{refuel} \quad \forall\ i \in S^{locations} \tag{6.2}$$

The constants $\bar{\Delta}_i^{drive}$ were substituted by the variables for driving durations $\Delta_{(i,i+1)}^{dr}$ and the constants $\bar{\Delta}_i^{service}$ were substituted by the variables for the duration of working time at locations Δ_i^{work} in all constraints of the MILP model of Bernhardt et al. (2016) where these constants appeared. This affected the constraints for the driving time left until the next break or daily rest when entering vertex i (variables E_i^{dt}), the daily driving time left until the next daily rest (variables E_i^{ddt}) and the overall time left until the next daily rest (variables E_i^t) upon arrival at a location i. Here, $\bar{\Delta}_i^{drive}$ had to be substituted by $\Delta_{(i,i+1)}^{dr}$. $\bar{\Delta}_i^{service}$ was replaced by Δ_i^{work} in the constraints for the time left until the next daily rest period when leaving i, L_i^t. In the constraints for the begin of service, both, $\bar{\Delta}_i^{drive}$ and $\bar{\Delta}_i^{service}$ have been substituted accordingly.

Time windows are only considered for customer locations and the final destination. The constraints which state that exactly one time window has to be chosen for each location are customized to a limited vertex set (see (6.3)). The start of loading and/or unloading is restricted by time windows. We decided to not consider time windows at gas stations (nTW_i, the number of time windows at location i, is equal to 0 for all i in the set of gas stations $S^{stations}$) and therefore, inequalities (6.4), that state that loading and/or unloading only can start after the the lower bound of the time window interval, can be adopted without modifications.[13] The variable tw_{iz} is equal to 1 if time window z at location i is chosen and 0 otherwise.

$$\sum_{z=0}^{nTW_i-1} tw_{iz} = 1 \qquad \forall\ i \in S^{customers} \cup \{r-1\} \tag{6.3}$$

$$start_i \geq \sum_{z=0}^{nTW_i-1} \overline{TW}_{iz}^{begin} tw_{iz} \qquad \forall\ i = 1, \ldots, r-1 \tag{6.4}$$

Lateness at location i is denoted by Δ_i^{late}. Since no lateness is considered at gas stations, only the vertex set $S^{customers} \cup \{r-1\}$ is covered by the modified lateness constraints (6.5). For gas station vertices, lateness is set to be equal to 0 by the new equations (6.6).

$$\Delta_i^{late} \geq start_i - \sum_{z=0}^{nTW_i-1} \overline{TW}_{iz}^{end} tw_{iz} \qquad \forall\ i \in S^{customers} \cup \{r-1\} \tag{6.5}$$

$$\Delta_i^{late} = 0 \qquad \forall\ i \in S^{stations} \tag{6.6}$$

[13] Although time windows can be used to model opening hours of gas stations, this adds more complexity to the model and does only make sense if opening hours of gas stations are maintained.

To restrict the solution space, we allow daily rest periods in vertices associated with gas stations only in cases where they are necessary for refueling because the time left until the next daily rest period is exhausted. The auxiliary binary variables λ_i^7 are introduced to model the corresponding constraints (see (6.7) to (6.11)).

$$900\ \lambda_i^7 \geq E_i^t - \bar{\Delta}^{refuel}\ \alpha_i^{refuel} \qquad \forall\ i \in S^{stations} \tag{6.7}$$

$$900\ \left(\lambda_i^7 - 1\right) \leq E_i^t - \bar{\Delta}^{refuel}\ \alpha_i^{refuel} \qquad \forall\ i \in S^{stations} \tag{6.8}$$

$$\alpha_i^{rest} \leq 1 - \lambda_i^7 \qquad \forall\ i \in S^{stations} \tag{6.9}$$

$$\alpha_i^{rest} \leq M \left(\bar{\Delta}^{refuel}\ \alpha_i^{refuel} - E_i^t\right) + (M+1)\ 900\ \lambda_i^7 \qquad \forall\ i \in S^{stations} \tag{6.10}$$

$$\lambda_i^7 = 0 \qquad \forall\ i \in S^{customers} \cup \{0, r-1\} \tag{6.11}$$

Constraints (6.7) set λ_i^7 to be equal to 1 if there is still time left until the next daily rest period after refueling without taking a daily rest period in advance. Constraints (6.8) ensure that λ_i^7 is equal to 0 in case that refueling takes place at gas station i and this is not possible without taking a daily rest period. Constraints (6.9) then state that a daily rest period in vertex i may only be taken if λ_i^7 is equal to 0. In case that the time needed for refueling suffices exactly without taking a daily rest period, λ_i^7 may take on both values, 1 or 0. To ensure that no daily rest period is taken in that case, constraints (6.10) are introduced with M chosen sufficiently large.[14] λ_i^7 is set to be 0 for all non gas station vertices by (6.11).

Waiting time, breaks and partial breaks at gas stations are prohibited by constraints (6.12), (6.13) and (6.14) as such activities from a mathematical point of view do not bring any benefits. Waiting time can always be postponed to the next customer location or be used to extend the duration of a daily rest period without worsening the solution value. If a

[14] Continuous variables that consider time aspects are the variables $start_i$, Δ_i^{late}, $\Delta_{(i,i+1)}^{dr}$, Δ_i^{work}, E_i^{dt}, E_i^{ddt}, E_i^t, L_i^{dt}, L_i^{ddt}, L_i^t, $\Delta_{(i,i+1)}^{rest}$, Δ_i^{rest} and Δ_i^{wait} (see the Appendix for details on these variables). If all input parameters except the parameters that consider refueling are integer, we can assume, because of the structure of the constraints, that if there is an optimal solution these variables are integer or there is another optimal solution in which this is the case. For example, let us consider the arrival time at a location $start_i$ (see Bernhardt et al. (2016) for corresponding constraints). The "arrival" at the origin, $start_0$, is defined as the sum of integer parameters multiplied with integer variables and the continuous variable for the duration of a rest period at the origin if a daily rest period is taken there. $start_{i+1}$ is again the sum of terms that are integer by definition, $start_i$, Δ_{i+1}^{wait}, $\Delta_{i,i+1}^{rest}$ and Δ_{i+1}^{rest}. Because the lower and upper bounds of all time windows are chosen to be integers (minutes from the beginning of the planning horizon), choosing the variables for resting and waiting not to be integer cannot improve the solution value considering objective function (6.24) (or (6.25)). Similarly, it can be assumed that if there is a solution, there is an optimal solution with all E_i^t being integer. Therefore, in (6.10) it suffices M to be equal to 1, as if there is a solution then there has to be an optimal one for which the value of $|\bar{\Delta}^{refuel}\ \alpha_i^{refuel} - E_i^t|$ is either 0, or greater than or equal to 1. Hence, in case the time does not suffice for refueling, λ_i^7 can be set to be zero by (6.8) without eliminating all optimal solutions by (6.10). If input parameters are chosen differently, M has to be adjusted accordingly not to miss optimal solutions.

break is needed to reset the time interval until the next break, it can also be taken later on the way from the gas station to the next location and therefore may be mapped onto the corresponding arc. As there are no additional waiting times considered at gas stations that can be compensated by breaks or partial breaks, α_i^{break}, the variable that is equal to 1 if a break is taken in vertex i and 0 otherwise, and α_i^{pbreak}, the variable that indicates if a partial break is taken in vertex i, are set to be zero for all gas stations $i \in S^{stations}$.[15]

$$\Delta_i^{wait} = 0 \qquad \forall\, i \in S^{stations} \tag{6.12}$$
$$\alpha_i^{break} = 0 \qquad \forall\, i \in S^{stations} \tag{6.13}$$
$$\alpha_i^{pbreak} = 0 \qquad \forall\, i \in S^{stations} \tag{6.14}$$

A partial daily rest period α_i^{prest} for $i \in S^{stations}$ is only allowed if it substitutes a break on the preceding arc (see (6.15)). Note that if the variable μ_i^{prest} is equal to one, this indicates that the partial daily rest period planned α_i^{prest} is not taken upon arrival at gas station i but "substitutes" a break between location $i-1$ and gas station i. That means that the last resting activity between location $i-1$ and gas station location i is a partial daily rest period although the number of breaks on the arc $A_{(i-1,i)}^{break}$ would suggest another break.

$$\alpha_i^{prest} \le \mu_i^{prest} \qquad \forall\, i \in S^{stations} \tag{6.15}$$

6.2 Refueling constraints

The refueling constraints (6.16) to (6.20) originate from the standard fuel optimizer model described by Suzuki (2008, 2009) and reflect the two following decisions:

- where to refuel (i.e. determination of refueling locations) and
- how much to refuel (i.e. determination of refueling quantities).

$$T_i \ge \bar{T}^{min} \qquad \forall\, i \in S^{locations} \tag{6.16}$$
$$T_{r-1} \ge \bar{f}^{end} \tag{6.17}$$
$$\Delta_i^{refuel} \ge \bar{\Delta}^{min}\ \alpha_i^{refuel} \qquad \forall\, i \in S^{locations} \tag{6.18}$$
$$\Delta_i^{refuel} \le \bar{T}^{max}\ \alpha_i^{refuel} \qquad \forall\, i \in S^{locations} \tag{6.19}$$
$$\Delta_i^{refuel} \le \bar{T}^{max} - T_i \qquad \forall\, i \in S^{locations} \tag{6.20}$$

The amount of fuel in the tank, either at gas station i before purchasing fuel if gas station i is chosen for refueling ($\Delta_i^{refuel} = 1$) or at the corresponding leaving point (if $\Delta_i^{refuel} = 0$),

[15] Estimated waiting times at gas stations, such as waiting in line until refueling is possible or waiting in line for paying, are included in the parameter $\bar{\Delta}^{refuel}$ and are not intended to be used for a break by the MILP model.

is denoted by T_i. Constraints (6.16) ensure that the amount of fuel in the tank never falls below the defined reserve quantity $\bar{T}^{min}$. The minimum amount of fuel to be left in the tank at the final destination given by $\bar{f}^{end}$ is ensured by (6.17). Constraints (6.18) state that in case that refueling takes place at gas station i, the purchased amount Δ_i^{refuel} has to be higher or at least as much as the minimum purchase quantity $\bar{\Delta}^{min}$. These constraints serve to raise the acceptance of drivers as they may not be willing to stop frequently for refueling very small amounts.[16] In addition, they are important if minimum purchase quantities are necessary to get a discount. If no refueling takes place at location i, the refueling quantity is set to be 0 by constraints (6.19). Taking into account the maximum tank capacity, inequalities (6.20) ensure that the refueling quantity Δ_i^{refuel} at i always has to be equal to or less than the tank capacity minus the amount of fuel in the tank T_i when reaching gas station i.

Constraint (6.21) sets the starting fuel level $\bar{f}^{start}$ for the origin location 0.

$$T_0 = \bar{f}^{start} \tag{6.21}$$

The standard fuel optimizer model described by Suzuki (2008, 2009) considers linear consumptions by suggesting the use of consumption rates. In contrast to this, we consider individual fuel consumptions per arc ($\bar{\Delta}_{(i,i+1)}^{cons}$), i.e. for each pair of consecutive locations, to be able to integrate more precise consumption data relying, for example, on topographic data, route types and/or empirical values if available. Also for detours individual fuel consumptions are considered. Deviating from the standard fuel optimizer model, we assume that the fuel consumption from the point where the road is left to head for the gas station, $\bar{\Delta}_i^{consTo}$, may differ from the fuel consumption for the way back, $\bar{\Delta}_i^{consFrom}$.

Constraints (6.22) are adapted from the standard fuel optimizer model and are customized to individual consumptions and different detour consumptions depending on whether heading for the gas station or returning to the route. They state that the amount of fuel left in the tank upon arrival at location $i+1$ is equal to the amount of fuel left in the tank at location i plus the refueling amount at location i minus the consumption for the path back to the route if location i was chosen for refueling minus the consumption for the way from i to $i+1$ on the original route and minus the consumption for the path to location $i+1$ if it is a gas station chosen for refueling.

$$\begin{aligned} T_{i+1} = \; & T_i + \Delta_i^{refuel} - \bar{\Delta}_i^{consFrom} \; \alpha_i^{refuel} - \bar{\Delta}_{(i,i+1)}^{cons} - \bar{\Delta}_{i+1}^{consTo} \; \alpha_{i+1}^{refuel} \\ & \forall \; i \in S^{locations} \backslash \{r-1\} \end{aligned} \tag{6.22}$$

By (6.23), the variables that indicate if refueling takes place at location i are set to be 0 for all locations that do not correspond to a gas station.

$$\alpha_i^{refuel} = 0 \qquad \forall \; i \in S^{customers} \cup \{0, r-1\} \tag{6.23}$$

[16] Another or additional option would be to restrict the number of stops to a predefined maximum number which is, for example, dependent on the original length of the complete route. Note that the third objective function that is used in the last optimization step for a postprocessing penalizes the number of gas stations visited. The solution process and the objective function mentioned will be described in Section 7.

6.3 Objective functions

The most important objective is the minimization of lateness. To keep the overall travel time $start_{r-1}$ low, i.e. to arrive at the final destination as soon as possible, is the second objective. The third objective is to minimize the overall costs for refueling. Other criteria that are important to obtain comprehensible solutions and thus for the acceptance of drivers and dispatchers are accumulated in one objective function. Similar as in Bernhardt et al. (2016), a combination of lexicographic ordering and trade-off strategy was chosen when setting up the objective functions and determining the solution process for this multicriteria optimization problem.[17] The different objective functions are described in Sections 6.3.1, 6.3.2 and 6.3.3. The solution methodology to solve the MILP model is described in Section 7.

6.3.1 Objective function 1

For the first objective function, the trade-off strategy from Bernhardt et al. (2016) was chosen, giving most importance to the minimization of lateness. For the choice of the penalty factor P see Bernhardt et al. (2016).

$$\textbf{Minimize} \quad start_{r-1} + \sum_{i=1}^{r-1} P \cdot \Delta_i^{late} \tag{6.24}$$

6.3.2 Objective function 2

The second objective function minimizes the overall refueling costs.[18]

$$\textbf{Minimize} \quad \sum_{i=0}^{r-1} \bar{P}_i \cdot \Delta_i^{refuel} \tag{6.25}$$

6.3.3 Objective function 3

Objective function 3 is an extension of objective function 2 described in Bernhardt et al. (2016). Note that the last two components are added to the original objective function penalizing the number of refueling stops and the duration of the complete route. As the variable part of the route are the detours, the last component penalizes durations for detours. The different weights may be customized depending on user preferences.

[17] The trade-off strategy is only relevant within the first and the last objective function.

[18] Note that refueling quantities are set to be zero for non gas station locations by (6.19) and (6.23).

$$
\begin{aligned}
\textbf{Minimize}\quad & \sum_{i=1}^{r-1}\sum_{z=0}^{nTW} 10\ (z+r-i)\ tw_{iz} + \sum_{i=0}^{r-1} start_i \\
& + \sum_{i=0}^{r-2} 10\ (r-i)\ (\mu_{(i,i+1)}^{earlydr1} + \mu_{(i,i+1)}^{earlydr2}) \\
& + \sum_{i=0}^{r-1} 10\ (r-i)\ (\alpha_i^{pbreak} + \alpha_i^{prest}) \\
& + \sum_{i=0}^{r-1} 20\ \Delta_i^{wait} \\
& + \sum_{i=0}^{r-2} 30\ (r-i)\ \mu_{(i,i+1)}^{redrest} + \sum_{i=0}^{r-1} 40\ (r-i)\ \mu_i^{redrest} \\
& + \sum_{i=0}^{r-2} 50\ (r-i)\ \mu_{(i,i+1)}^{extd2} + 60\ (r-i)\ \mu_{(i,i+1)}^{extd1} + 60\ (r-i)\ \mu_{(i,i+1)}^{extd3} \\
& + \sum_{i=0}^{r-1} 60\ (r-i)\ \mu_i^{extd} \\
& + \sum_{i=0}^{r-1} 50\ \alpha_i^{refuel} \\
& + \sum_{i=0}^{r-2} 100\ \Delta_{(i,i+1)}^{dr}
\end{aligned}
\tag{6.26}
$$

7 The solution process

Since a lexicographical ordering of the different objective functions will be used, multiple optimization steps are necessary to solve the multicriteria optimization problem. In each optimization step, a submodel is solved which consists of the constraints described in the previous section and in Bernhardt et al. (2016) and a corresponding objective function. From step 2 onwards additional constraints need to be added. The composition in each step is described in the following.

Punctuality at customer locations is often more important than saving fuel costs, as customer satisfaction has a big impact on future requests and thus on the economic viability of a haulage company. We therefore may order objective functions 1 and 2 lexicographically giving highest importance to objective function 1 (6.24).

When setting up the solution process for the MILP model without consideration of refueling, we noticed that it was beneficial to have an additional submodel in which optional rules were deactivated and to use the optimal objective function value of this submodel as an upper cutoff for the submodel in which optional rules were allowed. Using this experience, we adopted the same approach and obtained two submodels and two optimization steps for the first objective function. For details see Bernhardt et al. (2016).

For the objective of minimizing fuel costs, we set up a third submodel with objective function (6.25). Two additional constraints are added to this submodel. The first one, (7.1), does not allow more lateness than the total lateness over all locations i obtained in optimization step 2.

$$\sum_{i=1}^{r-1} \Delta_i^{late} \leq \sum_{i=1}^{r-1} \Delta_i^{late*} \tag{7.1}$$

Note that a solution still has to exist if there was one in the previous steps as refueling already was considered even though not in an optimal way.

For more freedom, in optimization step 3 we allow the overall travel time $start_{r-1}$ to be at most 30 minutes more than in optimization step 2. This is expressed in constraint (7.2), where $start^*_{r-1}$ denotes the overall travel time of step 2. The time may be used for an additional and/or alternative refueling.

$$start_{r-1} \leq start^*_{r-1} + 30 \tag{7.2}$$

Similarly to the solution process for the model without refueling, an additional submodel and a corresponding optimization step was added to obtain more comprehensible solutions, to only use optional rules if this is advantageous and to keep the number of refueling stops and detour durations low.

For the additional optimization step, the objective function of the previous step is transformed to constraint (7.3) with the optimal objective function value z^* of step 3 as an upper bound such that the fuel costs are prevented from increasing.

$$\sum_{i=0}^{r-1} \bar{P}_i \cdot \Delta_i^{refuel} \leq z^* \tag{7.3}$$

Again, the constraint (7.1) was added to keep the optimal lateness determined in step two.

In step 3, we allow for more freedom for refueling decisions when adding constraint (7.2). In step 4, we do not allow an increase of the overall travel time and thus add constraint (7.4), where $start^*_{r-1}$ in that case represents the travel time determined in optimization step 3.

$$start_{r-1} \leq start^*_{r-1} \tag{7.4}$$

The objective function of optimization step 4 is given by (6.26).

The solution of optimization step 4 still needs to be transformed into a readable driver schedule. In Bernhardt et al. (2016), a transformation algorithm was developed for this task (see the appendices in Bernhardt et al. (2016)). The time for loading and/or unloading at customer location i that was taken from the input parameters of the MILP model has to be replaced by the value of the variable for general working time, Δ_i^{work}, for each customer location or gas station i. Similarly, the driving duration between a pair of consecutive locations i and $i+1$ is now variable and given by $\Delta_{(i,i+1)}^{dr}$. This has to be adopted for the input parameters of the algorithm accordingly.

Figure 11 gives an overview of the solution process.

In the next section, the test instances are presented. Afterward, in Section 9, a preprocessing heuristic is introduced which helps to reduce the number of gas stations to be considered during the solution process. In our numerical experiments, all subproblems are solved with a commercial optimization solver. The details on the test environment are described in Section 10. In Appendix B.1 it is shown for a test instance how the driver schedule evolves over the several optimization steps.

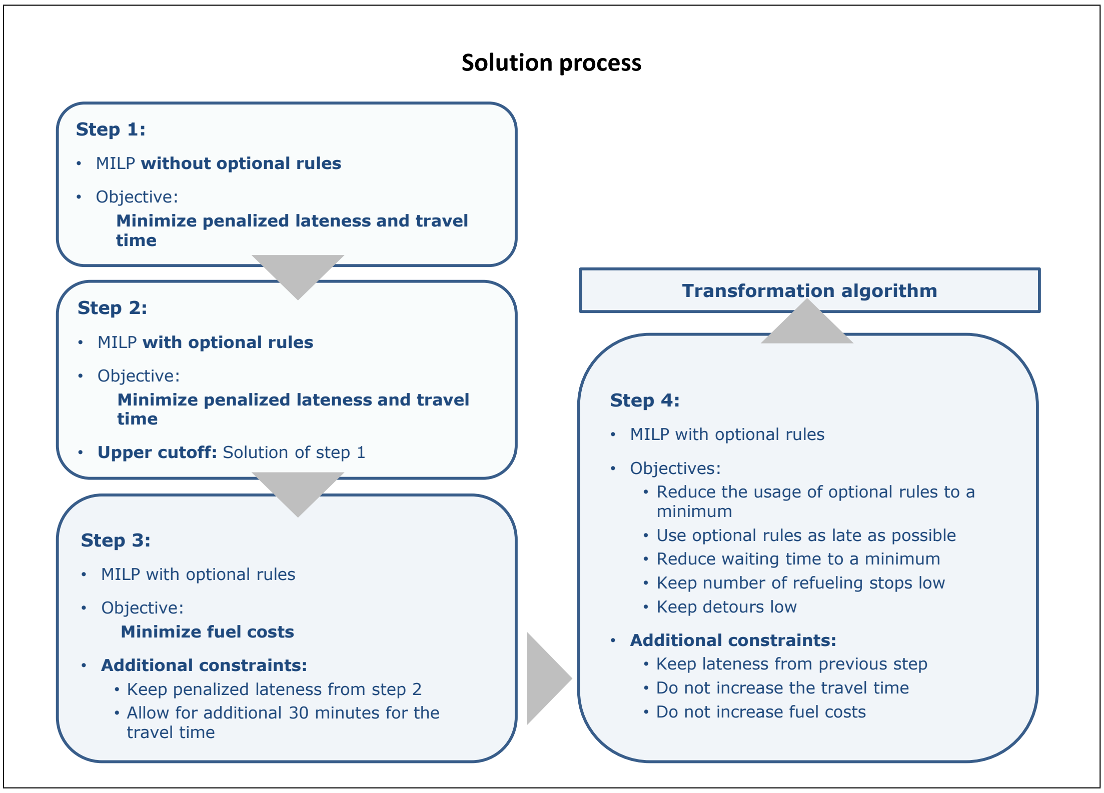

Figure 11: The four optimization steps in the solution process

8 Numerical experiments - The base instances

The data basis of the test instances of Bernhardt et al. (2016) were derived from real data provided by a German haulage company that operates a fleet of vehicles in Europe. The provided data comprised telematics data of the vehicles of the haulage company as well as arrival times at customer locations initially planned by dispatchers in the order management system. The arrival times at customer locations were used as a basis to add time windows to the test instances. One to three time windows were considered in Bernhardt et al. (2016) along with varying time window lengths. For determining driving durations, different road types were taken into account. To be as close to reality as possible, routes were adjusted to the real routes chosen by the drivers by setting support points.[19] For determining the routes, a planning horizon of one week was considered.

We extended the data basis of the test instances that were used to test the MILP models for planning time windows and rest periods and breaks (see Bernhardt et al. (2016)) by adding information about gas stations along the route. In a first step, we decided to consider gas stations with a straight line (i.e. Euclidean) distance of at most 30 km to the route.

To obtain driving durations and distances between locations and for detours, a modified A^* routing algorithm was used which was developed during the research project Dynaserv in which the haulage company mentioned above served as a partner. The real vehicle fuel consumptions for the one-week routes were used to determine distance dependent consumption rates. Real data on the tank capacities of the vehicles, starting and ending fuel levels at the beginning and the end of the planning horizon were adopted.

List prices per country of one of the main fuel card operators of the haulage company were provided. The fuel card operator considers different types of gas stations, among those the group of gas stations that are close to the motorway and therefore are more expensive and a group of gas stations that are less expensive. In reality, the fuel price at a gas station that is valid for the corresponding fuel card holder is dependent on the gas station type and on the list price of the corresponding country. Additionally, there were special discount arrangements for selected gas stations. For Spain, there was a contract with a different service station chain. For simplification reasons, fuel prices at gas stations were assumed to be equal to the list prices of the corresponding countries at the beginning of the corresponding planning horizons.

Table 1 gives an overview of the extended base instances.

In the first column, the base instance ID is displayed. In total, we considered 15 base instances. Distances and driving durations without consideration of detours are given in the second and third column, respectively. The number of customer locations including the origin and destination are given in the fourth column. The vehicles considered have different tank capacities that range from 900 to 1200 liters. The tank capacities are given in the fifth column. Starting and ending fuel levels are given in columns six and seven. Column eight shows the overall fuel consumption for the one-week routes. This fuel consumption divided

[19] Support points have to be part of the route determined by the routing algorithm (see Bernhardt et al. (2016) for more details).

base instance	overall distance (km)	overall driving duration (h)	# customer locations (incl. start & end)	tank capacity (l)	starting fuel level (l)	ending fuel level (l)	fuel consumption (l)	# gas stations (30 km straight line distance)
1	2914	36.85	4	925.00	925.00	619.75	670.30	276
2	3391	42.48	5	925.00	582.75	758.50	803.59	397
3	3653	46.95	6	900.00	360.00	837.00	1077.72	181
4	2831	36.45	6	900.00	873.00	468.00	781.30	447
5	1739	22.37	6	925.00	268.25	900.00	486.82	241
6	2944	37.47	6	900.00	729.00	396.00	889.12	326
7	2269	30.32	7	900.00	495.00	639.00	664.95	284
8	3142	39.77	7	900.00	666.00	891.00	816.86	374
9	3019	38.17	7	925.00	712.25	910.00	830.21	515
10	3436	43.77	8	900.00	747.00	693.00	896.92	383
11	3447	43.62	8	1200.00	504.00	444.00	1082.25	474
12	2475	31.85	9	900.00	873.00	684.00	737.67	298
13	2826	36.42	10	900.00	648.00	576.00	802.50	337
14	3055	40.85	11	900.00	666.00	576.00	837.06	368
15	3250	41.95	12	900.00	801.00	360.00	952.28	353

Table 1: Base instances

by the overall distance gave us the fuel consumption rate in liters per km. The number of gas stations within a straight line distance of at most 30 km along the route is shown in the last column. Note that gas stations were chosen per route between consecutive locations in $S^{customers} \cup \{0, r-1\}$. Gas stations that were within the chosen straight line distance for several of such routes were listed multiple times accordingly.

9 Heuristic preprocessing: Eliminating unattractive gas stations

As we will see later in our numerical experiments, the number of gas stations included in the list of potential gas stations for refueling strongly influences the duration needed by the optimization solver to find an optimal solution. Therefore, a preprocessing heuristic was developed to eliminate less promising gas stations from the list and thus reduce the computational efforts necessary in the following steps.

As mentioned earlier in Section 4, Suzuki (2014) also proposes a preprocessing procedure that reduces the number of gas stations to be considered. Note that this procedure applied to our problem may remove attractive gas stations as we also consider time factors which have a high priority in our problem definition. The time needed for a detour is not considered in the elimination process described by Suzuki (2014). Conversely, one criterion for the elimination of a gas station is that its detour distance is less than the detour distance of two other gas stations that represent the start and the end point of a subsequence of gas stations. Eliminating such gas stations with short detour distances can be disadvantageous if customer time windows are involved. Additionally, the average number of gas stations removed by the variable-reduction technique does not sufficiently reduce the problem size in preparation for the solution process for the MILP model provided in this study.[20]

The heuristic presented in this section is applied to each pair of consecutive customer locations (including origin and destination) and has a run time complexity of $\mathcal{O}(n^2)$, where n is the number of gas stations between the two locations. For each route between consecutive customer locations (or between origin and first customer location or last customer location and destination), gas stations within the chosen straight line distance of 30 km to the route were sorted ascending by price and by detour distance, where the price was chosen to be the first sorting criterion. To obtain the sorted list of gas stations, we used the sorting algorithm of the Collections package of Java (java.util.Collections) together with a comparison function (using java.util.Comparator). For two gas stations i and j, the comparison function returns -1 if gas station i according to the sorting criteria has to stand higher in the list than gas station j, 1 if j has to stand higher in the list than i and 0 otherwise. Nevertheless, the sorting algorithm can be chosen independently. As sorting algorithms are broadly discussed in the literature, we only present the comparison function, Algorithm 1, used for the sorting in Java. Note that in the following we assume that the detour to a gas station is equal to the detour back to the route. Additionally, linear fuel consumption rates per distance unit are assumed.

Algorithm 1 Compare price and detour

compare$(\bar{P}_i, \bar{P}_j, detour_i, detour_j)$
Input:

$\bar{P}_i(\bar{P}_j)$:	Fuel price at gas station i (j)
$detour_i(detour_j)$:	Detour distance if gas station i (j) is visited

Output:

$$\textbf{return} \begin{cases} -1 & \text{if gas station } i \text{ should stand higher in the list than gas station } j, \\ 1 & \text{if gas station } j \text{ should stand higher in the list than gas station } i \\ 0 & \text{otherwise} \end{cases}$$

// First sorting criterion: Fuel price (ascending)

if $\bar{P}_i < \bar{P}_j$ **then**
  return -1
else if $\bar{P}_i > \bar{P}_j$ **then**
  return 1
else

  // Second sorting criterion: Detour distance (ascending)

  if $detour_i < detour_j$ **then**
    return -1

[20] For the instances considered by Suzuki (2014), the number of gas stations was reduced by 54.8 % on average.

    else if $detour_j < detour_i$ **then**
        return 1
    else
        return 0
    end if
end if

The following comparison function (Algorithm 2) is necessary to sort the gas stations by the sequence in which the points where the route has to be left to reach the gas stations are traversed. The corresponding sorting is done as a preprocessing step to determine input parameters for the MILP model. In the MILP model, the driving durations and fuel consumptions between consecutive locations are needed. If i and j are two consecutive gas stations, the distance between them can for example be determined by subtracting the distance on the route between the last customer location and gas station i ($dist_i$) from the distance on the route between the last customer location and gas station j ($dist_j$). For the driving durations and fuel consumptions this can be done analogously.

Algorithm 2 Compare on-route distance

compare($dist_i, dist_j$)
Input:

$dist_i(dist_j)$: Distance on the route between the last customer location and gas station i (j)

Output:

$$\textbf{return} \begin{cases} -1 & \text{if gas station } i \text{ should stand higher in the list than gas station } j, \\ 1 & \text{otherwise} \end{cases}$$

if $dist_i < dist_j$ **then**
    return -1
else
    return 1
end if

We want to keep the "best" gas stations considering the two criteria, fuel price and detour distance. Those gas stations for which there is a "better" gas station considering both criteria, fuel price and detour distance, in a predefined on-route distance are eliminated. In the following, we call this predefined distance "filter distance". Algorithm 3 shows the elimination process. Note that if gas station j stands higher in the list than gas station i and the list has been sorted using comparison function 1, j has definitely a fuel price that is lower than or equal to the fuel price of i. We therefore only compare the detour distances of gas stations i and j in Algorithm 3.

Algorithm 3 Filter gas stations

Input:

$\bar{P}_i(\bar{P}_j)$	Fuel price at gas station i (j)
$detour_i(detour_j)$	Detour distance (one-way) if gas station i (j) is visited
$dist_i$	Distance on the route from the last customer vertex (or origin) to the point where the route has to be left if gas station i should be visited
n	Number of gas stations between the two customer locations considered
$S^{stations}_{(c,c+1)}$	Set of all gas stations i with selected straight line distance to the route between customer c and customer $c+1$.
$radius$	Filter distance

Output:

$\hat{S}^{stations}_{(c,c+1)}$	Remaining list of gas stations between customer locations c and $c+1$

Sort the gas stations in $S^{stations}_{(c,c+1)}$ by price (first criterion) and by detour distance (second criterion) using Algorithm 1. Resulting list: $\hat{S}^{stations}_{(c,c+1)}$

// Go through the sorted list of gas stations, start by the second station.
// (The first station is kept in the list as it has the best price).

for $i = 1$ **to** $n-1$ **do**

  // Go through the list of gas stations that were kept and only keep gas station i
  // in the list if for one of the two criteria (this can only be the detour distance)
  // it is better than all gas stations kept so far or no kept gas station lies in the
  // filter distance of gas station i.

  for $j = 0$ **to** $i-1$ **do**

    if $detour_i \geq detour_j$ **then**

      if $|dist_i - dist_j| < radius$ **then**

        $\hat{S}^{stations}_{(c,c+1)} \leftarrow \hat{S}^{stations}_{(c,c+1)} \setminus \{i\}$

$i = i - 1$
break
end if
end if
$j = j + 1$
end for
$i = i + 1$
end for
Sort the gas station list $\hat{S}^{stations}_{(c,c+1)}$ by the distance on the route between the last customer location c and the gas station using Algorithm 2.
Result: Newly sorted list $\hat{S}^{stations}_{(c,c+1)}$.
return $\hat{S}^{stations}_{(c,c+1)}$

Figure 12 illustrates Algorithm 3 by means of an example. The numbering of gas stations depicted is the numbering obtained by sorting the gas stations with Algorithm 1. The sequence in which the gas stations are considered is equal to this numbering. The gas station with number 1 is kept, as it is the cheapest (and it is the first one in the list) among all gas stations between customer c and customer $c + 1$. The second gas station is kept because the detour distance is less than the detour distance of gas station 1. Gas station 3 is eliminated because it is within the filter distance of gas station 1 (and 2) and it has a higher fuel price and a larger detour distance than gas station 1. Gas station 4 is within the filter distance of gas station 1, but it has a smaller detour distance, so it is kept. Gas station 5 is removed as it is within the filter distance of gas station 4 and has a larger detour distance (and equal fuel price). Gas station 6 is in no filter distance of any of the gas stations already considered and thus is kept in the list. Gas station 7 has a shorter detour distance than gas station 6, which is the only remaining gas station with lower index within the filter distance of gas station 7. Thus, it is not eliminated. Gas station 8 is within the filter distance of gas station 4, 6 and 7 and is removed because it has a higher fuel price and a larger detour distance than gas station 7.

Note that for the choice of the filter distance the tank capacity has to be taken into account. If the filter distance is larger than one half of the distance that can be traveled with a full tank, no solution might be found for the corresponding MILP model. The maximum detour distance also may have an influence when it is large. Additionally, the starting and ending fuel levels have to be taken into account. Therefore, for the filtering of gas stations between the origin and the first customer and between the last customer and the final

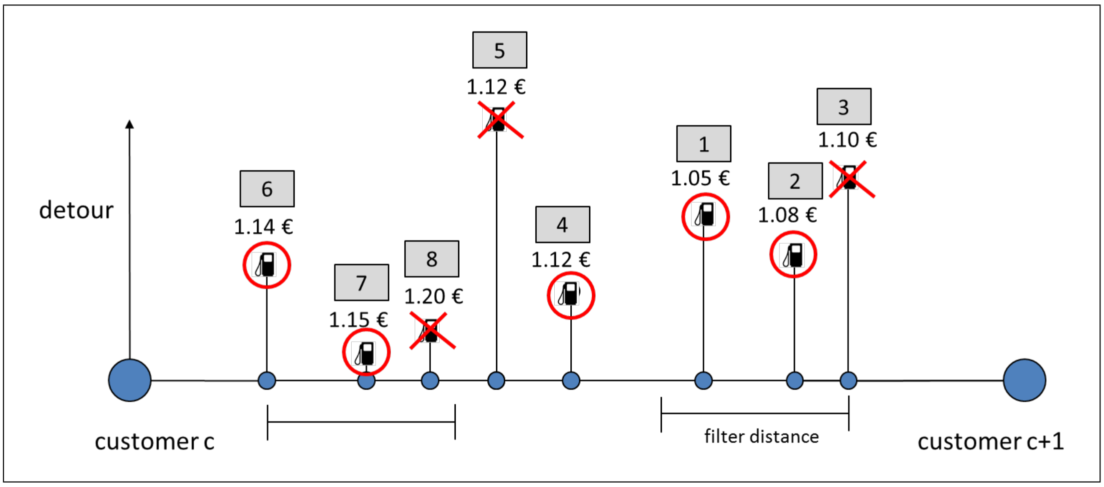

Figure 12: Example of filtering gas stations

destination, the filtering algorithm is slightly modified. To ensure that the first gas station on the route is reachable, we use Algorithm 4. At least one gas station is kept by the "normal" filtering algorithm. If the range with the starting fuel level is at least as far as the route length between origin and the first customer plus the maximum detour distance to a gas station, any gas station chosen on this arc is reachable. The original algorithm, Algorithm 3, is executed. Otherwise, in each iteration, we only remove a gas station from the list if a first gas station was found that is reachable with the starting fuel level.

Algorithm 4 Filter gas stations between origin and first customer location

Input:

$\bar{P}_i(\bar{P}_j)$	Fuel price at gas station i (j)
$detour_i(detour_j)$	Detour distance (one-way) if gas station i (j) is visited
$dist_i$	Distance on the route from the last customer vertex (or origin) to the point where the route has to be left if gas station i should be visited
n	Number of gas stations between origin and first customer location
$S_{(0,1)}^{stations}$	Set of all gas stations i with selected straight line distance to the route between origin 0 and first customer 1.
$radius$	Filter distance
$rangeStartingFuel$	Maximum distance traveled by the vehicle with the starting fuel level
$routeLength$	Length of the route between origin and first customer
$maxDetour$	Maximum detour distance to a gas station (one-way)

Output:

$\hat{S}_{(0,1)}^{stations}$	Remaining list of gas stations between origin 0 and customer location 1

// Check if the execution of algorithm 3 suffices.

if $(routeLength + maxDetour \leq rangeStartingFuel)$ **then**

    return $\hat{S}_{(0,1)}^{stations}$ obtained by Algorithm 3

else

    Sort the gas stations in $S_{(0,1)}^{stations}$ by price (first criterion) and by detour distance (second criterion) using Algorithm 1. Resulting list: $\hat{S}_{(0,1)}^{stations}$

    $gasStationInRange$ = **false**

// Go through the sorted list of gas stations, start by the second station
// (the first station is kept).

for $i = 1$ **to** $n - 1$ **do**

  for $j = 0$ **to** $i - 1$ **do**

    if not $gasStationInRange$ **then**

      if $(dist_j + detour_j) \leq rangeStartingFuel$ **then**

        $gasStationInRange \leftarrow$ **true**

      end if

    end if

    if $detour_i \geq detour_j$ **then**

      if $|dist_i - dist_j| < radius$ **then**

        if $gasStationInRange$ **then**

          $\hat{S}^{stations}_{(0,1)} \leftarrow \hat{S}^{stations}_{(0,1)} \setminus \{i\}$

          $i = i - 1$

          break

        end if // $gasStationInRange$

      end if // $|dist_i - dist_j| < radius$

    end if // $detour_i \geq detour_j$

    $j = j + 1$

  end for // $j = 0$ **to** $i - 1$

  $i = i + 1$

end for // $i = 1$ **to** $n - 1$

Sort the gas station list $\hat{S}^{stations}_{(0,1)}$ by the distance on the route between the origin 0 and the gas station using Algorithm 2.
Result: newly sorted list $\hat{S}^{stations}_{(0,1)}$.

61: **return** $\hat{S}_{(0,1)}^{stations}$

62:

63: **end if** // ELSE (not $routeLength + maxDetour \leq rangeStartingFuel$)

For the filtering of gas stations between the last customer and the final destination, we have to make sure that an arrival with the given ending fuel level is possible. This can be done similarly as in Algorithm 4 with the difference that gas stations are only removed from the list if a gas station has been found with a distance to the final destination that is less than the range with a full tank minus the range with the ending fuel level. We therefore replace line 23 by an if-statement that checks if the distance of gas station j to the destination is less than or equal to the range with a full tank minus the range with the ending fuel level ($routeLength - dist_j + detour_j \leq range - rangeEndingFuel$, $rangeEndingFuel$: range of the vehicle with the ending fuel level, $routeLength$: Length of the route between the last customer and the final destination). The if-statement in line 3 is modified with the inequality $routeLength + maxDetour \leq range - rangeEndingFuel$.

In the tests described in the following sections, different filter distances were considered. The real maximum average fuel consumption over all base instances was less than 32 l per 100 km distance traveled. The minimum tank capacity was 900 l and we decided for a minimum fuel level which had to be maintained in the tank at all times to not run out of fuel in case of unforeseen events (e.g. traffic jam) of 100 l. With this information, we computed a minimum range with a full tank of $\frac{900\,l - 100\,l}{\frac{0.32\,l}{km}} = 2500\,\text{km}$. We decided to consider a filter distance of at most 1000 km such that, provided that detour distances to gas stations are not "too large"[21], the filtering most likely does not lead to infeasibility of the MILP model set up later. Infeasibility after filtering may occur if even when considering all gas stations along the route there is no possibility to find a choice of gas stations where the minimum fuel level in the tank can be maintained. It also theoretically may happen that the maximum weekly driving time or the maximum time between two weekly rest periods is exceeded because of unfavorable positions of the remaining gas stations as far as the resulting time schedule is considered. But this may not be predicted easily. In all of our test instances, feasibility was preserved for all of the filter distances used.

Table 2 shows the remaining number of gas stations (left hand side) and the overall number of locations (right hand side) depending on the filter distance used. Note that the number of gas stations per pair of consecutive customer locations (including start and end) were added up for the complete route. Some of the gas stations may occur more than once between different customer locations.

[21] In the worst case, two remaining consecutive gas stations (with a customer in between) after filtering may have an on-route distance of at most 2000 km even though before filtering there were gas stations "in between". In such a case where the distance between two consecutive gas stations exceeds 1000 km, the gas stations belong to two consecutive route segments, where a route segment is defined to be the route between two customer locations including origin and destination. Thus, for the detour distance from the last gas station to the subsequent gas station, at least $(2500 - 2000)\,km = 500\,km$ are remaining from the range with a full tank for the detour from the first of the two gas stations to the route and the detour to the second one. In none of the considered cases, one-way detour distances were larger than 192 km. But if different input parameters are chosen or different properties are observed, this may have to be taken into account.

			Price and detour filter: filter distance													
Base instance	# customer locations incl. start and end	#gas stations	no price and detour filter	100 km	200 km	300 km	400 km	500 km	1000 km	# locations	no price and detour filter	100 km	200 km	300 km	400 km	500 km	1000 km
1	4		276	22	10	9	9	7	7		280	26	14	13	13	11	11
2	5		397	24	13	10	9	9	8		402	29	18	15	14	14	13
3	6		181	26	15	11	9	9	7		187	32	21	17	15	15	13
4	6		447	23	14	12	10	10	8		453	29	20	18	16	16	14
5	6		241	15	10	8	7	6	6		247	21	16	14	13	12	12
6	6		326	24	12	11	9	8	7		332	30	18	17	15	14	13
7	7		284	22	14	12	10	10	9		291	29	21	19	17	17	16
8	7		374	32	19	14	13	12	12		381	39	26	21	20	19	19
9	7		515	25	15	11	11	10	8		522	32	22	18	18	17	15
10	8		383	25	19	16	15	14	13		391	33	27	24	23	22	21
11	8		474	27	18	13	11	11	8		482	35	26	21	19	19	16
12	9		298	23	16	13	12	12	11		307	32	25	22	21	21	20
13	10		337	20	13	11	10	10	8		347	30	23	21	20	20	18
14	11		368	27	18	16	14	13	13		379	38	29	27	25	24	24
15	12		353	30	22	21	19	17	15		365	42	34	33	31	29	27
Ø	7		350	24	15	13	11	11	9		358	32	23	20	19	18	17

Table 2: Remaining locations after filtering

On average, 350 gas stations were found within a straight line distance of 30 km to the route of which 24 (7 %) remained when considering a filter distance of 100 km. With a filter distance of 200 km, only 15 (4 %) gas stations remained, with 300 km, 13 (4 %) gas stations. The number of gas stations in none of the base instances differed by more than 2 when using the filter distances 400 km and 500 km, respectively, and the average number of filtered gas stations was about 11 (3 %) in both cases. Finally, a filter distance of 1000 km was considered, leaving on average 9 (3 %) gas stations for the optimization process.

10 Numerical experiments - Environment and settings

For the following numerical experiments, the same test environment was used as for testing the MILP models and the algorithm for determining time windows, rest periods and breaks in Bernhardt et al. (2016). That means, the MILP model was implemented in Java (Java 8, 64 bit) and solved with Cplex 12.6 (64 bit) with the ILOG Cplex Concert Technology. The test runs were made on an Intel Core i5 2500K with 8 GB RAM (DDR3-10700 (667 MHz)) running Windows 7 Professional Service Pack 1, 64 bit.

Table 3 shows the average number of variables and constraints over all test instances with one time window per customer location and depending on the filter distance chosen. In the last line, the values for the MILP model of Bernhardt et al. (2016) are listed for comparison. The number of binary variables in a test instance with more than one time window per customer location is raised by 1 for each additional time window.[22] If the number of time windows at customer locations is constant, the number of additional binary variables is

[22] The number of alternative time windows may differ among customer locations. For each time window z at customer location i, there is a binary decision variable tw_{iz} in the MILP model (for more details see Section 6.1).

equal to the number of customer vertices, $|S^{customers}|$, for each additional time window per customer location.

	# variables			# constraints			
filter distance (km)	**Ø continuous**	**Ø integer**	**Ø binary**	**Ø step 1**	**Ø step 2**	**Ø step 3**	**Ø step 4**
100	474	123	977	5536	5033	5035	5036
200	337	87	694	3920	3563	3565	3566
300	297	76	611	3448	3134	3136	3137
400	277	71	570	3212	2919	2921	2922
500	267	68	549	3094	2812	2814	2815
1000	249	63	512	2881	2618	2620	2621
no refueling:	80	26	208	1162	1049	1050	

Table 3: Average number of variables and constraints (one time window) depending on the filter distance

In Bernhardt et al. (2016), 225 test instances were generated from 15 base instances (real data) that were now enriched with information needed for refueling (see Section 8). For each of the 225 original test instances, 6 different filter distances (100, 200, 300, 400, 500 and 1000 km) were tested. Thus, 1350 complete test runs were performed.

While the preprocessing algorithm took less than one second for each of the instances, from previous test runs and from the experience we made in Bernhardt et al. (2016), it seemed reasonable to establish maximum run times for the different optimization steps to solve the MILP model. Figure 13 shows the allocated time for each step.

The idea was to not allow more than half an hour time for the overall solution process unless no solution could be found until then. In none of the optimization steps the solution process was stopped if no solution was found so far because we wanted to know the run times to find feasible solutions for these cases. As lateness is considered to be more crucial than fuel costs and because we knew that the first two steps were most time consuming, we decided to allow at most 25 minutes for step 1 and 2. If no optimal solution was found in step 1 within 25 minutes, the best solution until then was saved. If no solution at all was found during this time interval, the optimization solver was not stopped until the first solution was found. The same was done in step 2 allowing at most 25 minutes minus the duration of the first step in case at least one feasible solution was found. In step 3, a maximum of 30 minutes minus the durations of steps 1 and 2 were allowed. The remaining time was dedicated to step 4.

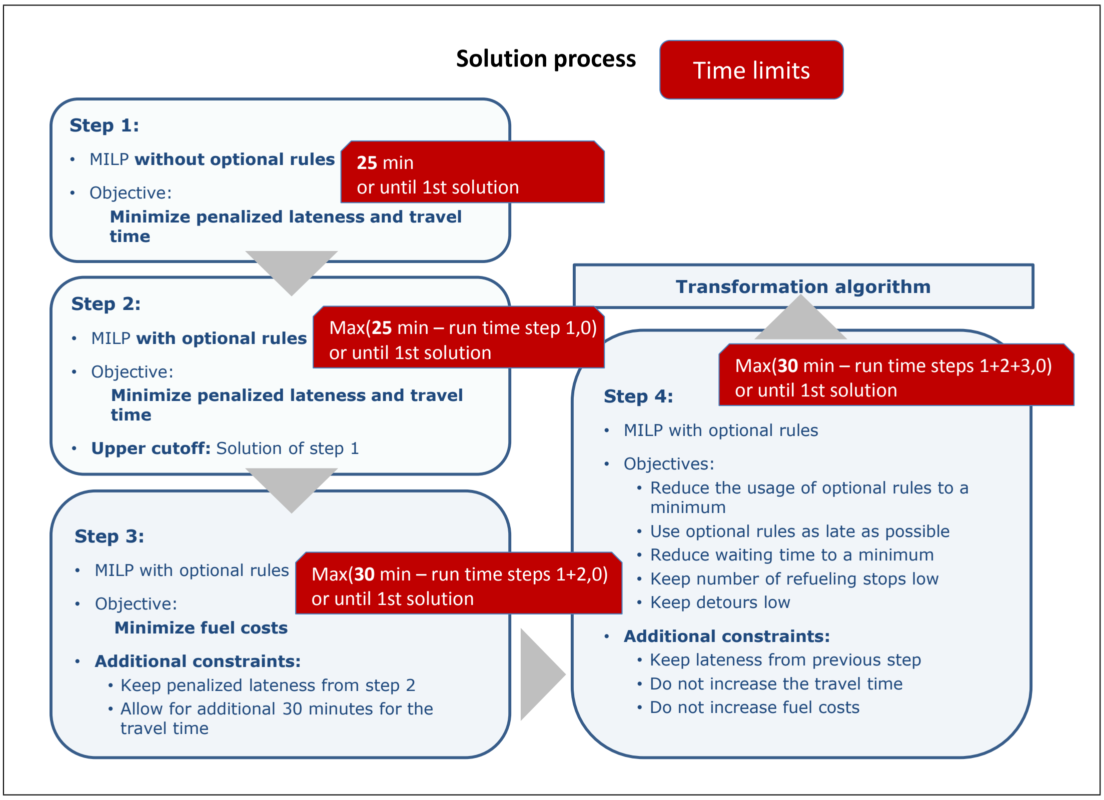

Figure 13: Time limits of each optimization step for the MILP model

11 Numerical experiments - Analysis

In the following, the results of the numerical experiments are presented and analyzed. In Section 11.1 the influence of the filter distance on the run time is discussed. Then, in Section 11.2, the impact of the number and the length of customer time windows on the run time is considered. Section 11.3 gives some managerial insights.

11.1 The influence of the filter distance on the run time

Figure 14 depicts the overall run times (sum of the run times of optimization steps 1-4) of all test runs[23] depending on the number of locations. The latter depend, in turn, on the chosen filter distance.[24] It clearly can be seen that the filter distance and thus the number of locations considered very strongly influences the run times. The figure shows three test runs with an overall run time of more than 30 minutes and 20 test runs (1.5 %) with a run time between 25 and 30 minutes. The other test runs (98.3 %) required less than 25 minutes.

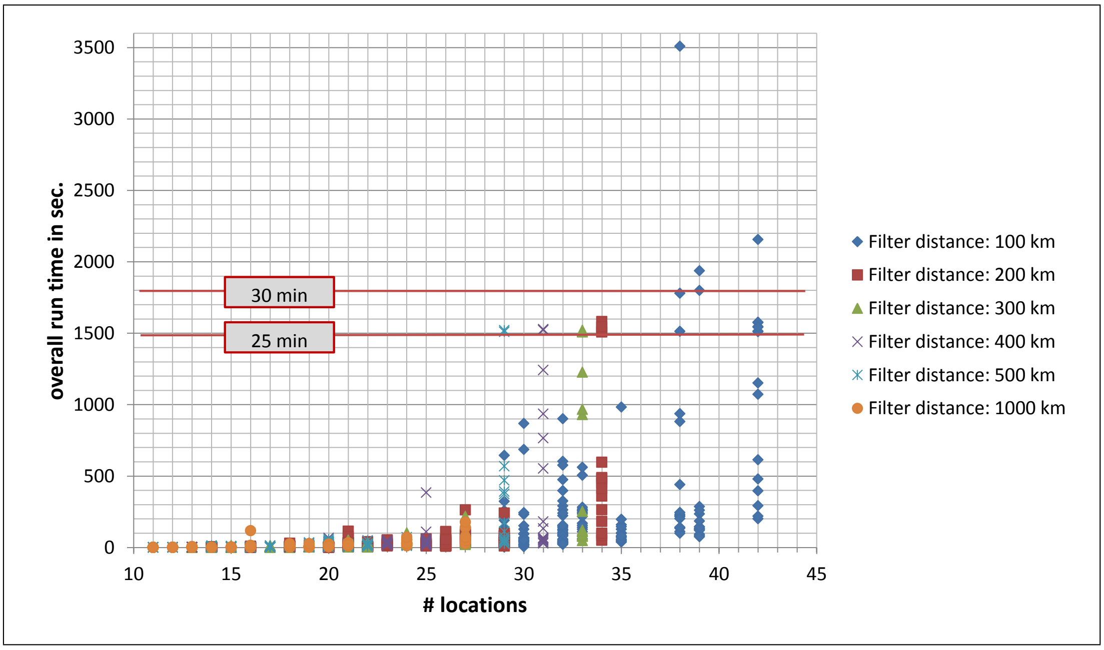

Figure 14: Run times depending on the number of locations and filter distances

The three test runs with a run time of more than 30 minutes belong to the cases with a filter distance of 100 km with the most locations to be considered. In the case of the

[23] For each test instance, 6 test runs were performed that differ by the chosen filter distance (see Section 10).

[24] The detailed results can be found in Appendix B.2.

longest run time of about 58 minutes (3509 sec) (base instance 14, 3 time windows with a length of 120 minutes), it took nearly 58 minutes (3464 sec) in the second step until a first feasible solution was found. In most of the cases in which in at least one of the steps no optimal solution was found the second step was the bottleneck. But in some of those cases with many locations the first optimization step was already very expensive. In the two other cases where the overall run time exceeded 30 minutes the first step was the most time consuming. In one of the two cases (base instance 8, 3 time windows with a length of 600 minutes) the first step took 32 minutes. In the other case (base instance 15, 3 time windows with a length of 600 minutes), more than 21 minutes were needed to obtain an optimal solution in this step (overall run time: 36 minutes). In the test run with a run time of exactly 30 minutes (base instance 8, two time windows with a length of 600 minutes) only feasible solutions were found in each step with a run time for step one of 25 minutes. For the test run (base instance 14, 3 time windows with a length of 600 seconds) with a run time very close to 30 minutes (1780 sec) an optimal solution was found in each step. For the other test runs with an overall run time of more than 25 minutes and a filter distance of 100 km, optimization step 2 by far had the longest run time. This occurred for 6 instances from base instance 15 and one instance from base instance 14. For each of these instances, steps 1 and 2 required together more than 25 minutes.

For a filter distance of 200 km, in step two the time limit was reached 4 times, for a filter distance of 300 km it was reached 3 times and for the filter distances of 400 and 500 km it was reached 2 times in each case. All of the corresponding instances were derived from base instance 15. Considering the filter distance of 1000 km, overall run times were below 3 minutes with an average run time of 11 seconds.

In total, 27.38 hours were needed for all 1350 test runs. Figure 15 shows the proportions of the different optimization steps on the overall run time for all test instances. The greatest impact has optimization step 2 with a cumulated duration of 18.44 hours (67 %). Optimization step 1 took 5.76 hours (21 %) to complete. Time limits for step 3 were only relevant in two cases (the second and third case with a run time of more than 30 minutes described above). The overall duration of 1.82 hours (7 %) is rather short which shows that the refueling subproblem is much faster to solve as the subproblem for planning time windows, rest periods and breaks. The last optimization step took 1.36 h in total (5 %).

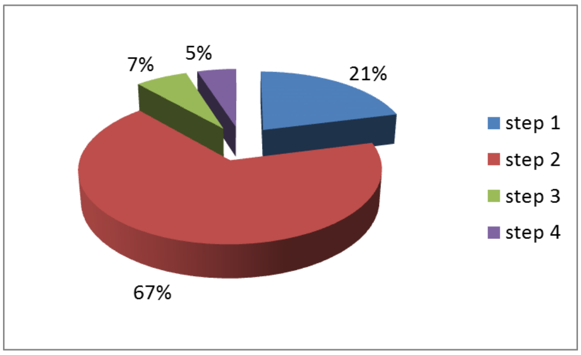

Figure 15: The proportions of the different optimization steps on the overall run time

Figures 16 and 17 show the proportions of the different steps on the overall (for all test instances) run time for a filter distance of 100 km and a filter distance of 1000 km, respectively. When reducing the filter distance, the proportions for the different optimization steps are comparable in their magnitude although the run times increase significantly. Note that the overall run time for all instances was 14.05 h if a filter distance of 100 km was chosen and only 0.71 h if a filter distance of 1000 km was selected. This means that there is a 95 % decrease of the overall run time with a filter distance of 1000 km compared to 100 km.

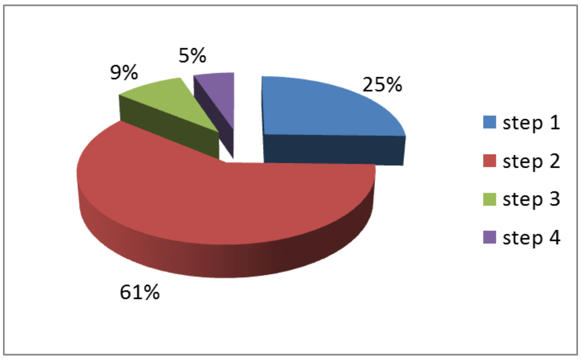

Figure 16: The proportions of the different optimization steps on the overall run time for a filter distance of 100 km (225 test instances)

Figures 18 and 19 depict the average run times per base instance and per optimization step for the filter distances of 100 km and 1000 km. Again, it can be seen that on average the run time of step two is the longest with the two exceptions for base instances 2 and 8 and filter distance 100 km. Note that the large values for optimization step 1 and a filter distance of 100 km of base instance 8 are mainly due to the two cases described before (see pages 44 et seq.). In total, there are three base instances for which the run time of step 1 is longer than that of step 2. For base instance 2, 11 of 15 test instances (73 %) show a longer run time for step 1.

In total, there were 263 test runs (19 %) for which step 1 took longer than step 2. In those cases, the shorter run time of step 2 may be attributed to the upper cutoff provided by step 1.

Although the run times of step 3 only account for 7 % of the overall run time for all test runs, the strong growth of the run time with the number of locations, especially when considering the maximum run time for a given number of locations, can be observed here as well (see Figure 20).

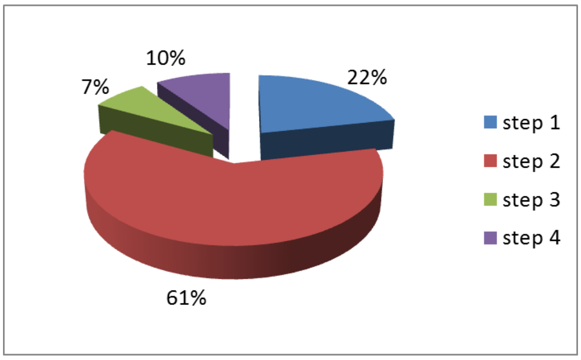

Figure 17: The proportions of the different optimization steps on the overall run time for a filter distance of 1000 km (225 test instances)

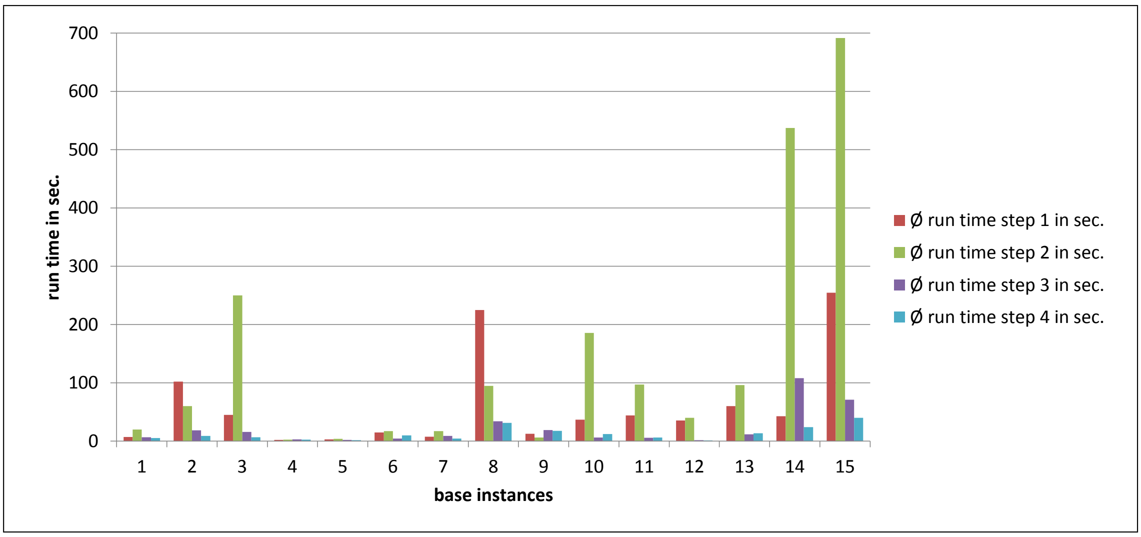

Figure 18: Average run times per base instance and per optimization step for a filter distance of 100 km

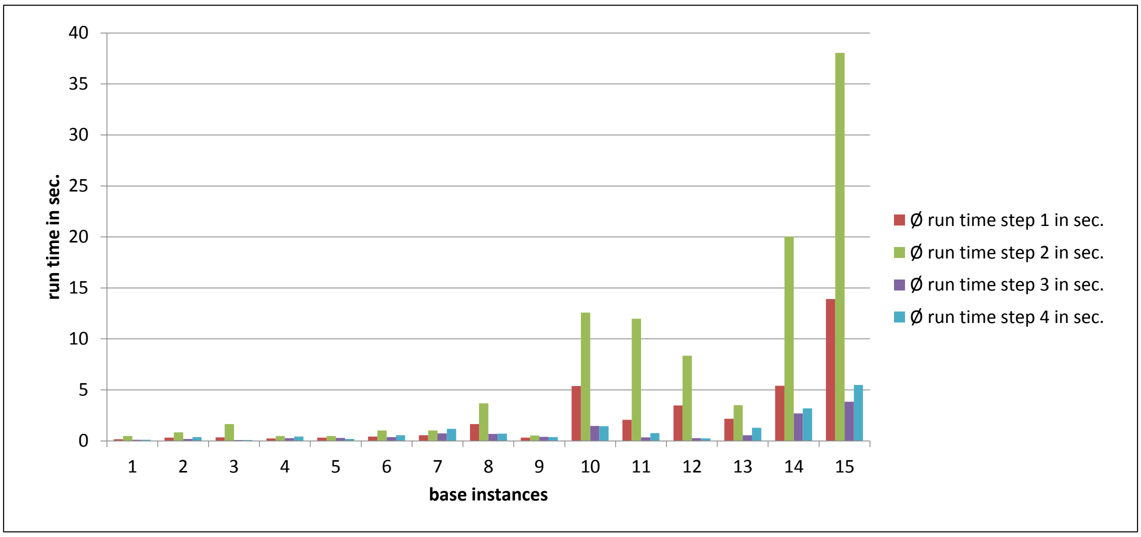

Figure 19: Average run times per base instance and per optimization step for a filter distance of 1000 km

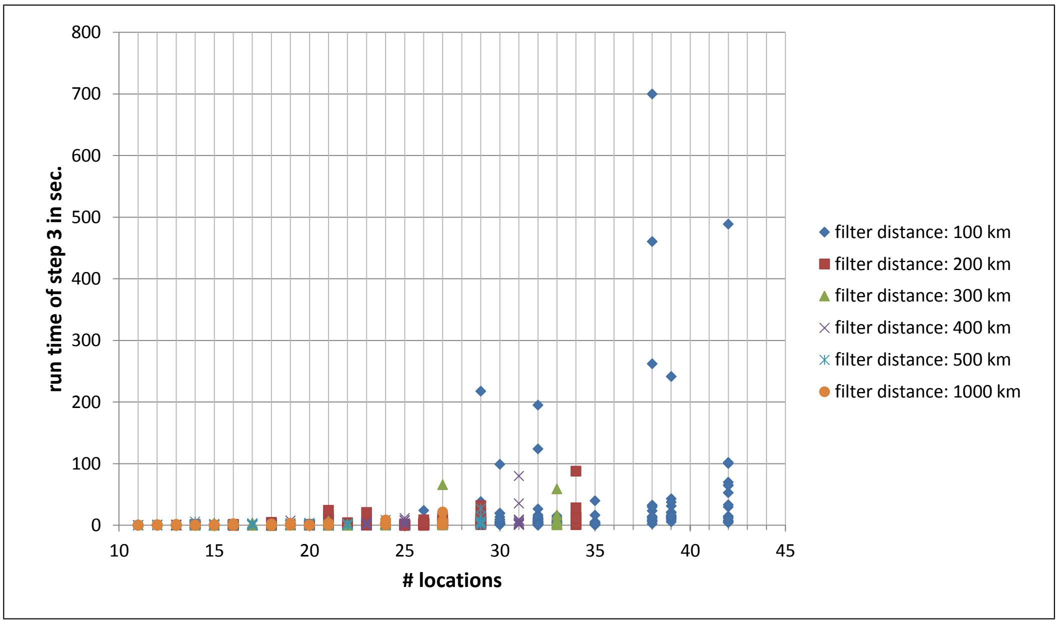

Figure 20: Run times of step 3

11.2 The influence of the number and length of time windows on the run time

It is interesting to see that the three instances with a run time of more than 30 minutes all have three time windows (see Figure 21). This suggests that the number of time windows influences the run time.

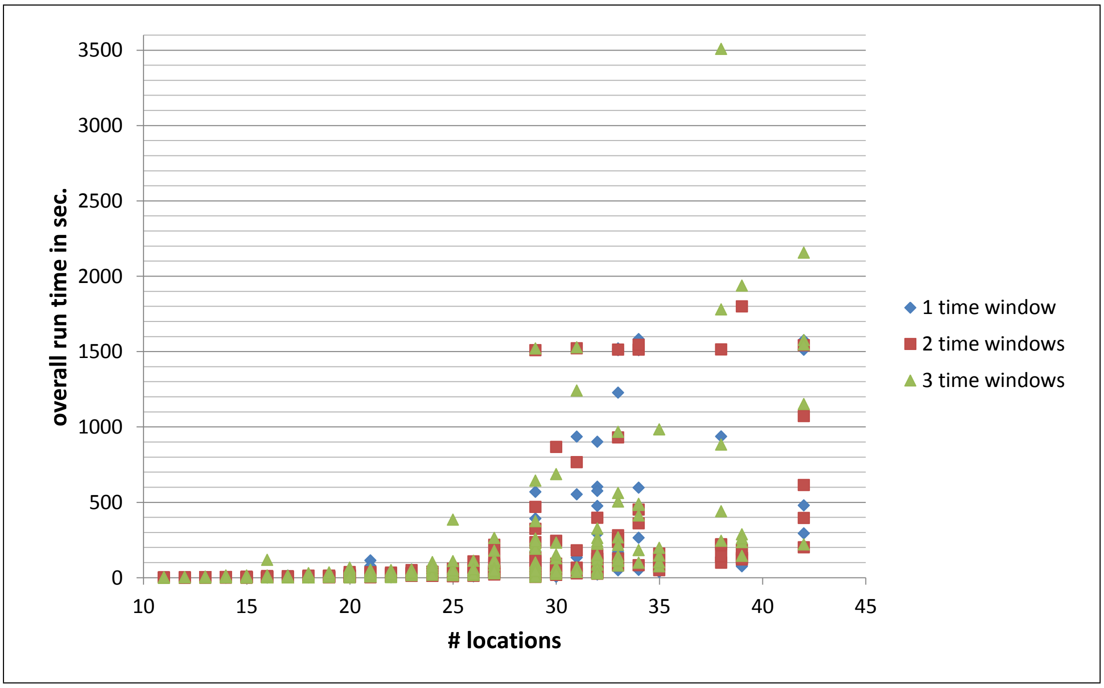

Figure 21: Run times depending on the number of locations and the number of time windows

A detailed analysis of the average run times depending on filter distances and the number of time windows (see Figure 22) did not provide a clear picture. While the longest run times for the filter distances of 100, 400, 500 and 1000 km were obtained for three time windows, for the filter distances of 200 and 300 km the instances with three time windows show on average the shortest run times.

We observe that the longest run time was obtained for an instance with a time window length of 120 minutes (see Figure 23), followed by five instances with a time window of 600 min.

Figure 24 shows the average run times per filter distance and per time window length. It can be seen that for each filter distance the average run time for instances with time windows of 600 min is always ranked first or second when considering the longest run times. For a time window length of 120 min, significant longer average run times than for

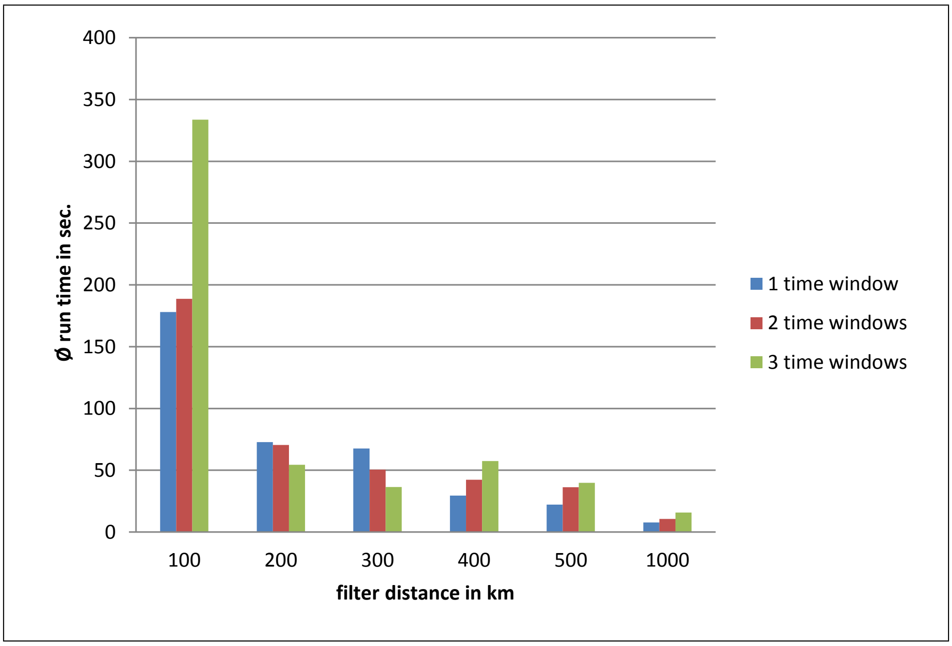

Figure 22: Average run time depending on the filter distance and the number of time windows

the time window lengths of 0, 30 and 60 min can be found for the filter distances 100, 400 and 500 km.

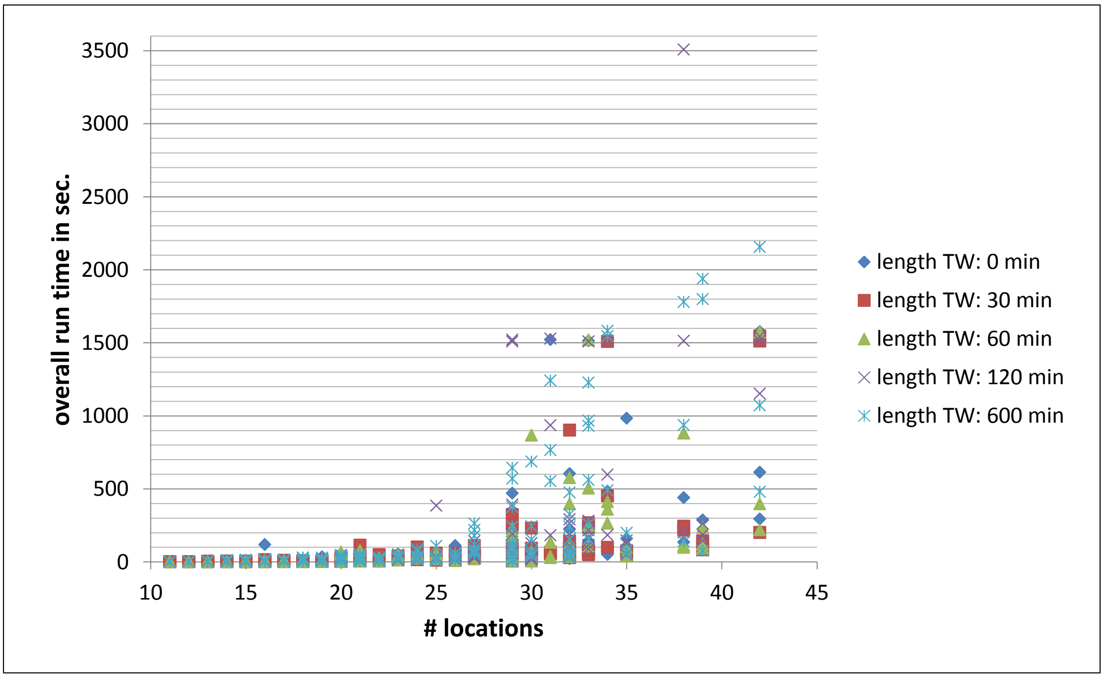

Figure 23: Run times depending on the time window (TW) length and on the number of locations

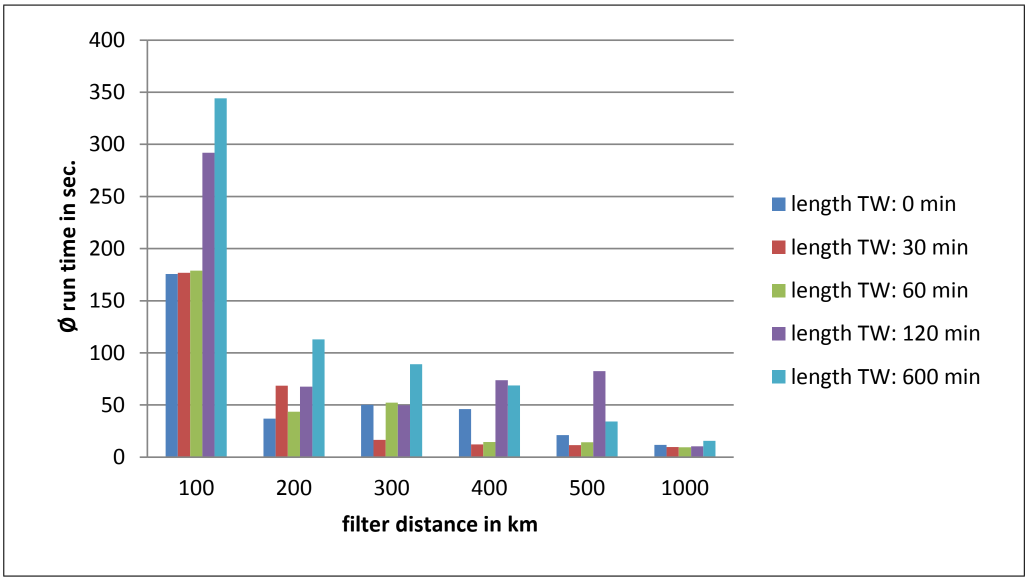

Figure 24: Average run times depending on the filter distance and the time window length

11.3 Managerial insights

In this section, valuable managerial insights are drawn from the results obtained. First, the solution quality is analyzed depending on the selected filter distance. Afterwards, general findings are discussed that may be relevant for practitioners.

11.3.1 Filter distance and solution quality

Figure 25 shows that the average lateness is constant for the filter distances of 200, 300, 400 and 500 km. For a filter distance of 1000 km there is an increase of 0.08 %, i.e. the difference in the overall lateness for all 225 test instances is 35 minutes, that means on average 9 seconds per instance. The increase in lateness stems from two instances. The first one has a total lateness of 18 minutes and is derived from base instance 11. Moreover, it has one time window per customer location with a time window length of zero. The other one also refers to base instance 11 with one time window per customer location but with a time window length of 30 minutes. For these two cases, even without considering the time for refueling, time windows cannot be met. The increase in lateness when considering a filter distance of 100 km can be explained by the non-optimal solution values obtained for the instances where time limits were reached and the solution process was stopped prematurely (see Figure 13 for the solution process and corresponding time limits and Section 11.1 for a description of test instances with non-optimal solution values).

Fuel costs increase marginally with the filter distance. The difference between choosing a filter distance of 100 km and a filter distance of 1000 km is on average 38 ct (0.04 %). Over all 225 instances, this amounts to 86 €. This means that the selection of a large filter distance has the advantage of reducing the problem size and thus the run time without negatively affecting the total fuel cost.

The average travel time is relatively constant and lies between 107.76 h (with a filter distance of 300 km) and 107.84 h (with a filter distance of 100 km) per week (see Figure 26).

Table 4 summarizes the findings considering four criteria for each of the filter distances. The filter distance with the best obtained value cumulated over all instances is chosen as reference. The corresponding value is set to be equivalent to 100 %.

filter distance (km)	% of minimum lateness	% of minimum fuel costs	% of minimum travel time	% of minimum run time
100	100.11%	100.01%	100.07%	2057.62%
200	100.00%	100.00%	100.00%	580.88%
300	100.00%	100.03%	100.00%	454.55%
400	100.00%	100.03%	100.01%	379.70%
500	100.00%	100.04%	100.01%	288.29%
1000	100.08%	100.04%	100.03%	100.00%

Table 4: Solution quality and run times depending on the filter distance

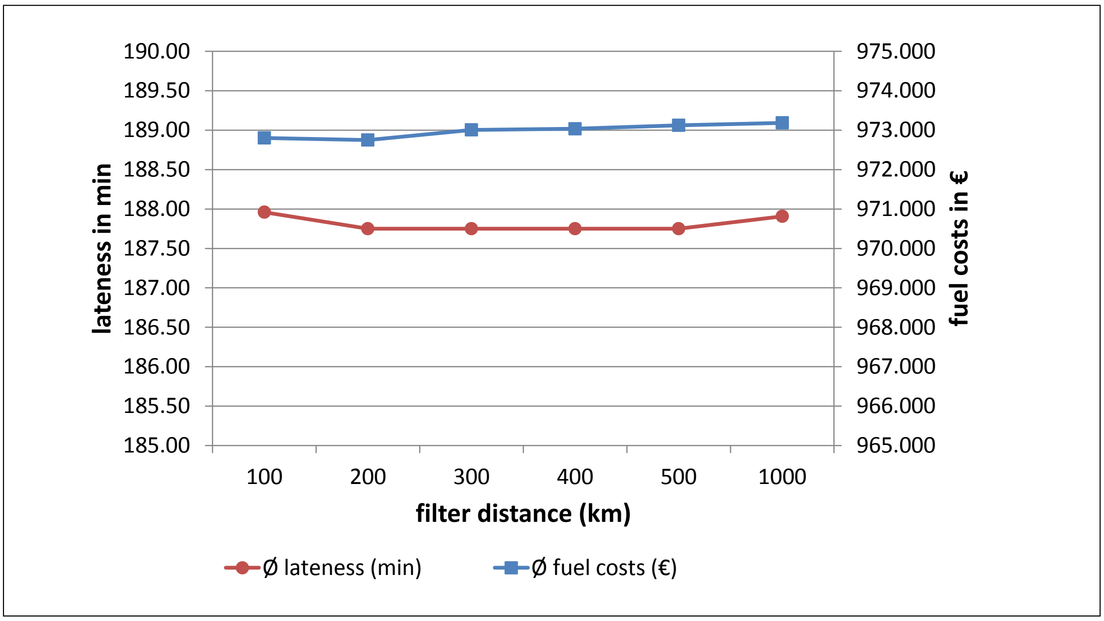

Figure 25: Lateness and fuel costs depending on the filter distance

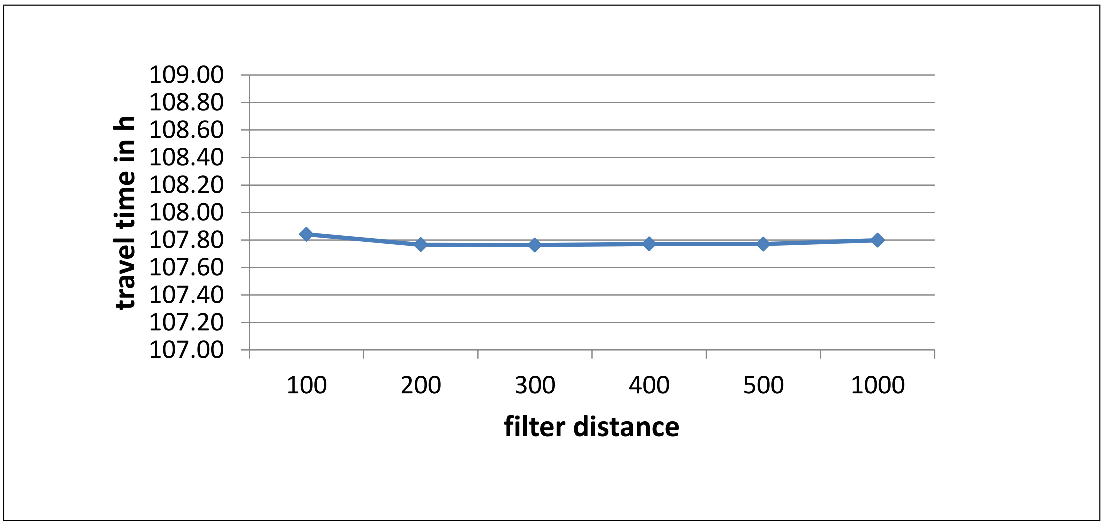

Figure 26: Average travel time and filter distance

It can be seen that slight increases in the solution quality correlate with a significant growth of the run time for the shorter filter distances. When considering the filter distance 100 km, the problem sizes are largest and so are the run times. Compared to the filter distance 200 km there are more instances for which no optimal solution is found in one or more of the optimization steps. Thus, a deterioration of lateness, fuel cost and travel time can be observed.

11.3.2 General findings

In this section, more general findings are discussed. The first table (Table 5) depicts the average detour distance[25] depending on the chosen filter distance. It can be noted that the best value is obtained for a filter distance of 1000 km. A reason for this may be the consideration of country prices. As prices often do not vary over long distances traveled, the detour distance in many cases is the filter criterion that eliminates gas stations within the given filter distance. With a filter distance of 1000 km more gas stations are eliminated than with a filter distance of 200 km, thereby removing more gas stations with large detour distances. As the choice of gas stations with large detour distances is reduced, it is quite probable that the optimal solutions for a filter distance of 1000 km incorporate shorter detour distances than those for a smaller filter distance. But this does not always have to be the case.

filter distance (km)	Ø detour distance (km)	% of minimum
100	6.34	119%
200	5.64	106%
300	5.37	101%
400	5.45	102%
500	5.37	101%
1000	5.34	100%

Table 5: Filter distance and average detour distance

Tables 6, 7 and 8 show the number of refueling stops depending on the filter distance, the number of time windows per customer location and their length.[26] It can be seen that for filter distances of 200 km and 300 km the average number of refueling stops is 2.12. For filter distances of 400, 500 and 1000 km the average number is about 2.19 which is 3 % more than the minimum value. For a filter distance of 100 km, the effects of non-optimal solution values have to be taken into account (recall Table 4). If time windows are not chosen optimally and thus more lateness is accepted, this leaves more time for additional refueling stops. This may explain why the average number of refueling stops is higher for a filter distance of 100 km than for a filter distance of 200 km.

[25] The average is taken over all instances where per instance the whole detour for all visited gas stations is considered.

[26] Note that the number of refueling stops was only considered in the objective function of optimization step 4.

filter distance (km)	Ø refueling stops	% of minimum
100	2.18	103%
200	2.12	100%
300	2.12	100%
400	2.19	103%
500	2.19	103%
1000	2.19	103%

Table 6: Filter distance and the number of refueling stops

The number of refueling stops lies between 1 and 4 for all test runs. Considering an arbitrary instance, the difference of the number of refueling stops between two filter distances is at most 1. On average over all test runs 2.16 refueling stops are made. With more time windows per customer location there are more possibilities for planning arrival times without worsening lateness and the overall travel time. This leaves more freedom and time for refueling and may be the reason why in Table 7 the average number of refueling stops slightly increases with the number of time windows per customer location.

# TW	Ø refueling stops	% of minimum
1	2.14	100%
2	2.17	101%
3	2.19	102%

Table 7: The number of time windows and the number of refueling stops

For a time window length of zero, an additional refueling stop will more easily lead to lateness than for a time window length greater than zero. Therefore, the average number of stops for those instances is the lowest (see Table 8). For time window lengths of 30, 60 and 120 minutes the average number of refueling stops is relatively constant. When considering a time window length of 600 minutes which in reality corresponds to opening hours for many of the considered instances this is nearly the same as planning arrival times freely without time windows. This means that additional refueling stops have a relatively direct impact on the overall travel time which is a component of the objective functions of optimization steps 1, 2 and 4. This may be the reason why the average number of refueling stops for a time window length of 600 minutes is less than for time window lengths of 30, 60 and 120 minutes.

length TW (min)	Ø refueling stops	% of minimum
0	2.13	100.0%
30	2.18	102.1%
60	2.18	102.1%
120	2.17	101.9%
600	2.16	101.2%

Table 8: Time window length and the number of refueling stops

12 Summary and future research

In this section we summarize the findings of this study and outline opportunities for future research.

12.1 Summary

Fuel is a main cost driver in the European road haulage sector. Strongly varying diesel prices across different European countries allow a high cost saving potential, especially when considering long-haul trips that involve crossing several national borders. We described why it is not recommendable to consider fuel optimization as an isolated problem. When choosing appropriate customer time windows and determining driver rest periods and breaks, the time needed for refueling stops has to be taken into account as otherwise the time needed for an unplanned refueling may lead to changes in the driver schedule and thus jeopardize a punctual arrival. When planning refueling stops, it is required, among other things, to determine between which pairs of consecutive customer locations refueling should be done not only considering fuel costs as decision criterion but also minimizing lateness.

On the basis of the MILP model of Bernhardt et al. (2016) and extending constraints from the standard fuel optimizer model presented by Suzuki (2008) we developed a MILP model that plans driver activities in accordance with Regulation (EC) No 561/2006 simultaneously considering the choice of customer time windows, refueling stops and refueling quantities. Similarly as in Bernhardt et al. (2016), we considered "soft" time windows to even find solutions if lateness cannot be avoided completely. The main objectives were the minimization of overall lateness, travel time and fuel expenditures. The solution process presented to solve the resulting multicriteria optimization problem consists of four optimization steps and a transformation algorithm that is needed to obtain a readable driver schedule.

We observed that often list prices were constant over several days. As future prices currently cannot be predicted exactly several days in advance the approach to plan with the current price is justifiable. However, online replanning is recommended and this step can be carried out with the solution process presented.

The test instances described in Bernhardt et al. (2016) were derived from real data provided by a German haulage company that operates a fleet of vehicles in Europe. The instances comprise between 2 and 10 customer locations and in addition, stops for start and end location. The number of time windows and their length were varied. We extended the database of the test instances by adding information about gas stations along the route based on real data. To obtain driving durations and distances between locations and for detours to gas stations, a modified A^* routing algorithm was used. Vehicle consumption rates, tank capacities and starting and ending fuel levels were taken from the real data from the one-week trips considered. The locations of gas stations considered all over Europe were provided by the service station chain the partner haulage company had fuel cards for.

Fuel prices at gas stations were assumed to be equal to the list prices provided by the fuel card operator for the corresponding countries.

From the experience made in Bernhardt et al. (2016) we anticipated significantly long run times if many locations were involved. To reduce the run times for the solution process, a preprocessing heuristic was developed to eliminate unattractive gas stations and thus reduce the solution space. The filter distance was the control parameter which was varied in our numerical experiments to obtain different numbers of remaining gas stations per instance. The larger the filter distance, the more gas stations were eliminated. Additionally, time limits were set up for the different optimization steps to restrict the overall run time to 30 minutes. Here, we made the exception that in each optimization step at least a feasible solution had to be found before stopping the solution process.

Numerical experiments were conducted for the 225 test instances of Bernhardt et al. (2016) with the extended data described above. For each of the instances test runs were performed for the filter distances of 100, 200, 300, 400, 500 and 1000 km to analyze the influence of the filter distance on the run time. As expected, run times, especially when considering worst case scenarios, strongly increased with decreasing filter distance and thus with an increasing number of locations considered. On the other hand, the analysis of the solution quality showed that there are only slight improvements in lateness, overall travel time and fuel expenditures when considering more gas stations. The test results suggest that it is legitimate to choose a rather large filter distance. Using a filter distance of 100 km does not seem to be reasonable at all as the run time on average was 233.47 seconds with 12 instances that had a run time of more than 25 minutes. Of these 12 instances 11 (4.89 %) were not solved to optimality, that means only a feasible solution was obtained in at least one of the 4 optimization steps. With a filter distance of 1000 km all instances were solved optimally in reasonable times. On average, here the solution process took 11.35 seconds. When considering a filter distance of 500 km, the solution process took on average 32.71 seconds (288.19 %) and for two cases (0.89 %) the run time was more than 25 minutes and no optimal solution was found. For a filter distance of 1000 km, the overall lateness was only 0.08 % worse than the best overall lateness, which amounts to an average difference of only 9 seconds per instance. On average, the fuel costs were only 0.04 % higher than those obtained for the filter distance of 200 km with the lowest fuel cost, that means 38 ct per test instance. The overall travel time only takes 0.03 % longer which corresponds to only 2.11 minutes more in the mean. Thus, among the filter distances considered the filter distance of 1000 km for our test setting seems to be the most reasonable one.

12.2 Future research

In our numerical experiments, prices at gas stations were considered to be constant per country. This approach may be reasonable if only a specific group of gas stations with identical prices per country is considered to be suitable for the refueling of trucks and the driver needs. This in turn may depend on existing contracts with fuel card operators. In Section 11.3.2 we noticed that the overall detour distance on average was best for a filter distance of 1000 km. As described there, we assume that this may be due to the consideration of country prices. It would also be interesting to analyze if the number of refueling stops would change when considering different price structures. Therefore, the analysis of the effect of varying prices within countries would be interesting.

In our mathematical experiments, ending fuel levels were taken from real data and thus were input data. In reality, ending fuel levels have to be determined prior to the start of the solution process. They should orientate on the fuel price trend and on the price structure of countries expected to be visited in the future. The determination of a good ending fuel level can be a future field of research.

Fuel consumption rates depend on the road type, geographical properties of the road, vehicle characteristics, and the driving behavior. Thus, it would be interesting to incorporate additional information, for example, from a geographic information system (GIS), from the standardized FMS interface of the vehicle and other historical data when determining fuel consumptions between locations for the MILP model input.

As described in Bernhardt et al. (2016), truck drivers face difficult working conditions and there is a shortage of qualified drivers. When making a preselection of gas stations with special amenities such as restrooms and restaurants, a first improvement on working conditions is possible.

The use of a commercial optimization solver can be an obstacle for a company due to cost reasons. Thus, the development of a heuristic solution process can be attractive for smaller companies.

Appendices

A Parameters and variables of the complete MILP model

In this appendix a complete overview of all parameters (Appendix A.1) and variables (Appendix A.2) of the MILP model for the combined planning of time windows, rest periods and breaks and vehicle refueling is given.

A.1 Parameters of the MILP model

$r \in \mathbb{N}$	Total number of vertices representing origin (vertex 0) and destination (vertex $r-1$), customer locations and gas stations. The vertices are numbered from 0 to $r-1$ according to the sequence of customer locations to be visited and gas stations that are passed.
$S^{locations}$	Set of all vertices including all vertices for customer locations, gas stations, origin and destination
$S^{customers} \subset S^{locations}$	Set of all vertices that correspond to customer locations
$S^{stations} \subset S^{locations}$	Set of all vertices that correspond to gas stations
$\bar{f}^{start} \in \mathbb{N}_0$	Amount of fuel in the tank at start location 0 in liters
$\bar{f}^{end} \in \mathbb{N}_0$	Minimum amount of fuel to be left in the tank at the final destination $r-1$ in liters
$\bar{P}_i \in \mathbb{R}_0^+$	Fuel price at gas station $i \in S^{stations}$ in € per liter
$\bar{\Delta}^{min} \in \mathbb{R}_0^+$	Minimum amount of fuel to purchase at a gas station in liters
$\bar{T}^{max} \in \mathbb{R}_0^+$	Vehicle tank capacity in liters
$\bar{T}^{min} \in \mathbb{R}_0^+$	Lower bound fuel, i.e. the minimum amount of fuel to be maintained in the tank at all times in liters
$\bar{\Delta}^{dr}_{(i,i+1)} \in \mathbb{N}_0$	Driving time in minutes needed to travel from i to $i+1$, $i = 0, \ldots, r-2$ not including the time needed for out of route distances to and from gas stations
$\bar{\Delta}^{drTo}_i \in \mathbb{N}_0$	Driving time in minutes needed to travel from the point of departure to the corresponding gas station i (equals 0 if $i \notin S^{stations}$)
$\bar{\Delta}^{drFrom}_i \in \mathbb{N}_0$	Driving time in minutes needed to travel from the gas station i to the corresponding point of return (equals 0 if $i \notin S^{stations}$)
$\bar{\Delta}^{cons}_{(i,i+1)} \in \mathbb{R}_0^+$	Fuel consumption in liters when traveling from i to $i+1$, $i = 0, \ldots, r-2$ not including the consumption for out of route distances to and from gas stations

$\bar{\Delta}_i^{consTo} \in \mathbb{R}_0^+$	Fuel consumption in liters when traveling from the point of departure to the corresponding gas station i (equals 0 if $i \notin S^{stations}$)
$\bar{\Delta}_i^{consFrom} \in \mathbb{R}_0^+$	Fuel consumption in liters needed to travel from the gas station i to the corresponding point of return (equals 0 if $i \notin S^{stations}$)
$\bar{\Delta}^{refuel} \in \mathbb{N}_0$	Time needed for refueling in minutes
$\bar{\Delta}_i^{service} \in \mathbb{N}_0$	Time needed for loading and/or unloading at vertex i, $i \in S^{customers}$, in minutes, $\bar{\Delta}_0^{service} = 0$ and $\bar{\Delta}_i^{service} = 0 \; \forall \; i \in S^{stations}$
$nTW_i \in \mathbb{N}_0$	Number of time windows at customer location i, $i \in S^{customers}$
$\overline{TW}_{iz}^{begin} \in \mathbb{N}_0$	Lower limit of the time window z at vertex i, $i \in S^{customers}$, $z = 1, \ldots, nTW$ in minutes counted from start time 0
$\overline{TW}_{iz}^{end} \in \mathbb{N}_0$	Upper limit of the time window z at vertex i, $i \in S^{customers}$, $z = 1, \ldots, nTW_i$ in minutes counted from start time 0
$\overline{udt} \in \mathbb{N}_0$	Driving time since the last daily rest period or break at the beginning of the planning horizon in minutes
$\overline{ddt} \in \mathbb{N}_0$	Cumulated daily driving time since the end of the last daily rest period at the beginning of the planning horizon in minutes
$\overline{ptr} \in \mathbb{N}_0$	Passed time since the end of the last daily rest period at the beginning of the planning horizon in minutes
$\overline{ptwr} \in \mathbb{N}_0$	Passed time since the end of the last weekly rest period at the beginning of the planning horizon in minutes
$\overline{urt} \in \mathbb{N}_0$	If a daily rest period takes place at start time, this parameter expresses its duration since the start of the rest period in minutes
$\overline{ubt} \in \mathbb{N}_0$	If a break takes place at start time, this parameter expresses its duration since the start of the break in minutes
$\overline{dte} \in \{0,1\}$	Is equal to 1 if a driving time extension is currently used when the planning horizon begins, 0 otherwise
$\overline{hpb} \in \{0,1\}$	Is equal to 1 if the first part of a break with a duration of at least 15 minutes has already been taken before the beginning of the planning horizon, 0 otherwise
$\overline{hpr} \in \{0,1\}$	Is equal to 1 if the first part of a daily rest period with a duration of at least 3 hours has already been taken before the beginning of the planning horizon, 0 otherwise
$\overline{noRed} \in \{0,1,2,3\}$	The number of reduced daily rest periods that have already been taken in the current week
$\overline{noExt} \in \{0,1,2\}$	The number of extended daily driving times that have already been taken in the current week

A.2 Variables of the MILP model

Variables needed to define the objective functions

$start_i \in \mathbb{R}_0^+$	Start of loading and/or unloading if vertex $i \in S^{customers}$, start of refueling if $i \in S^{stations}$, start of driving (after a potential break or rest period) if $i = 0$
$\Delta_i^{late} \in \mathbb{R}_0^+$	Lateness in vertex i, $i = 1, \ldots, r-1$. Is set to be zero if the considered vertex does not correspond to a customer location.
$\Delta_i^{refuel} \in \mathbb{R}_0^+$	Amount of fuel to purchase at gas station $i \in S^{stations}$ in liters

Variables for the integration of refueling decisions

$$\alpha_i^{refuel} = \begin{cases} 1 & \text{if } i \in S^{stations} \text{ and } i \text{ is selected for refueling} \\ 0 & \text{otherwise} \end{cases}$$

$T_i \in \mathbb{R}_0^+$	Amount of fuel in the tank either at truck stop i before purchasing fuel ($\Delta_i^{refuel} = 1$) or at the corresponding leaving point ($\Delta_i^{refuel} = 0$)
$\Delta_{(i,i+1)}^{dr} \in \mathbb{R}_0^+$	Driving duration between locations. If i is a gas station and refueling takes place at i, $\Delta_{(i,i+1)}^{dr}$ includes the out of route driving duration from gas station i. If refueling takes place at $i+1$, the out of route driving duration to gas station i+1 is added.
$\Delta_i^{work} \in \mathbb{R}_0^+$	Time needed for loading and/or unloading at location i in minutes if i is associated with a gas station, i.e. $i \in S^{customers}$. Time needed for refueling if $i \in S^{stations}$. Δ_i^{work} is set to be 0 if $i \in S^{stations}$ and no refueling takes place in i.

Variables that indicate which time window is chosen at customer i

$$tw_{iz} = \begin{cases} 1 & \text{if time window z is chosen at destination } i \in S^{customers} \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, \ldots, r-1, z = 1, \ldots, nbTW_i$

The following set comprises the continuous status variables for each vertex i.

E_i^{dt} Driving time left until the next break or daily rest period when entering vertex i, $i = 0, \ldots, r-1$ in minutes
$0 \leq E_i^{dt} \leq 270$

E_i^{ddt} Driving time left until the next daily period rest when entering vertex i
$i = 0, \ldots, r-1$ in minutes
$0 \leq E_i^{ddt} \leq 540$

E_i^{t} Time left until the next daily rest period when entering vertex i
$i = 0, \ldots, r-1$ in minutes
$0 \leq E_i^{t} \leq 900$

L_i^{dt} Driving time left until the next break or daily rest period when leaving vertex i, $i = 0, \ldots, r-1$ in minutes
$0 \leq L_i^{dt} \leq 270$

L_i^{ddt} Driving time left until the next daily rest period when leaving vertex i
$i = 0, \ldots, r-1$ in minutes
$0 \leq L_i^{ddt} \leq 540$

L_i^{t} Time left until the next daily rest period when leaving vertex i
$i = 0, \ldots, r-1$ in minutes
$0 \leq L_i^{t} \leq 900$

The following variables indicate for each arc $(i, i+1)$ if a daily rest is made, the number of daily rests and their cumulated duration.

$$\alpha_{(i,i+1)}^{rest} = \begin{cases} 1 & \text{if at least one daily rest is taken on arc } (i, i+1) \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$A_{(i,i+1)}^{rest} \in \mathbb{N}_0$ The number of daily rest periods taken on arc $(i, i+1)$,
$i = 0, \ldots, r-2$

$\Delta_{(i,i+1)}^{rest} \in \mathbb{R}_0^+$ The cumulated duration of all daily rest periods on arc $(i, i+1)$,
$i = 0, \ldots, r-2$

Regarding daily rests at vertices, the following variables indicate if a daily rest is made and its duration.

$$\alpha_i^{rest} = \begin{cases} 1 & \text{if a daily rest is made in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$\Delta_i^{rest} \in \mathbb{R}_0^+$ The duration of a daily rest in vertex i,

$i = 0, \ldots, r-1$

The next set of variables are needed to determine if breaks are taken on arc $(i, i+1)$ and their number.

$$\alpha_{(i,i+1)}^{break} = \begin{cases} 1 & \text{if at least one break is taken on arc } (i, i+1) \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$A_{(i,i+1)}^{break} \in \mathbb{N}_0$ The number of breaks taken on arc $(i, i+1)$, $i = 0, \ldots, r-2$

The following variables indicate if breaks are taken in vertices.

$$\alpha_i^{break} = \begin{cases} 1 & \text{if a break is taken in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

Each variable Δ_i^{wait} gives the waiting time in vertex i:

$\Delta_i^{wait} \in \mathbb{R}_0^+$ Waiting time in vertex i, $i = 0, \ldots, r-1$

The next variables specify if an early daily rest is made on an arc, meaning that the daily driving time is not completely used up.

$$\mu_{(i,i+1)}^{earlydr1} = \begin{cases} 1 & \text{if a break is replaced by a daily rest period on arc } (i, i+1) \\ & \text{ and this rest is the first rest on this arc} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$$\mu_{(i,i+1)}^{earlydr2} = \begin{cases} 1 & \text{if a break is replaced by a daily rest period on arc } (i, i+1) \\ & \text{ and this rest is \textit{not} the first rest on this arc} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

When arriving in vertex i, in case a daily rest period was taken on arc $(i-1, i)$, the following variable indicates if a break was taken since the last daily rest period.

$$e_i^{bt} = \begin{cases} 1 & \text{if the last rest activity on the preceding arc } (i-1, i) \text{ was a break} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

The next variables indicate if a break is still necessary to completely use up the daily driving time left when leaving vertex i.

$$l_i^{bn} = \begin{cases} 1 & \text{if a break would be necessary to completely exploit} \\ & \text{ the daily driving time left when leaving vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

The following variables are needed to model the optional rules.

$$\alpha_i^{pbreak} = \begin{cases} 1 & \text{if the first part of a break is taken in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$\mu_{(i,i+1)}^{upbreak} = \begin{cases} 1 & \text{if the second part of a break is taken on arc } (i, i+1) \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$$\mu_i^{upbreak} = \begin{cases} 1 & \text{if the second part of a break is taken in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$l_i^{pbreak} = \begin{cases} 1 & \text{if when leaving vertex } i \text{ a partial break of 15 minutes was taken} \\ & \text{since the last rest period} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$\alpha_i^{prest} = \begin{cases} 1 & \text{if the first part of a daily rest is taken in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$l_i^{prest} = \begin{cases} 1 & \text{if when leaving vertex } i \text{ a partial rest period of 3 h was taken} \\ & \text{since the last rest period} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$\mu_i^{prest} = \begin{cases} 1 & \text{if the last break on arc } (i-1,i) \text{ is substituted by a} \\ & \text{first partial rest} \\ 0 & \text{otherwise} \end{cases}$$

$i = 1, \ldots, r-1$

$$\mu_i^{dredrest} = \begin{cases} 1 & \text{if in vertex } i \text{ the decision is made that the next} \\ & \text{daily rest after leaving vertex } i \text{ will be a reduced one} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$\mu_{(i,i+1)}^{redrest} \in \{0,1,2,3\}$ The number of reduced daily rests made on arc $(i,i+1)$, $i = 0, \ldots, r-2$

$$\mu_i^{redrest} = \begin{cases} 1 & \text{if a reduced daily rest is taken in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$l_i^{dredrest} = \begin{cases} 1 & \text{if the next daily rest is a reduced one and is taken} \\ & \text{after leaving vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-1$

$$\mu_{(i,i+1)}^{extd1} = \begin{cases} 1 & \text{if a driving time extension is used on arc } (i,i+1) \text{ before the} \\ & \text{first daily rest} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$\mu_{(i,i+1)}^{extd2} \in \{0,1,2\}$ The number of driving time extensions used on arc $(i,i+1)$ between the first and the last daily rest, $i = 0, \ldots, r-2$

$$\mu_{(i,i+1)}^{extd3} = \begin{cases} 1 & \text{if a driving time extension is used on arc } (i,i+1) \text{ after the} \\ & \text{last daily rest} \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \ldots, r-2$

$$\mu_i^{extd} = \begin{cases} 1 & \text{if a driving time extension is decided in vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \dots, r-1$

$$l_i^{extd} = \begin{cases} 1 & \text{if a decision concerning a driving time extension was made} \\ & \text{before leaving vertex } i \\ 0 & \text{otherwise} \end{cases}$$

$i = 0, \dots, r-1$

Auxiliary variables:

$$\begin{aligned} \lambda_i^1,\ \lambda_i^2,\ \lambda_i^3,\ \lambda_i^4,\ \lambda_i^6,\ \lambda_i^7 &\in \{0,1\}, \qquad i = 0, \dots, r-1 \\ \lambda_i^5 &\in \{0,1\}, \qquad i = 0, \dots, r-2 \end{aligned}$$

B Detailed results of numerical experiments

This appendix presents detailed results of the mathematical experiments conducted. In Appendix B.1 results of the optimization steps of the solution process are illustrated for one of the test runs. Appendix B.2 shows the overall run times for all test runs.

B.1 Solution process - An example

In the following, we will show by means of an example how the driver schedule evolves over the several optimization steps. We chose the base instance 3 with three time windows per customer location and a time window size of 30 minutes (see Table 10) and considered a filter distance of 1000 km. The same instance (without considering information concerning refueling) was used in Bernhardt et al. (2016) to compare different planning techniques. Note that the driver starts his work week at the first customer (marked by a green dot in Figure 27), i.e. there is no driving duration between start location 0 and the customer location numbered 1. The final destination is chosen to be equal to the final destination reached by the driver in real life and is marked by a red dot in Figure 27. The driver was heading for a stop in Wolfsburg when his weekly rest period had to be started.

start	Mon 07:47						
target location			Rastatt (DE)	Kirkel (DE)	Madrid (ES)	Duenas (ES)	
stops		[0]	[1]	[2]	[3]	[4]	[5]
dur. loading/unloading (h)			2:00	2:00	2:00	2:00	0:00
time windows	start		Mon 06:30	Mon 05:30	Wed 05:30	Thu 03:30	Mon 00:00
	end		Mon 07:00	Mon 06:00	Wed 06:00	Thu 04:00	Sun 23:59
	start		Mon 09:00	Mon 08:00	Wed 08:00	Thu 06:00	
	end		Mon 09:30	Mon 08:30	Wed 08:30	Thu 06:30	
	start		Mon 11:30	Mon 10:30	Wed 10:30	Thu 08:30	
	end		Mon 12:00	Mon 11:00	Wed 11:00	Thu 09:00	

Table 10: Time windows

The problem has 6 stops[27] associated with customer locations, origin and final destination here numbered from 0 to 5. Note that the numbering in the model depends on the number and locations of the gas stations. After the execution of the filter algorithm we obtained 7 gas stations located in France, Spain and Belgium. Gas stations in Germany and the Netherlands were in the original list of gas stations along the route but were eliminated because of their high prices. Note that we consider list prices per country which are depicted in Figure 27. For more details on the example see Table 1 on page 31. The

[27] Note that even though the driver starts his week with loading and/or unloading at the first customer location, in the MILP model, there is an additional vertex for the start location. This vertex has been added for modeling purposes and for reasons of standardization (e.g. the origin location never has time windows) and thus is also depicted in Table 10. The distance between the artificial origin and the first customer location is zero.

transformation algorithm was executed after each optimization step to obtain visually comparable results.[28]

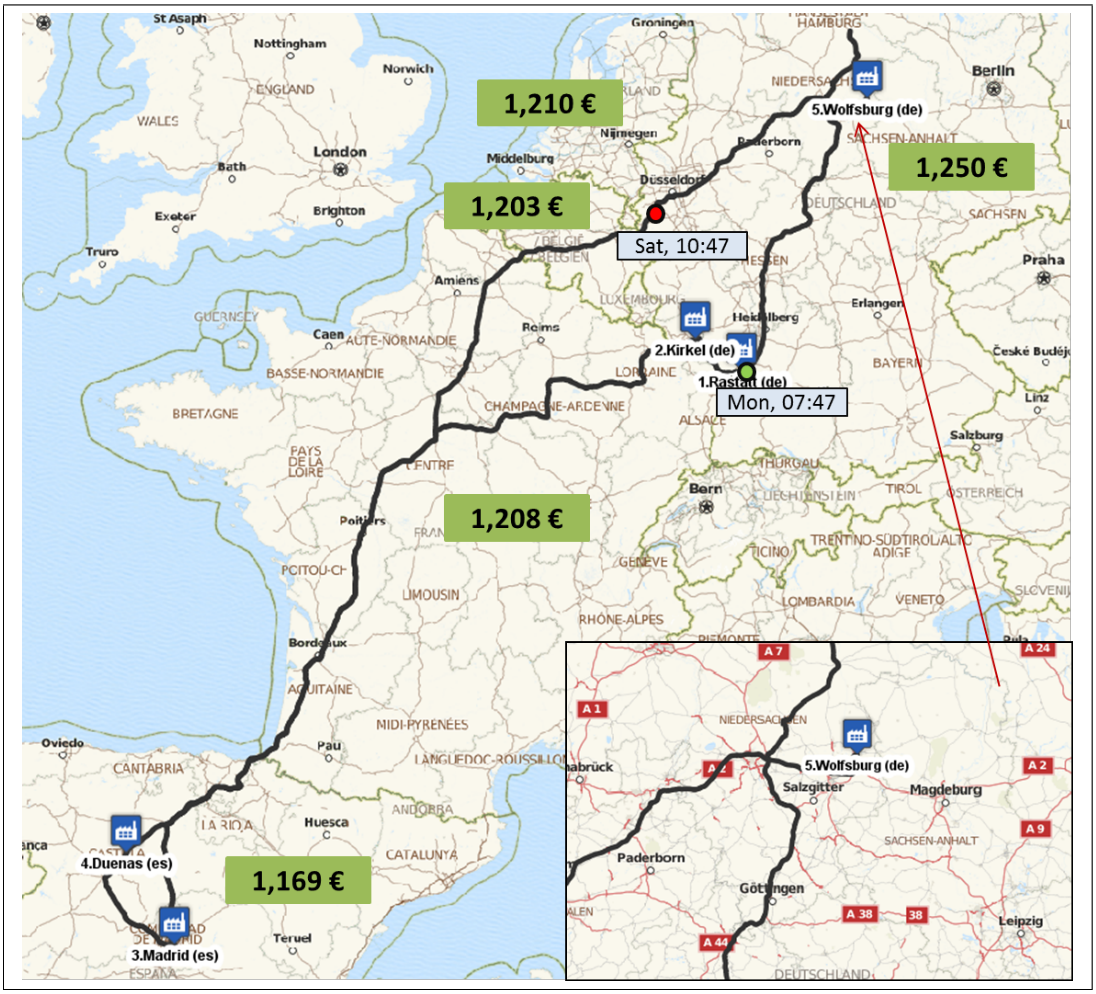

Figure 27: Base instance 3

In optimization step 1 (see Table 11), the MILP submodel without optional rules was solved, minimizing lateness and travel time. Refueling was already considered but not in an optimal way. With the filtered list of gas stations as possible choices, the driver has to refuel the vehicle in France at the beginning of his trip since with the remaining amount of fuel in the tank he cannot reach a gas station in Spain. He also has a refueling stop in Belgium at the end of his trip as otherwise it is not possible to obtain the high ending fuel level of 837 l at the final destination. An additional refueling stop was chosen to take place in Spain. The fuel costs amount to 1893 € for the whole trip where the final destination is reached with 40 l more in the tank than actually needed for the minimum ending fuel level.

[28] For the representation of the schedule as depicted in Tables 11 to 14, the mapping of customer locations to stops as described in Section 6 on page 19 in the MILP model is used.

The optimal objective function value is chosen as an upper cutoff for the MILP submodel in step 2 (see Table 12), where the optional rules are considered. If we compare the two schedules, we can see that the overall lateness in step 2 is reduced by the use of the optional rules by 1:27 h (44 %). Two reduced daily rest periods between customer stop 2 and 3 allow a punctual arrival at an earlier customer time window. The choice of an earlier time window without causing lateness was also possible for customer stop 4. With the help of a splitted break with one part before loading and/or unloading at customer location 4 and the other part on the way to customer location 5, two extended daily driving times and another reduced daily rest period, the driver arrives at the final destination 14:15 h earlier. The choice of gas stations did not change between steps 1 and 2, only the refueling quantities which still are non-optimal. This time, the final destination is reached with the minimum ending fuel level given by the input data. The overall fuel costs are 1857 €.

In step 3 (see Table 13), the fuel costs have been optimized while the lateness from the previous step was prevented from increasing. The travel time was allowed to increase by a maximum of half an hour. In the schedule, the second daily rest period between customer locations 2 and 3 is turned to a regular one. The replacement of the time window chosen for customer stop 3 by the one chosen in step 1 allows another refueling stop on the way to customer 3 without causing lateness. Thus, refueling can take place at the first reachable gas station in Spain (second refueling stop). As the arrival at the final destination is allowed to be 30 minutes later, there is time for an additional refueling stop in Spain between customer location 4 and the final destination. The chosen gas station is the last one in Spain in the filtered list. The complete refueling plan can be described as follows: the driver refuels in France as the next cheaper gas station in Spain would not be reachable, otherwise. The refueling amount is just enough to reach the first gas station in Spain with the minimum fuel quantity allowed in the tank. He fills up completely as there is no cheaper gas station along the route. The last gas station before leaving Spain is used for an additional refueling stop and again, refueling is done until the tank capacity is reached. The last refueling stop is necessary to comply with the predefined ending fuel level. The total fuel costs are 1841 €.

The last optimization step (see Table 14) serves as a postprocessing with the purpose to obtain more comprehensible solutions and to allow more freedom for replanning if necessary or for the continuation after the current planning horizon. Constraints are set up to not worsen lateness, travel time and costs for refueling in this step. Optional rules should only be used if this is advantageous and as late as possible. Waiting time should be reduced to a minimum and arrival times at customer locations should be as early as possible. Additionally, the number of refueling stops and detour durations should be kept low. The last objective function takes account of these criteria. Thus, the second daily rest period is reduced to the minimum duration of a regular daily rest period and waiting time at customer location 3 is omitted. The daily rest period on the route to customer location 4 is extended by 3 minutes such that loading and/or unloading can start at the lower bound of the chosen time window. The arrival at the final destination is 10 minutes earlier than in step 3. The refueling strategy remains the same.

		day	type	from	until	duration
			Step 1			
		day	**type**	**from**	**until**	**duration**
		Mon	(un)load	07:47	09:47	2:00
stop: 1						
chosen TW:	start:	Mon	06:30	end:	Mon	07:00
	lateness:		0:47			
		Mon	drive	09:47	12:03	2:16
		Mon	(un)load	12:03	14:03	2:00
stop: 2						
chosen TW:	start:	Mon	10:30	end:	Mon	11:00
	lateness:			1:03		
		Mon	drive	14:03	16:17	2:14
		Mon	rest	16:17	17:02	0:45
		Mon	drive	17:02	17:58	0:56
		Mon	refuel	17:58	18:18	0:20
		Mon	drive	18:18	20:47	2:29
		Mon	rest	20:47	07:47	11:00
		Tue	drive	07:47	12:17	4:30
		Tue	rest	12:17	13:02	0:45
		Tue	drive	13:02	17:32	4:30
		Tue	rest	17:32	04:32	11:00
		Wed	drive	04:32	09:02	4:30
		Wed	rest	09:02	09:47	0:45
		Wed	drive	09:47	12:27	2:40
		Wed	(un)load	12:27	14:27	2:00
stop: 3						
chosen TW:	start:	Wed	10:30	end:	Wed	11:00
	lateness:		1:27			
		Wed	drive	14:27	16:17	1:50
		Wed	rest	16:17	04:15	11:58
		Thu	drive	04:15	05:00	0:45
		Thu	refuel	05:00	05:20	0:20
		Thu	drive	05:20	06:00	0:40
		Thu	(un)load	06:00	08:00	2:00
stop: 4						
chosen TW:	start:	Thu	06:00	end:	Thu	06:30
	lateness:		0:00			
		Thu	drive	08:00	11:05	3:05
		Thu	rest	11:05	11:50	0:45
		Thu	drive	11:50	16:20	4:30
		Thu	rest	16:20	03:20	11:00
		Fri	drive	03:20	07:50	4:30
		Fri	rest	07:50	08:35	0:45
		Fri	drive	08:35	13:05	4:30
		Fri	rest	13:05	00:05	11:00
		Sat	drive	00:05	02:15	2:10
		Sat	refuel	02:15	02:35	0:20
		Sat	drive	02:35	03:27	0:52
stop: 5						
chosen TW:	start:	Mon	00:00	end:	Sun	23:59
	lateness:		0:00			

Table 11: Schedule from optimization step 1

		Step 2				
		day	**type**	**from**	**until**	**duration**
		Mon	(un)load	07:47	09:47	2:00
stop: 1						
chosen TW:	start:	Mon	06:30	end:	Mon	07:00
	lateness:		0:47			
		Mon	drive	09:47	12:03	2:16
		Mon	(un)load	12:03	14:03	2:00
stop: 2						
chosen TW:	start:	Mon	10:30	end:	Mon	11:00
	lateness:		1:03			
		Mon	drive	14:03	16:17	2:14
		Mon	rest	16:17	17:02	0:45
		Mon	drive	17:02	17:58	0:56
		Mon	refuel	17:58	18:18	0:20
		Mon	drive	18:18	21:52	3:34
		Mon	rest	21:52	06:52	9:00
		Tue	drive	06:52	11:22	4:30
		Tue	rest	11:22	12:07	0:45
		Tue	drive	12:07	16:37	4:30
		Tue	rest	16:37	01:40	9:03
		Wed	drive	01:40	06:10	4:30
		Wed	rest	06:10	06:55	0:45
		Wed	drive	06:55	08:30	1:35
		Wed	(un)load	08:30	10:30	2:00
stop: 3						
chosen TW:	start:	Wed	08:00	end:	Wed	08:30
	lateness:		0:00			
		Wed	drive	10:30	13:05	2:35
		Wed	refuel	13:05	13:25	0:20
		Wed	drive	13:25	13:45	0:20
		Wed	rest	13:45	01:25	11:40
		Thu	drive	01:25	01:45	0:20
		Thu	rest	01:45	02:00	0:15
		Thu	wait	02:00	03:30	1:30
		Thu	(un)load	03:30	05:30	2:00
stop: 4						
chosen TW:	start:	Thu	03:30	end:	Thu	04:00
	lateness:		0:00			
		Thu	drive	05:30	09:40	4:10
		Thu	rest	09:40	10:10	0:30
		Thu	drive	10:10	14:40	4:30
		Thu	rest	14:40	15:25	0:45
		Thu	drive	15:25	16:25	1:00
		Thu	rest	16:25	01:25	9:00
		Fri	drive	01:25	05:55	4:30
		Fri	rest	05:55	06:40	0:45
		Fri	drive	06:40	11:10	4:30
		Fri	rest	11:10	11:55	0:45
		Fri	drive	11:55	12:00	0:05
		Fri	refuel	12:00	12:20	0:20
		Fri	drive	12:20	13:12	0:52
stop: 5						
chosen TW:	start:	Mon	00:00	end:	Sun	23:59
	lateness:		0:00			

Table 12: Schedule from optimization step 2

		day	type	from	until	duration
Step 3						
		Mon	(un)load	07:47	09:47	2:00
stop: 1						
chosen TW:	start:	Mon	06:30	end:	Mon	07:00
	lateness:		0:47			
		Mon	drive	09:47	12:03	2:16
		Mon	(un)load	12:03	14:03	2:00
stop: 2						
chosen TW:	start:	Mon	10:30	end:	Mon	11:00
	lateness:		1:03			
		Mon	drive	14:03	16:17	2:14
		Mon	rest	16:17	17:02	0:45
		Mon	drive	17:02	17:58	0:56
		Mon	refuel	17:58	18:18	0:20
		Mon	drive	18:18	21:52	3:34
		Mon	rest	21:52	06:52	9:00
		Tue	drive	06:52	11:22	4:30
		Tue	rest	11:22	12:07	0:45
		Tue	drive	12:07	16:37	4:30
		Tue	rest	16:37	03:40	11:03
		Wed	drive	03:40	08:10	4:30
		Wed	rest	08:10	08:55	0:45
		Wed	drive	08:55	09:29	0:34
		Wed	refuel	09:29	09:49	0:20
		Wed	drive	09:49	10:50	1:01
		Wed	wait	10:50	11:00	0:10
		Wed	(un)load	11:00	13:00	2:00
stop: 3						
chosen TW:	start:	Wed	10:30	end:	Wed	11:00
	lateness:		0:00			
		Wed	drive	13:00	15:55	2:55
		Wed	rest	15:55	03:05	11:10
		Thu	drive	03:05	03:25	0:20
		Thu	rest	03:25	03:40	0:15
		Thu	(un)load	03:40	05:40	2:00
stop: 4						
chosen TW:	start:	Thu	03:30	end:	Thu	04:00
	lateness:		0:00			
		Thu	drive	05:40	07:02	1:22
		Thu	refuel	07:02	07:22	0:20
		Thu	drive	07:22	10:10	2:48
		Thu	rest	10:10	10:40	0:30
		Thu	drive	10:40	15:10	4:30
		Thu	rest	15:10	15:55	0:45
		Thu	drive	15:55	16:55	1:00
		Thu	rest	16:55	01:55	9:00
		Fri	drive	01:55	06:25	4:30
		Fri	rest	06:25	07:10	0:45
		Fri	drive	07:10	11:40	4:30
		Fri	rest	11:40	12:25	0:45
		Fri	drive	12:25	12:30	0:05
		Fri	refuel	12:30	12:50	0:20
		Fri	drive	12:50	13:42	0:52
stop: 5						
chosen TW:	start:	Mon	00:00	end:	Sun	23:59
	lateness:		0:00			

Table 13: Schedule from optimization step 3

		day	type	from	until	duration
			Step 4			
		day	**type**	**from**	**until**	**duration**
		Mon	(un)load	07:47	09:47	2:00
stop: 1						
chosen TW:	start:	Mon	06:30	end:	Mon	07:00
	lateness:		0:47			
		Mon	drive	09:47	12:03	2:16
		Mon	(un)load	12:03	14:03	2:00
stop: 2						
chosen TW:	start:	Mon	10:30	end:	Mon	11:00
	lateness:		1:03			
		Mon	drive	14:03	16:17	2:14
		Mon	rest	16:17	17:02	0:45
		Mon	drive	17:02	17:58	0:56
		Mon	refuel	17:58	18:18	0:20
		Mon	drive	18:18	21:52	3:34
		Mon	rest	21:52	06:52	9:00
		Tue	drive	06:52	11:22	4:30
		Tue	rest	11:22	12:07	0:45
		Tue	drive	12:07	16:37	4:30
		Tue	rest	16:37	03:37	11:00
		Wed	drive	03:37	08:07	4:30
		Wed	rest	08:07	08:52	0:45
		Wed	drive	08:52	09:26	0:34
		Wed	refuel	09:26	09:46	0:20
		Wed	drive	09:46	10:47	1:01
		Wed	(un)load	10:47	12:47	2:00
stop: 3						
chosen TW:	start:	Wed	10:30	end:	Wed	11:00
	lateness:		0:00			
		Wed	drive	12:47	15:42	2:55
		Wed	rest	15:42	02:55	11:13
		Thu	drive	02:55	03:15	0:20
		Thu	rest	03:15	03:30	0:15
		Thu	(un)load	03:30	05:30	2:00
stop: 4						
chosen TW:	start:	Thu	03:30	end:	Thu	04:00
	lateness:		0:00			
		Thu	drive	05:30	06:52	1:22
		Thu	refuel	06:52	07:12	0:20
		Thu	drive	07:12	10:00	2:48
		Thu	rest	10:00	10:30	0:30
		Thu	drive	10:30	15:00	4:30
		Thu	rest	15:00	15:45	0:45
		Thu	drive	15:45	16:45	1:00
		Thu	rest	16:45	01:45	9:00
		Fri	drive	01:45	06:15	4:30
		Fri	rest	06:15	07:00	0:45
		Fri	drive	07:00	11:30	4:30
		Fri	rest	11:30	12:15	0:45
		Fri	drive	12:15	12:20	0:05
		Fri	refuel	12:20	12:40	0:20
		Fri	drive	12:40	13:32	0:52
stop: 5						
chosen TW:	start:	Mon	00:00	end:	Sun	23:59
	lateness:		0:00			

Table 14: Schedule from optimization step 4

B.2 Run times

The next six tables depict the run times for all 225 test instances for the different filter distances chosen for the preprocessing heuristic. In each table, the instances are categorized according to the number and length of time windows considered. Additionally, the number of customer locations and the overall number of locations is given on the left hand side.

# time windows		base inst.	# loc.	length of time windows in min: 0	30	60	120	600	Σ
	1	1	26	36.85	48.56	35.62	25.30	33.31	179.64
		2	29	101.46	113.79	211.16	114.51	97.31	638.23
		3	32	603.63	901.81	575.85	291.52	475.21	2848.02
		4	29	4.45	5.18	16.86	5.63	4.87	36.98
		5	21	11.48	11.75	9.05	6.29	7.13	45.69
		6	30	16.12	17.93	5.21	14.46	38.07	91.78
		7	29	34.29	21.85	12.00	19.28	19.45	106.87
		8	39	127.13	83.26	92.74	145.24	75.10	523.46
		9	32	21.34	45.23	96.40	35.16	57.02	255.14
		10	33	258.87	61.04	110.33	169.40	154.22	753.86
		11	35	100.20	39.28	38.19	51.26	58.30	287.23
		12	32	35.23	44.84	42.28	86.55	82.42	291.30
		13	30	97.63	64.69	22.92	51.31	69.23	305.78
		14	38	135.58	232.05	111.49	201.99	937.24	1618.35
		15	42	293.27	1511.20	1574.46	1514.27	479.55	5372.74
	Σ			1877.50	3202.45	2954.54	2732.17	2588.41	13355.08
	2	1	26	42.85	29.19	24.70	44.23	28.19	169.15
		2	29	104.71	323.67	69.05	235.05	135.63	868.10
		3	32	128.20	98.31	397.38	106.88	156.05	886.82
		4	29	8.27	18.33	9.48	4.76	11.65	52.49
		5	21	4.68	6.43	4.87	7.10	11.22	34.29
		6	30	18.36	93.13	63.34	54.90	51.12	280.85
		7	29	56.60	14.39	11.31	18.72	19.58	120.59
		8	39	133.68	131.14	120.57	185.80	1799.46	2370.64
		9	32	28.27	28.41	34.87	25.82	34.23	151.59
		10	33	127.47	235.97	246.15	281.28	258.73	1149.60
		11	35	159.59	71.08	50.25	127.36	81.42	489.69
		12	32	126.10	81.12	57.64	39.12	33.99	337.98
		13	30	49.03	39.52	868.01	22.90	244.30	1223.75
		14	38	221.40	216.90	99.42	1513.80	140.95	2192.47
		15	42	614.10	201.04	396.62	1542.69	1071.84	3826.28
	Σ			1823.30	1588.61	2453.64	4210.40	4078.33	14154.28
	3	1	26	33.92	53.51	29.97	45.77	53.92	217.08
		2	29	209.12	258.18	31.34	190.77	644.16	1333.57
		3	32	223.66	85.88	121.41	263.19	325.78	1019.92
		4	29	12.06	9.33	6.29	12.76	8.96	49.39
		5	21	14.82	12.17	7.52	11.40	21.30	67.21
		6	30	24.93	50.92	40.84	35.62	152.60	304.91
		7	29	47.11	16.13	228.62	13.93	26.69	332.48
		8	39	286.98	145.00	235.83	261.43	1938.03	2867.27
		9	32	47.08	29.78	37.46	56.92	240.55	411.80
		10	33	141.67	267.84	505.21	217.08	561.43	1693.22
		11	35	983.09	75.36	101.64	146.22	198.01	1504.32
		12	32	93.10	141.87	121.13	149.30	27.17	532.57
		13	30	67.11	231.57	68.16	128.12	686.59	1181.55
		14	38	440.24	244.58	881.94	3508.53	1780.16	6855.44
		15	42	1576.56	1546.64	219.87	1151.88	2155.95	6650.90
	Σ			4201.44	3168.75	2637.21	6192.91	8821.30	25021.61

Table 15: Filter distance 100 km: Run times in seconds for the MILP model solution process

# time windows		base inst.	# loc.	0	30	60	120	600	Σ
				length of time windows in min					
	1	1	14	2.43	2.26	3.12	2.75	2.98	13.54
		2	18	9.92	8.57	7.35	4.57	4.48	34.88
		3	21	64.09	114.75	77.48	19.75	23.24	299.32
		4	20	2.51	2.45	1.76	1.64	2.57	10.94
		5	16	3.32	2.48	2.35	2.17	2.68	13.01
		6	18	1.76	1.86	0.94	2.95	6.52	14.02
		7	21	25.30	13.48	7.02	8.32	34.94	89.06
		8	26	12.00	9.84	24.38	12.12	10.20	68.55
		9	22	3.57	6.86	5.06	6.99	9.94	32.42
		10	27	51.29	30.33	61.45	59.06	67.07	269.20
		11	26	31.14	38.10	9.16	39.39	55.13	172.91
		12	25	20.72	10.34	20.52	19.11	17.85	88.53
		13	23	53.31	32.71	11.84	27.91	20.87	146.64
		14	29	23.76	36.08	52.32	48.16	45.79	206.11
		15	34	50.76	1508.83	264.31	597.50	1582.46	4003.86
	Σ			355.89	1818.94	549.06	852.38	1886.72	5462.98
	2	1	14	2.50	2.20	2.79	3.43	3.12	14.04
		2	18	3.14	3.29	3.57	3.20	3.85	17.05
		3	21	17.75	36.04	36.30	11.01	23.45	124.55
		4	20	3.34	3.28	1.79	11.22	2.81	22.43
		5	16	2.09	1.47	2.47	2.28	5.24	13.54
		6	18	1.90	3.79	4.18	7.75	9.08	26.71
		7	21	8.05	7.08	4.68	6.26	3.29	29.36
		8	26	14.46	14.42	11.69	20.83	32.98	94.37
		9	22	5.21	6.80	7.57	6.54	8.64	34.76
		10	27	47.05	36.16	41.34	31.75	90.01	246.31
		11	26	107.91	58.56	27.71	43.01	27.16	264.34
		12	25	39.78	30.64	33.79	19.06	10.00	133.27
		13	23	12.46	34.23	17.32	11.73	24.67	100.40
		14	29	34.87	34.23	49.42	10.78	82.78	212.07
		15	34	82.57	451.55	360.27	1512.62	1544.85	3951.85
	Σ			383.07	723.72	604.88	1701.46	1871.92	5285.04
	3	1	14	2.72	2.79	2.76	3.45	3.43	15.15
		2	18	12.78	5.73	9.55	4.79	29.59	62.43
		3	21	24.21	34.53	22.65	17.11	37.92	136.42
		4	20	3.60	4.21	3.67	4.68	3.37	19.53
		5	16	4.52	2.60	2.60	3.93	9.52	23.18
		6	18	4.40	8.19	7.33	5.40	15.26	40.58
		7	21	7.58	12.26	9.00	12.82	32.15	73.82
		8	26	24.38	42.96	30.65	16.66	58.95	173.61
		9	22	3.51	41.57	5.24	9.06	12.15	71.54
		10	27	104.60	111.65	82.42	87.98	263.80	650.45
		11	26	111.74	41.82	28.22	23.37	56.18	261.33
		12	25	23.54	26.46	60.44	32.89	15.35	158.67
		13	23	16.96	19.53	47.55	16.88	52.26	153.18
		14	29	100.32	87.25	83.84	61.93	241.36	574.71
		15	34	482.48	99.59	412.98	182.97	489.19	1667.21
	Σ			927.35	541.16	808.89	483.93	1320.48	4081.81

Table 16: Filter distance 200 km: Run times in seconds for the MILP model solution process

# time windows		base inst.	# loc.	0	30	60	120	600	Σ
				length of time windows in min					
	1	1	13	1.67	1.97	1.92	1.45	2.20	9.20
		2	15	2.39	3.01	3.01	1.42	1.78	11.61
		3	17	10.27	10.55	11.62	3.98	10.31	46.72
		4	18	2.70	2.03	2.34	3.07	2.76	12.90
		5	14	2.23	2.53	1.51	1.96	3.23	11.46
		6	17	1.87	1.90	1.47	2.17	6.69	14.10
		7	19	8.49	4.68	4.09	3.37	3.28	23.90
		8	21	6.02	6.85	9.70	4.76	5.71	33.04
		9	18	1.95	3.74	2.64	2.39	2.09	12.80
		10	24	23.46	23.60	26.30	26.55	29.22	129.14
		11	21	10.61	12.31	10.75	8.89	24.20	66.75
		12	22	7.58	8.86	3.46	10.17	12.18	42.26
		13	21	13.14	14.46	18.08	8.75	11.59	66.02
		14	27	37.78	49.95	38.07	33.38	52.89	212.07
		15	33	77.49	48.81	1520.62	1509.11	1227.73	4383.75
	Σ			207.64	195.24	1655.58	1621.42	1395.85	5075.73
	2	1	13	1.62	1.97	1.69	2.58	2.22	10.07
		2	15	1.51	1.01	1.06	2.03	3.07	8.69
		3	17	2.70	3.96	3.24	4.84	8.75	23.49
		4	18	2.04	2.28	2.06	1.76	1.78	9.92
		5	14	1.83	1.92	2.59	1.86	3.79	11.98
		6	17	2.15	3.73	5.94	9.10	6.58	27.50
		7	19	4.32	4.87	3.54	6.05	3.53	22.31
		8	21	4.60	5.31	7.00	8.56	24.38	49.86
		9	18	2.00	1.84	2.98	2.84	4.84	14.49
		10	24	24.73	37.89	33.63	40.20	36.89	173.35
		11	21	26.05	13.98	11.62	8.36	12.32	72.34
		12	22	22.03	26.33	12.48	32.84	9.41	103.09
		13	21	6.82	6.32	18.60	9.75	17.61	59.10
		14	27	37.53	21.90	21.00	44.02	218.43	342.89
		15	33	1513.61	80.36	252.28	87.53	930.39	2864.18
	Σ			1653.55	213.66	379.71	262.32	1284.00	3793.24
	3	1	13	2.04	2.57	2.56	3.17	2.04	12.39
		2	15	3.62	2.32	2.36	2.31	14.82	25.43
		3	17	7.74	4.74	3.79	9.31	13.14	38.72
		4	18	3.10	3.99	3.25	4.74	3.04	18.13
		5	14	2.03	3.64	2.53	2.81	4.43	15.43
		6	17	4.59	6.58	7.38	5.21	9.34	33.10
		7	19	4.06	5.26	6.37	5.71	6.49	27.88
		8	21	9.23	16.16	12.28	9.50	20.47	67.64
		9	18	3.64	2.34	2.76	12.79	5.62	27.14
		10	24	45.80	102.34	52.48	86.89	86.99	374.49
		11	21	45.02	21.39	24.60	22.82	23.82	137.66
		12	22	29.77	17.60	25.88	23.74	7.58	104.57
		13	21	35.72	12.14	10.30	13.23	51.98	123.37
		14	27	82.85	50.34	52.90	33.31	117.20	336.60
		15	33	121.84	79.80	109.34	115.35	966.83	1393.15
	Σ			401.05	331.21	318.76	350.89	1333.79	2735.69

Table 17: Filter distance 300 km: Run times in seconds for the MILP model solution process

# time windows	base inst.	# loc.	0	30	60	120	600	Σ
			length of time windows in min					
1	1	13	2.09	2.59	2.40	2.00	2.34	11.42
1	2	14	2.08	2.21	3.37	1.76	2.53	11.95
1	3	15	5.07	4.85	8.06	3.10	7.61	28.70
1	4	16	2.17	1.20	1.06	1.81	2.06	8.30
1	5	13	0.91	1.45	1.87	1.12	2.43	7.78
1	6	15	1.44	1.19	0.83	1.25	4.35	9.05
1	7	17	2.90	3.35	2.92	12.86	3.82	25.85
1	8	20	5.23	5.10	5.10	3.67	3.99	23.09
1	9	18	2.06	2.04	1.93	2.40	3.28	11.72
1	10	23	35.71	17.38	18.24	14.77	31.92	118.02
1	11	19	7.60	5.40	7.41	4.80	9.55	34.76
1	12	21	5.86	6.49	7.65	4.45	22.31	46.76
1	13	20	11.23	9.45	3.68	3.14	11.84	39.34
1	14	25	23.85	18.80	28.45	24.12	20.00	115.22
1	15	31	47.00	55.02	133.35	935.40	552.93	1723.70
Σ			155.19	136.53	226.32	1016.65	680.96	2215.65
2	1	13	1.89	2.17	1.90	2.67	2.28	10.91
2	2	14	3.21	1.45	1.58	1.72	2.15	10.11
2	3	15	2.28	2.08	0.91	1.12	4.20	10.58
2	4	16	2.26	1.89	1.31	1.45	1.65	8.57
2	5	13	1.50	1.61	1.37	1.98	2.85	9.31
2	6	15	1.08	3.20	3.53	4.59	4.67	17.05
2	7	17	3.46	1.95	2.89	3.92	3.51	15.72
2	8	20	6.21	4.20	3.87	4.87	12.67	31.81
2	9	18	2.76	2.06	2.25	1.78	3.98	12.82
2	10	23	49.97	28.52	14.06	28.03	27.50	148.08
2	11	19	11.87	8.64	6.90	5.87	7.50	40.78
2	12	21	17.63	8.97	14.43	19.97	7.00	68.00
2	13	20	2.54	13.03	16.64	7.13	12.68	52.03
2	14	25	35.12	60.06	41.51	11.43	26.55	174.67
2	15	31	1521.43	66.47	28.56	181.44	765.70	2563.61
Σ			1663.20	206.28	141.69	277.96	884.90	3174.04
3	1	13	3.04	2.95	2.73	2.92	1.75	13.38
3	2	14	3.15	2.03	1.54	2.23	9.64	18.60
3	3	15	3.76	1.19	4.45	1.65	7.82	18.86
3	4	16	2.47	2.50	2.14	2.59	3.48	13.17
3	5	13	1.97	2.67	1.98	2.55	2.57	11.73
3	6	15	2.31	2.98	3.90	5.07	6.58	20.84
3	7	17	3.64	3.45	6.35	4.46	4.20	22.09
3	8	20	6.43	14.24	24.91	12.23	39.16	96.97
3	9	18	2.79	2.32	2.67	4.48	7.49	19.75
3	10	23	49.98	39.17	49.45	30.00	34.57	203.18
3	11	19	28.11	9.27	10.41	12.17	17.52	77.47
3	12	21	20.67	14.82	19.05	18.10	8.83	81.47
3	13	20	9.84	26.38	66.75	15.21	33.40	151.58
3	14	25	50.09	27.21	48.00	384.82	109.34	619.47
3	15	31	68.53	59.20	37.74	1528.03	1241.88	2935.38
Σ			256.78	210.37	282.07	2026.50	1528.22	4303.93

Table 18: Filter distance 400 km: Run times in seconds for the MILP model solution process

# time windows		base inst.	# loc.	0	30	60	120	600	Σ
				length of time windows in min					
# time windows	1	1	11	1.16	1.04	0.77	0.67	0.72	4.35
		2	14	2.34	2.89	2.82	1.54	1.89	11.48
		3	15	2.97	4.56	8.83	4.37	3.73	24.45
		4	16	1.45	0.83	1.22	1.42	1.93	6.85
		5	12	0.64	0.98	0.58	0.84	2.04	5.09
		6	14	1.12	0.90	0.91	1.11	3.93	7.97
		7	17	2.82	3.37	2.86	12.84	3.84	25.73
		8	19	4.42	4.99	4.31	4.54	3.70	21.95
		9	17	1.97	1.78	1.48	1.95	3.10	10.28
		10	22	5.32	19.66	19.92	15.30	27.05	87.25
		11	19	8.11	6.33	3.46	6.15	9.69	33.75
		12	21	5.80	6.55	7.64	4.42	23.90	48.31
		13	20	16.29	13.51	3.70	5.31	13.38	52.18
		14	24	16.10	15.48	26.58	27.82	15.66	101.64
		15	29	25.85	57.30	175.67	391.70	569.29	1219.81
	Σ			96.35	140.17	260.74	479.97	683.86	1661.08
	2	1	11	0.59	0.64	0.56	0.91	0.72	3.42
		2	14	1.70	0.97	1.90	1.25	1.67	7.49
		3	15	2.62	2.40	5.99	1.67	5.54	18.22
		4	16	1.76	2.18	1.55	1.69	1.37	8.55
		5	12	0.72	0.86	1.20	1.00	2.43	6.21
		6	14	0.78	3.17	2.65	3.09	3.64	13.32
		7	17	3.46	1.98	2.90	3.92	3.48	15.74
		8	19	2.48	2.90	2.48	3.03	13.71	24.60
		9	17	1.50	1.95	1.61	2.15	3.56	10.76
		10	22	19.58	18.27	19.94	19.33	21.98	99.09
		11	19	11.83	9.84	4.63	6.12	6.82	39.24
		12	21	17.74	8.96	14.27	19.91	6.99	67.86
		13	20	5.31	37.11	27.97	15.27	15.40	101.06
		14	24	35.41	27.72	29.41	11.53	31.47	135.53
		15	29	469.66	31.17	42.98	1509.76	112.51	2166.07
	Σ			575.13	150.12	160.04	1600.60	231.27	2717.17
	3	1	11	1.14	1.14	0.98	1.08	0.90	5.24
		2	14	1.53	2.73	2.03	1.67	14.23	22.18
		3	15	2.76	2.03	2.70	2.23	9.66	19.38
		4	16	1.90	3.32	1.76	2.15	1.97	11.11
		5	12	1.73	1.81	1.62	1.59	1.67	8.42
		6	14	2.12	4.48	3.01	2.23	9.47	21.31
		7	17	3.59	3.43	6.36	4.48	4.24	22.11
		8	19	4.77	9.05	5.45	7.82	28.22	55.30
		9	17	2.18	2.54	2.57	2.95	6.12	16.36
		10	22	26.05	49.33	27.96	23.15	21.92	148.40
		11	19	34.68	10.90	12.31	7.30	17.74	82.93
		12	21	20.59	14.95	19.00	19.56	8.77	82.87
		13	20	43.87	21.75	58.72	17.85	48.23	190.41
		14	24	56.69	52.90	33.45	19.59	71.15	233.78
		15	29	71.43	50.95	43.91	1520.84	374.92	2062.05
	Σ			275.04	231.30	221.83	1634.48	619.20	2981.85

Table 19: Filter distance 500 km: Run times in seconds for the MILP model solution process

# time windows		base inst.	# loc.	0	30	60	120	600	Σ
				length of time windows in min					
	1	1	11	1.12	1.03	0.76	0.66	0.72	4.29
		2	13	2.37	1.73	2.32	1.11	0.69	8.22
		3	13	2.58	2.56	2.11	1.17	2.67	11.08
		4	14	1.06	1.09	0.79	1.00	1.41	5.35
		5	12	0.62	1.00	0.56	0.83	1.98	4.99
		6	13	0.89	0.89	0.72	0.90	3.12	6.52
		7	16	3.23	4.04	2.65	3.88	2.46	16.27
		8	19	4.04	4.95	4.29	4.56	3.65	21.49
		9	15	1.16	1.23	1.33	0.97	1.31	5.99
		10	21	17.68	14.37	16.76	10.31	21.17	80.28
		11	16	5.69	9.16	3.46	4.18	5.05	27.55
		12	20	5.72	7.85	6.05	5.76	16.61	42.00
		13	18	4.90	3.98	2.39	2.82	7.61	21.70
		14	24	15.99	15.38	26.69	27.71	15.71	101.48
		15	27	39.83	32.88	57.52	26.18	67.91	224.31
	Σ			106.88	102.13	128.41	92.03	152.07	581.51
	2	1	11	0.59	0.62	0.58	0.89	0.70	3.39
		2	13	0.84	0.87	0.97	0.98	1.36	5.02
		3	13	2.54	1.33	3.49	1.19	3.68	12.23
		4	14	1.37	1.54	1.11	1.56	1.58	7.16
		5	12	0.70	0.89	1.17	1.00	2.44	6.19
		6	13	1.08	2.61	2.36	1.55	3.04	10.62
		7	16	3.63	3.84	5.41	3.73	1.95	18.57
		8	19	2.45	2.92	2.47	3.01	13.73	24.57
		9	15	1.59	1.01	1.28	1.34	2.74	7.97
		10	21	23.65	17.11	14.98	22.47	15.79	93.99
		11	16	7.72	10.62	7.75	5.34	3.57	35.01
		12	20	14.91	9.28	17.41	15.43	6.10	63.14
		13	18	3.85	3.62	7.21	5.37	13.29	33.34
		14	24	35.44	27.77	29.42	11.45	30.61	134.69
		15	27	40.20	49.20	31.31	141.82	73.94	336.48
	Σ			140.59	133.24	126.91	217.11	174.52	792.36
	3	1	11	1.14	1.14	0.97	1.11	0.87	5.23
		2	13	1.20	1.41	1.53	1.79	6.57	12.50
		3	13	1.78	2.18	0.94	1.50	3.18	9.58
		4	14	1.50	2.40	1.27	1.78	1.76	8.70
		5	12	1.75	1.76	1.59	1.59	1.65	8.35
		6	13	2.90	4.28	2.93	2.18	6.02	18.31
		7	16	5.01	2.75	2.43	2.90	4.40	17.49
		8	19	4.70	9.08	5.43	7.69	28.08	54.97
		9	15	1.76	1.17	1.29	2.18	4.18	10.59
		10	21	32.34	22.92	32.95	30.12	20.31	138.63
		11	16	118.19	14.90	9.77	8.27	13.70	164.82
		12	20	12.62	16.63	17.88	24.26	8.56	79.95
		13	18	6.96	4.17	14.06	5.93	26.46	57.57
		14	24	56.50	52.74	33.49	19.69	71.26	233.69
		15	27	28.48	62.98	41.43	47.32	178.54	358.75
	Σ			276.82	200.50	167.95	158.31	375.55	1179.12

Table 20: Filter distance 1000 km: Run times in seconds for the MILP model solution process

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