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# Designing a resilient and responsive supply chain network under disruption risks: A bi-objective stochastic programming approach

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## Abstract

We propose a bi-objective two-stage stochastic formulation for the problem of designing a resilient and agile supply chain network comprising suppliers, potential locations for facilities and customers. As suppliers and facilities are vulnerable to disruption risks, various preparedness and reactive measures are considered. These include contingency procurement by switching to backup sourcing, investment in facility fortification and deferral of customer demand. The latter enhances the supply chain's ability to respond not only to unforeseen disruptions, but also to other sources of uncertainty, such as demand and costs. First-stage decisions define a schedule for facility deployment, the choice of fortification levels in unreliable locations and the selection of primary suppliers as well as backup suppliers. Once uncertainty is disclosed, second-stage decisions determine the activation of backup suppliers and the material flows across the network. The latter may result in delayed deliveries to customers, provided that the delay does not exceed a given threshold. Two conflicting objectives are considered, namely minimising the total expected cost and minimising the total expected unmet demand. We develop a tailored two-phase heuristic procedure that is embedded in the  $\varepsilon$ -constraint method. Our numerical study with randomly generated instances demonstrates the effectiveness of the proposed methodology. Furthermore, a comparative analysis of a representative subset of Pareto-optimal solutions reveals a strong trade-off between alternative network configurations, thereby facilitating the decision-making process.

**Keywords:** Resilient and flexible supply chains, disruption risks, network design, two-stage stochastic programming, MIP-based heuristic

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# 1 Introduction

The risks and uncertainties that supply chain networks face require a planning approach that strikes a balance between resilience and flexibility. While cost efficiency is valuable, it often cannot be achieved at the expense of adaptability (Chopra and Sodhi, 2014). This has been demonstrated by the aftermath of numerous events, whether caused by natural disasters (e.g. hurricanes, floods) or by intentional/unintentional human actions (e.g. pandemic outbreaks, geopolitical conflicts, cyber attacks). So-called *black swan* events (Taleb, 2007) have significant and far-reaching consequences that result in the disruption of large parts (or even the entire) supply chain.

At the strategic planning level, the OR/MS community has increasingly focused over the last two decades on the design of supply chain networks that can withstand disruptive events (Katsaliaki et al., 2022). The recent reviews by Aldrighetti et al. (2021) and Suryawanshi and Dutta (2022) demonstrate the progress made in this field, both in modelling problems and developing methodologies for solving them. In particular, mathematical models have become more comprehensive due to the increased consideration of realistic features. To hedge against disruption risks, two main approaches are usually integrated into these models, namely proactive and reactive strategies. Prominent proactive risk mitigation measures include contracting with multiple suppliers, fortifying facilities and suppliers to reduce their vulnerability to disruption and buffering capacities at various levels of the supply chain (Snyder et al., 2016; Katsaliaki et al., 2022). In contrast, reactive contingency plans provide for adjustments to the operation of the supply chain, taking into account the severity of the disruption. These plans often involve the re-routing of material flows depending on the availability of the remaining resources, such as sharing goods between disrupted and non-disrupted facilities and reallocating customers (Snyder et al., 2016; Nazemi and Parragh, 2022).

In the face of black swan events, designing responsive and agile supply chains has proven to be more valuable than strictly minimising costs or maximising profits (Aldrighetti et al., 2021). Changes in supply and demand require flexible delivery patterns and fulfilment rates, both before and after a disruption occurs. In this paper, we address the design of a resilient supply chain network by combining robustness and flexible strategies to prepare for and handle unforeseen disruptions. Figure 1 shows the general structure of the three-echelon network we consider and highlights the components that are subject to preparedness, recovery and flexibility measures.

At the upstream level (i.e. the supplier layer), backup sourcing is adopted in case of partial or even total failure of the primary suppliers. Contracts with backup suppliers are assumed to be in place prior to any disruption but are activated only for the periods of time during which the primary suppliers face capacity losses. Once the original situation has been restored, backup sourcing is discontinued. The advantages of this form of dual sourcing have been generally recognised (Huang and Xu, 2015). Facility location planning takes place at the intermediate echelon using fortification as a resilience strategy. To this end, two sets of potential

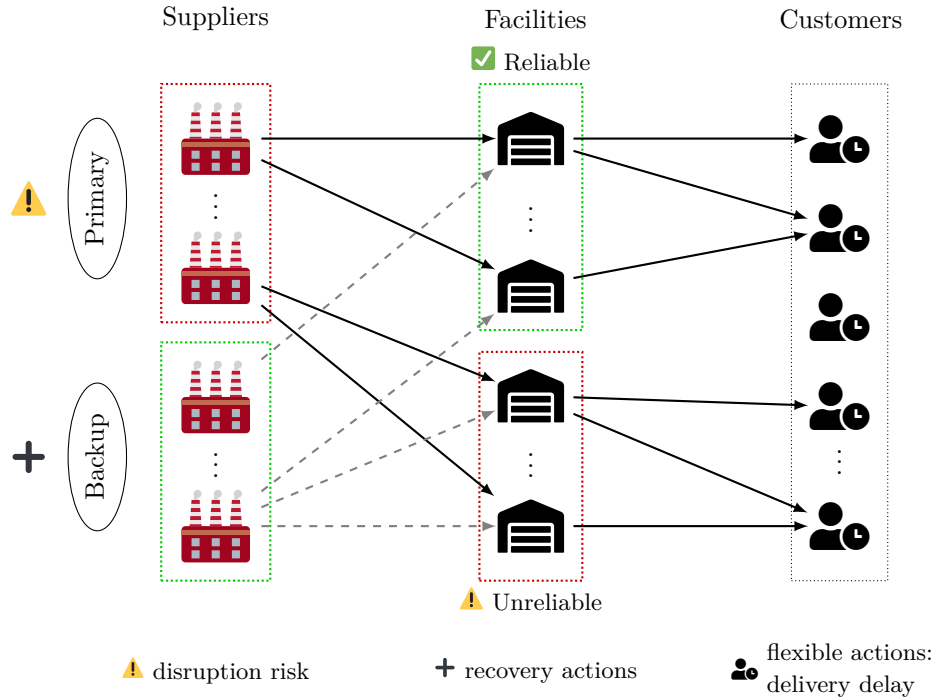


Figure 1: Supply chain network structure.

locations are considered. Some locations are not prone to disruption (reliable set), while others may experience failures (unreliable set), with the extent and duration of their loss of capacity depending on the level of investment in their protection. In their study, Gholami-Zanjani et al. (2021) have shown that facility fortification and contracting with backup suppliers are among the resilience actions having the greatest effect on supply chain robustness. At the lower network level (i.e. customer layer), various measures are introduced to enhance the flexibility and responsiveness of the supply chain. These include reallocating customer demands among the operating facilities, enforcing a minimum customer service level and (partially) meeting demand requirements through delayed deliveries. In particular, Correia and Melo (2021) have found that delays in fulfilling customer orders contribute to the network's ability to adjust to shifting conditions.

In addition to random disruptions at primary suppliers and unreliable facilities over a planning horizon spanning multiple periods, two other sources of uncertainty are also considered in our problem. These involve customer demand and distribution costs across the network. Typically, when making strategic decisions about the choice of suppliers and the deployment of facilities, future demands and shipment costs are not known. Uncertainty is captured by a finite set of scenarios with known probabilities, whereby each scenario represents a joint realisation of the

random parameters over all periods, i.e. demand, distribution costs and disruptive events. The latter specify the network nodes that are affected, the start and end periods of the disruption at each node and the impact on their capacity loss and recovery.

We develop a two-stage stochastic programming formulation to design a supply chain network with the aforementioned characteristics. Two conflicting objectives are considered, namely minimising total expected cost and maximising the network's responsiveness, where the latter translates into minimising total expected unmet demand. Decisions in the first stage include selecting (primary and backup) suppliers, determining a schedule for opening (reliable and unreliable) facilities over the planning horizon and choosing the fortification levels at unreliable locations. These decisions are made before the realisation of the uncertain parameters. Decisions on the flow of goods across the network, late deliveries for (partial) demand fulfilment and the activation of backup suppliers during periods of disruption are deferred to the second stage (post-disruption phase).

The main contributions of our study are fourfold:

- We propose a new mathematical model for a stochastic supply chain network design (SCND) problem capturing a number of features that together have a significant impact on the design of a resilient and flexible network, but which have not received sufficient attention in the OR/MS literature to date. Specifically, the model includes different time scales for decision-making, backup sourcing, various investment options to protect unreliable facilities, a minimum level of customer service and the opportunity to delay shipments to customers. As for the first feature, the strategic decisions involving supplier selection, facility location and facility fortification can be made at a subset of the periods in the planning horizon, while all other tactical decisions concerning the material flow and the activation of backup suppliers can be made at any time period. These settings result in a very challenging problem for which finding Pareto-optimal solutions using an off-the-shelf optimisation solver within acceptable time is rather limited, especially for large-scale instances.
- Therefore, the second contribution of our work is to propose an effective MIP-based heuristic scheme that is embedded in the  $\varepsilon$ -constraint method. At each iteration, the associated  $\varepsilon$ -constraint problem is solved in two phases, first by constructing a feasible solution and then improving it through reverting some of the decisions made previously, using two customised procedures.
- The third contribution of our study is to assess the validity of the new model and the effectiveness of the two-phase heuristic scheme. To this end, we discuss the results of an extensive computational study and compare the heuristic solutions with those obtained by running the  $\varepsilon$ -constraint method exclusively with CPLEX. To facilitate the decision-making process, we also provide managerial insights that highlight the trade-offs achieved

by alternative schedules for designing a resilient and flexible network.

- Finally, the fourth contribution is to demonstrate the potential benefit of a stochastic programming approach over a deterministic counterpart.

The remainder of this paper is organized as follows. Section 2 reviews the relevant literature on resilient SCND problems and identifies research gaps. In Section 3, we formally describe the problem we study and formulate a bi-objective two-stage stochastic programme. Section 4 presents the solution methodology we have developed. Computational results are reported in Section 5. Finally, Section 6 summarises the main findings and discusses directions for future research.

## 2 Review of related literature

The OR/MS literature on supply chain resilience and SCND problems is extensive. In this section, we review relevant articles, focusing on the most recent research that shares several similarities with our problem. To this end, Table 1 presents a list of selected papers (in chronological order) and summarises their main characteristics. For comparison purposes, the last row of the table classifies our study according to the nine categories indicated in the table. All the articles listed in this table have in common the integration of facility location planning into the decision space, as this is a key feature. This can be seen in column 4, which gives the number of layers in the supply chain network at which location decisions are made. This information is also related to column 2, which shows the total number of tiers in the network. Most authors consider three-echelon networks (12), as is the case in our problem, followed by two-echelon networks (9), which is in line with the findings of Aldrighetti et al. (2021). While the intermediate and lower tiers concern facilities (e.g. warehouses, distribution centres) and customers, respectively, the articles dealing with three-tier networks differ in the way they handle the upper echelon. When the latter represents plants (Baghalian et al., 2013; Jalali et al., 2016; Ramshani et al., 2019), processing centres (Gholami-Zanjani et al., 2021) or other types of facilities (Cheng et al., 2018), these can also be the subject of location planning decisions along with the facilities in the intermediate layer. On the other hand, when suppliers are included in the upper tier, as in our case, it is natural for facility location decisions to concern the intermediate echelon (Benyoucef et al., 2013; Toloie et al., 2020; Namdar et al., 2021; Nayeri et al., 2022; Aldrighetti et al., 2023).

Column 3 in Table 1 reveals that about one-third of the selected articles define the SCND problem over a multi-period finite planning horizon. This feature is of great importance for modelling the occurrence, frequency, duration and impact of disruptive events on the network. However, in the multi-period models developed by Benyoucef et al. (2013), Toloie et al. (2020), Sabouhi et al. (2021) and Gholami-Zanjani et al. (2021), all (strategic) facility location

Table 1: Classification of SCND problems under disruption risks.

Reference	Network layers		Multiple periods		Decision planning		Occurrence of		Proactive/reactive measures		Uncertain parameters			Modelling approach <sup>3</sup>	Solution approach <sup>4</sup>
	layers	layers	location	other <sup>1</sup>	of	disruptions <sup>2</sup>	sourc.*	measures	demand	costs	other	Number of objectives			
													fort.**		
Baghalian et al. (2013)	3		2	Dist, LS		Links				*			SP	Solver	
Benyoucef et al. (2013)	3		1	Prc, Inv, Dist		Sup			*				SP	SAA	
Azad et al. (2014)	2	*	1	Dist, TM		Fac, Links			*				CP	H	
Jalali et al. (2016)	3		2	Dist, LS		Fac			*				SP	H	
Jabbarzadeh et al. (2016)	2		1	Dist		Fac			*				L-R	Solver	
Fattahi et al. (2017)	3		1	Prc, Inv, Dist, LS		Fac			*				SP	CC	
Cheng et al. (2018)	3		2	Dist, LS		Fac			*				SP, RO	CC	
Afiy et al. (2019)	2		1	Dist		Fac			*				MILP	H	
Azad and Hassini (2019)	2		1	Dist, LS		Fac, Dem			*				SP	BD	
Ramshani et al. (2019)	3		2	Dist, LS		Fac, Links			*				SP	BD	
Snoeck et al. (2019)	2	*	1	Dist, Inv, LS		Fac			*				SP	SAA, H	
Bhuiyan et al. (2020)	2		1	Dist, L, LS		Fac, Links			*				SP	BD	
Sabouhi et al. (2020)	4		1	Prc, Dist, LS	*	Sup, Fac, Links			*				SP	BD	
Tolooie et al. (2020)	3	*	1	Prc, Dist, LS		Fac			*				SP	BD	
Alikhani et al. (2021)	4		2	Prc, Dist, L, LS		Fac, Links			*				SP	Solver	
Cheng et al. (2021)	2		1	Dist, LS		Fac			*				RO	CC	
Gholami-Zanjani et al. (2021)	3	*	2	Dist, Inv, LS		Fac			*				SP	BD	
Hasani et al. (2021)	4	*	2	Prc, Prd, Dist, Inv, TM, LS		Sup, Fac			*				RO	H	
Namdar et al. (2021)	3		1	Prc, Dist, LS		Sup, Fac			*				FP	Solver	
Nayeri et al. (2021)	4		2	Prc, Prod, Dist, TM, LS		Fac			*				FP	MCGP	
Sabouhi et al. (2021)	5	*	2	Prc, Prod, Dist, Inv, LS		Fac, Sup, Links			*				SP	BD	
Nayeri et al. (2022)	3		1	Prc, Dist, Inv, LS		Fac			*				FP	H	
Nazemi and Parragh (2022)	2		1	Dist, LS		Fac			*				SP	$\epsilon$	
Alidrihetti et al. (2023)	3	*	1	Prc, Dist, LS		Sup, Fac			*				SP	BD, H	
Alikhani et al. (2023)	4		2	Prc, Dist, Inv, L, LS		Sup, Fac			*				SP	SAA	
Badyal et al. (2023)	3	*	1	Dist, Inv, LS		Fac			*				SP	BD, LD	
Chen and Chen (2023)	4		2	Prc, Dist, Inv, L, LS		Sup, Fac			*				RO	Solver	
Koca (2023)	2		1	Dist, LS		Fac			*				SP	BD	
Pu et al. (2024)	4	*	1	Prc, Dist, Inv, LS		Sup, Fac			*				SP	H	
<b>New model</b> (cf. Section 3)	3	*	1	Prc, Dist, LS		Sup, Fac			*				SP	$\epsilon$ , BD, H	

sourc.\*: measures involving suppliers; fort.\*\*: facility fortification; delays\*\*\*: late deliveries to customers  
<sup>1</sup> Dist: Distribution; Inv: Inventory; L: Arc selection; LS: Lost sales (unmet demand); Prc: Procurement; Prd: Production; TM: Transport mode selection  
<sup>2</sup> Dem: Customer demand; Fac: Facilities; Links: Arcs in the network; Sup: Suppliers  
<sup>3</sup> CP: Cone programming; FP: Fuzzy programming; MILP: Mixed-integer linear programming; MINLP: Mixed-integer non-linear programming; RO: Robust optimisation; SP: Stochastic programming  
<sup>4</sup> BD: Benders decomposition; CC: Column-and-constraint method;  $\epsilon$ :  $\epsilon$ -constraint method; H: (Math/Meta)heuristic; L-R: Level decomposition; L-R: Lagrangian relaxation; MCGP: Multi-choice goal programming; SAA: Sample Average Approximation; Solver: General-purpose optimisation solver

decisions are made at a single point in time, namely at the beginning of the planning horizon, while tactical decisions, such as the allocation of customer demand to facilities, can be made over time. We note that this strategy strongly limits the design of the network as opposed to a multi-period setting for strategic decisions. In the model to be presented in Section 3, we consider distinct time scales for strategic and tactical decisions. To this end, tactical decisions regarding the flow of goods across the network and the deployment of backup suppliers can be made in any time period, while strategic supplier selection and location decisions can only be made at selected time periods. This feature has not been captured by any of the multi-period models listed in Table 1. Correia and Melo (2021) have shown that this approach is valid when decisions involving sizeable investments (e.g. for opening new facilities and protecting them) are combined with medium-term decisions.

In addition to decisions about the location and distribution of goods, other supply chain operations are also modelled, such as procurement and production planning, inventory management, and transport mode selection (see column 5). In particular, holding safety stock and transferring inventory from non-disrupted to disrupted facilities are used as reactive resilience actions by some authors (Nazemi and Parragh, 2022; Alikhani et al., 2023). A few studies also address the construction of arcs in the network in preparation for a possible failure of transport routes (Bhuiyan et al., 2020; Alikhani et al., 2021, 2023; Chen and Chen, 2023). Decisions on supplier selection and procurement are primarily considered when the upper echelon of the network represents suppliers. However, sourcing disruptions are not often modelled as shown in column 6. As a result, only a few authors include preparedness measures in their models in anticipation of supply losses (see column 7, 'sourc.'). One straightforward action is to subcontract an external entity from which materials can be purchased at a higher cost (Chen and Chen, 2023). Supplier fortification is another option (Hasani et al., 2021; Sabouhi et al., 2021; Alikhani et al., 2023), but one we have not adopted in our work because we assume that the supply chain is operated by a retailer who owns the facilities where products purchased from various suppliers are stored (cf. Figure 1). It is therefore not the retailer's responsibility to invest in measures to protect suppliers against disruption risks. Instead, we resort to backup sourcing, a strategy that is also followed by Sabouhi et al. (2020), Hasani et al. (2021), Namdar et al. (2021), Aldrighetti et al. (2023) and Pu et al. (2024). Unlike these authors, who use multiple sourcing, we adopt a dual sourcing policy during periods of disruption. This means that each facility is assigned to a single primary supplier and a second (backup) supplier is only activated when the former undergoes capacity losses. While many companies adopt this resilience strategy (Snyder et al., 2016; Düerkop and Huth, 2023), to the best of our knowledge, our work is the first to integrate these conditions into an SCND problem, which involves formulating a set of specific constraints.

According to column 6 in Table 1, in almost all the papers reviewed, facilities are exposed to disturbances that may cause their total or partial failure. Facility fortification is therefore a proactive measure that is often adopted (see column 8, 'fort.'). The impact of the disruption

in terms of capacity loss depends on the amount of investment for strengthening the facility, which in some works is limited by an available budget (Jabbarzadeh et al., 2016; Afify et al., 2019; Bhuiyan et al., 2020; Cheng et al., 2021; Badyal et al., 2023). Similar to Azad et al. (2014), Jabbarzadeh et al. (2016), Azad and Hassini (2019), Bhuiyan et al. (2020), Toloie et al. (2020) and Nazemi and Parragh (2022), we also divide the potential locations for opening new facilities into two sets, i.e. reliable and unreliable locations. Reliable locations require a higher investment to avoid failure, while protection systems are deployed at unreliable locations in the pre-disruption phase. In addition to the disruption of facilities and/or suppliers, a small number of articles model the failure of transport routes (see column 6). In this case, the reallocation of material flows and the selection of alternative links or other modes of transport are among the most frequently used reactive contingency strategies.

Supply chain networks should respond to customer needs before and after a disruption. As can be seen in column 5, most of the models examined allow for demand shortfalls in the event of disruptions, but incur penalty costs to mitigate this situation. However, as Sabouhi et al. (2020) and Koca (2023) point out, this approach does not limit the amount of unmet demand and could lead to insufficient responsiveness from the supply chain. To overcome this shortcoming, these authors impose a minimum level of customer service. Nayeri et al. (2021, 2022) follow a similar strategy, but minimise deviations from a pre-specified target for satisfying demand. By contrast, Nazemi and Parragh (2022) minimise the conditional value at risk associated with unfilled demand. In our study, we include three responsiveness measures, namely we allow delays in demand fulfilment provided lateness does not exceed a given threshold, set a minimum service level on every time period and minimise the total amount of shortages over the planning horizon. As highlighted in column 9 ('delays'), the first measure has not yet been incorporated into any model, and our work represents the first contribution to further enhancing the network's responsiveness with this strategy.

In addition to disruption risks, supply chain networks are also exposed to other sources of uncertainty, such as demand, supply and cost rates (Suryawanshi and Dutta, 2022). Therefore, columns 10–12 in Table 1 provide information on the uncertain parameters considered in the selected papers. Interestingly, only around half of the authors assume future demand to be unknown. Note that demand fluctuations also affect the network configuration, an aspect that we take into account (see Section 3). Moreover, the vast majority of the proposed models rely on complete information on cost factors. The category 'other' (column 12) includes all the exogenous sources of uncertainty that are usually triggered by disruptive events, e.g. total/partial loss of capacity at various tiers of the network (nodes and/or arcs). Due to the complexity of designing a supply chain in the event of unforeseen disturbances, it is not surprising that 86 percent (25/29) of the mathematical formulations reviewed have a single objective function (column 13). Some authors convert multiple decision criteria into one objective by assigning weights to the individual criteria (Sabouhi et al., 2021). In our work, we go further by considering two conflicting objectives, namely the minimisation of the total expected cost and the

maximisation of the network's responsiveness, and handling them accordingly.

Finally, stochastic programming is a widely used modelling technique (column 14), and in particular scenario-based two-stage stochastic programmes with recourse are a common approach, as in our model. In far fewer problems, robust optimisation or fuzzy programming are employed to address uncertainty. The last column in Table 1 shows that a variety of exact and heuristic solution methods have been developed for this family of SCND problems. Algorithms based on Benders decomposition or heuristic techniques are used in more than half of the works. Due to the bi-objective nature of our problem, we resort to the  $\varepsilon$ -constraint method and solve the individual sub-problems by Benders decomposition with an embedded heuristic scheme (see Section 4).

Despite the significant advances in modelling and solving SCND problems under disruption risks, there are still relevant features that have not been considered together so far, which makes our contribution original. The last row of Table 1 and the review conducted in this section highlight the research gaps we address in this paper.

### **3 Problem statement and mathematical modelling**

The general setting under which our problem is studied and the assumptions made are described and motivated in the following.

- A greenfield planning approach is adopted for designing the three-echelon network depicted in Figure 1 over a multi-period finite planning horizon. The network is operated by a retailer who purchases a range of products from various suppliers, stores them in facilities (e.g. warehouses) and delivers them to different customers. Without loss of generality, these products are grouped into a single product family since they share the same storage, handling and transport resources.
- Location decisions concern the second tier in the network. Following the setting proposed by Snyder and Daskin (2007) and Lim et al. (2010), and that has also been adopted by other authors (Jabbarzadeh et al., 2016; Nazemi and Parragh, 2022; Tolooie et al., 2020), two types of candidate locations are considered: reliable locations and unreliable locations. While the latter are likely to be partially or totally disrupted after they have been established, the former are not subject to disruption, but are more costly. Candidate sites are assumed to be geographically dispersed, especially in the case of unreliable locations, to reduce the vulnerability of the network to potential disruption risks. In addition, an investment can be made in fortifying unreliable facilities (e.g. by installing cyber security equipment or constructing earthquake-resistant buildings) as a proactive measure against future disruptions. The higher the level of fortification, the lower the capacity loss of the facility in the event of a disruption. However, a high level of fortification requires

greater investment in both its deployment and operation, but it reduces the time needed to restore the original operational capacity of the facility. A reliable facility, on the other hand, is located in a safe area and its capacity is assumed to be available at all times.

- The supplier level includes primary and backup suppliers. Each operating facility negotiates a contract in advance with a primary supplier to deliver a family of products over the planning horizon. Contracts are also signed with backup suppliers in preparation for possible disturbances at the primary suppliers. Disruptions are reflected in a partial or even total loss of capacity of the latter. A dual-sourcing policy is therefore adopted in order to avoid supply shortages even though it comes at a higher cost. Typically, supplier monitoring, transaction and quality control costs are higher with dual sourcing than with single sourcing (Düerkop and Huth, 2023). It is thus assumed that distribution costs from backup suppliers are more expensive than from primary suppliers. Moreover, supplier segregation (Hosseini et al., 2019; Jabbarzadeh et al., 2018; Namdar et al., 2021) is assumed by geographically separating primary and backup suppliers. The latter are located in safe locations and are only deployed to supplement capacity shortages at primary suppliers.
- The last tier in the network includes a set of customers with unknown demand for a single product family. Disruptions to facilities can be handled by the late delivery of customer orders, provided that the delay does not exceed a certain threshold. In this case, however, penalty costs are incurred for the late fulfilment of demand. Despite this reactive measure, in a more extreme situation of a massive loss of capacity of primary suppliers after a disaster, it may not be possible - even resorting to backup suppliers - to fully satisfy the demand of one or several customers. This will inevitably lead to lost sales. Nevertheless, a minimum level of customer service must be guaranteed at all times, which means that a pre-specified proportion of total demand must be met at every period of the planning horizon. Although the assumption of lost sales is often considered in the literature dedicated to resilient SCND problems (Aldrighetti et al., 2021), delays in deliveries to customers is an option that has not been included as a reactive measure in any study until now.
- Customer demand is uncertain, as is the extent to which capacity losses at primary suppliers and unreliable facilities occur over the planning horizon as a result of disruptions. Moreover, distribution costs across the network are also uncertain, as transportation routes may be affected, thereby resulting in higher costs for choosing alternative routes or alternative transport modes. The uncertain parameters are represented by a random vector, say  $\xi$ , with a finite discrete support. Accordingly,  $\xi$  has a joint probability distribution function that is assumed to be known in advance (e.g. estimated from historical data using predictive modelling techniques, Bhuiyan et al. (2020)). The joint realisations of the uncertain parameters are thereby represented by a finite set of scenarios with known

probabilities.

- Due to the strategic nature of location decisions, the latter can be made in a subset of the periods in the planning horizon, which are termed *strategic periods*. All other decisions (i.e. deployment of backup suppliers and the material flow across the network) can be made in any time period. This setting has also been adopted by other authors (e.g., Correia and Melo (2021); Mohamed et al. (2020)), but not in the context of SCND in the presence of disruptions.
- Besides minimising the total expected cost of designing the supply chain network, the minimisation of the total unmet demand to improve the network's responsiveness is also considered. These two objectives are conflicting because delivery delays and demand shortfalls can only be avoided through higher capital expenditure in facility fortification and procurement from backup suppliers.

The aim of our problem is to define a schedule for opening facilities, selecting suppliers and distributing a product family from suppliers to customers via the operating facilities over the planning horizon.

### 3.1 Notation

Sets:

$T$	Set of discrete time periods in the planning horizon, with $n_T$ denoting the last period.
$T_L$	Set of <i>strategic</i> time periods, $T_L \subset T$ .
$I^1$	Set of primary suppliers ( $I^1 \neq \emptyset$ ).
$I^2$	Set of backup suppliers ( $I^2 \neq \emptyset$ ).
$I$	Set of all suppliers, $I = I^1 \cup I^2$ , $I^1 \cap I^2 = \emptyset$ .
$J^r$	Set of candidate sites for locating reliable facilities ( $J^r \neq \emptyset$ ).
$J^u$	Set of candidate sites for locating unreliable facilities ( $J^u \neq \emptyset$ ).
$J$	Set of all candidate sites, $J = J^r \cup J^u$ , $J^r \cap J^u = \emptyset$ .
$F_j$	Set of fortification levels for an unreliable facility $j \in J^u$ , sorted from lowest to highest.
$K$	Set of customers.
$S$	Set of scenarios.

Without loss of generality, we assume that  $t = 1$  is always the first strategic period and define  $\Delta$  as the total number of periods between two consecutive strategic periods, i.e.  $\Delta = |T|/|T_L|$ , considering that  $|T|$  is a multiple of  $|T_L|$ . For example, for a 12-month time horizon with quarterly opportunities for opening facilities, we have  $|T| = 12$ ,  $T_L = \{1, 4, 7, 10\}$  and  $\Delta = 3$ .

A further assumption is that the two location sets  $J^r$  and  $J^u$  are geographically distant from each other, thereby precluding the opening of all facilities in the same region, especially when there is a significant disruptive risk in that region. Furthermore, a geographical dispersion within set  $J^u$  is also assumed for the same reason.

*Capacities and related parameters:*

- $\bar{Q}_i$  Regular capacity of supplier  $i \in I$  in a time period.
- $a_{its}$  Proportion of the regular capacity of primary supplier  $i \in I$  that is lost in time period  $t \in T$  under scenario  $s \in S$  ( $0 \leq a_{its} \leq 1$ ).
- $Q_j^r$  Regular capacity of reliable facility  $j \in J^r$  in a time period.
- $Q_{jf}^u$  Regular capacity of unreliable facility  $j \in J^u$  with fortification level  $f \in F_j$  in a time period.
- $b_{jfts}$  Proportion of the regular capacity of unreliable facility  $j \in J^u$  with fortification level  $f \in F_j$  that is lost in time period  $t \in T$  under scenario  $s \in S$  ( $0 \leq b_{jfts} \leq 1$ ).
- $\pi_s$  Probability of scenario  $s \in S$  occurring,  $\sum_{s \in S} \pi_s = 1$ .

*Demand-related parameters:*

- $d_{kts}$  Demand of customer  $k \in K$  in time period  $t \in T$  under scenario  $s \in S$ .
- $\rho_k$  Maximum number of periods tolerated by customer  $k \in K$  for late deliveries.
- $sl_t$  Minimum customer service level in time period  $t$ , expressed as the minimum proportion of total demand in period  $t$  that must be satisfied in any scenario,  $0 < sl_t \leq 1$ .

*Costs:*

- $FC_{jt}^r$  Fixed cost of opening a reliable facility at candidate location  $j \in J^r$  at the beginning of the strategic time period  $t \in T_L$ .
- $FC_{jft}^u$  Fixed cost of opening an unreliable facility at candidate location  $j \in J^u$  with fortification level  $f \in F_j$  at the beginning of the strategic time period  $t \in T_L$ .
- $OC_{jt}^r$  Fixed cost of operating the reliable facility  $j \in J^r$  in time period  $t \in T$ .
- $OC_{jft}^u$  Fixed cost of operating the unreliable facility  $j \in J^u$  with fortification level  $f \in F_j$  in time period  $t \in T$ .
- $AC_{jft}^u$  Additional fixed cost associated with the damage caused by the total disruption of unreliable facility  $j \in J^u$  with fortification level  $f \in F_j$  in time period  $t \in T$ .
- $SC_{ijts}$  Cost of distributing one unit of product from supplier  $i \in I$  to facility  $j \in J$  at time period  $t \in T$  under scenario  $s \in S$ .
- $DC_{jkts}$  Cost of distributing one unit of product from facility  $j \in J$  to customer  $k \in K$  at time period  $t \in T$  under scenario  $s \in S$ .

$TC_{ktt'}$  Tardiness penalty cost for delivering one unit of product to customer  $k \in K$  in time period  $t'$  that was originally demanded for period  $t \in T$ , with  $t' = t, t + 1, \dots, \min\{t + \rho_k, n_T\}$ . Naturally,  $TC_{ktt'} = 0$  for  $t' = t$ .

Observe that the distribution costs ( $SC, DC$ ) are indexed in the scenario set. This makes it possible to take into account the impact of disruptions on transport links. For example, if both a primary supplier  $i$  and an unreliable facility  $j$  experience capacity losses in some period  $t$  under a given scenario  $s$  (i.e.  $0 < a_{its} < 1$  and  $0 < b_{jfts} < 1$ ) then it may be necessary to use a more costly alternative transport route or transport mode.

### 3.2 Two-stage stochastic formulation

We cast our problem as a bi-objective two-stage stochastic programme. The *ex ante* or first-stage decisions determine the timing of the opening of facilities, the choice of the fortification level for unreliable facilities and the selection of (primary and backup) suppliers over the planning horizon, before the uncertainty about disruptions, customer demand and distribution costs is revealed. Once disturbances have been observed at primary suppliers and unreliable facilities, and demand realisation has occurred, *ex post* or second-stage decisions are made that specify the material flow across the network and the amount of unserved demand in each time period.

Considering all the features previously described about the uncertain parameters, as well as the notation introduced above, we directly present the extended form of the deterministic equivalent model and describe it as a bi-objective mixed-binary linear programme. The decision variables in this model are divided into two groups, as shown below.

First-stage decision variables:

$$z_{ijt}^1 = 1 \text{ if primary supplier } i \text{ is assigned to facility } j \text{ in strategic time period } t, \text{ and } 0 \text{ otherwise } (i \in I^1, j \in J, t \in T_L). \quad (1)$$

$$z_{ijt}^2 = 1 \text{ if backup supplier } i \text{ is assigned to facility } j \text{ in time period } t, \text{ and } 0 \text{ otherwise } (i \in I^2, j \in J, t \in T). \quad (2)$$

$$y_{jt}^r = 1 \text{ if a reliable facility is opened in candidate location } j \text{ at the beginning of the strategic time period } t, \text{ and } 0 \text{ otherwise } (j \in J^r, t \in T_L). \quad (3)$$

$$y_{jft}^u = 1 \text{ if an unreliable facility is opened in candidate location } j \text{ at the beginning of the strategic time period } t \text{ with fortification level } f, \text{ and } 0 \text{ otherwise } (j \in J^u, f \in F_j, t \in T_L). \quad (4)$$

Second-stage decision variables:

$$v_{ijts} : \text{Amount of product distributed from supplier } i \text{ to facility } j \text{ at time period } t \text{ under scenario } s \ (i \in I, j \in J, t \in T, s \in S). \quad (5)$$

$$x_{jkt't's} : \text{Amount of product distributed from facility } j \text{ to customer } k \text{ at time period } t' \text{ to (fully/partially) satisfy demand of period } t \text{ under scenario } s \ (j \in J, k \in K, t \in T, t' = t, t+1, \dots, \min\{t + \rho_k, n_T\}, s \in S). \quad (6)$$

Our problem is modeled with the following objective functions.

$$\begin{aligned} \text{Min } w_1 = & \sum_{j \in J^r} \sum_{t \in T_L} FC_{jt}^r y_{jt}^r + \sum_{j \in J^u} \sum_{f \in F_j} \sum_{t \in T_L} FC_{jft}^u y_{jft}^u + \\ & \sum_{j \in J^r} \sum_{t \in T} OC_{jt}^r \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{j\tau}^r + \sum_{j \in J^u} \sum_{f \in F_j} \sum_{t \in T} OC_{jft}^u \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{jft\tau}^u + \\ & \sum_{s \in S} \pi_s \left( \sum_{j \in J^u} \sum_{f \in F_j} \sum_{t \in T} b_{jfts} (AC_{jft}^u - OC_{jft}^u) \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{jft\tau}^u + \right. \\ & \left. \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} SC_{ijts} v_{ijts} + \right. \\ & \left. \sum_{j \in J} \sum_{k \in K} \sum_{t \in T} \sum_{t' = \max\{1, t - \rho_k\}}^t (DC_{jkt's} + TC_{kt't}) x_{jkt't's} \right) \end{aligned} \quad (7)$$

$$\text{Min } w_2 = \sum_{s \in S} \pi_s \sum_{k \in K} \sum_{t \in T} \left( d_{kts} - \sum_{j \in J} \sum_{t'=t}^{\min\{t+\rho_k, n_T\}} x_{jkt't's} \right) \quad (8)$$

The objective function (7) minimises the total expected cost over the planning horizon. This comprises fixed costs for opening and operating (reliable and unreliable) facilities as well as costs associated with damages to disrupted facilities and distribution costs across the network. Without loss of generality, damage costs are assumed to be proportional to the scale of the disruption, in which case the facility operating costs decrease in the same proportion. If an unreliable facility  $j$  with fortification level  $f$  fails completely in a given period  $t$  under scenario  $s$ , i.e.  $b_{jfts} = 1$ , then its fixed operating cost is not charged and only the damage cost is incurred. The objective function (8) is a responsiveness-related criterion that minimises the total expected amount of unmet demand. The competing nature of objectives (7) and (8) arises from the trade-off between investing in resilience and contingency measures and meeting customer demand.

Next, we present the various constraints in the mathematical formulation grouped by type of activity in the distribution network.

### Choice of facility locations

$$\sum_{t \in T_L} y_{jt}^r \leq 1 \quad j \in J^r \quad (9)$$

$$\sum_{t \in T_L} \sum_{f \in F_j} y_{jft}^u \leq 1 \quad j \in J^u \quad (10)$$

Constraints (9) and (10) ensure that at most one facility is opened at a candidate location over the planning horizon. Constraints (10) also state that a single level of fortification can be installed in an unreliable facility. Furthermore, and due to the sizeable investment, once opened, a facility cannot be closed and later reopened.

### Dual sourcing

$$\sum_{i \in I^1} z_{ijt}^1 = y_{jt}^r \quad j \in J^r, t \in T_L \quad (11)$$

$$\sum_{i \in I^1} z_{ijt}^1 = \sum_{f \in F_j} y_{jft}^u \quad j \in J^u, t \in T_L \quad (12)$$

$$\sum_{i \in I^2} z_{ijt}^2 \leq \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{j\tau}^r \quad j \in J^r, t \in T \quad (13)$$

$$\sum_{i \in I^2} z_{ijt}^2 \leq \sum_{\tau \in T_L: 1 \leq \tau \leq t} \sum_{f \in F_j} y_{jft}^u \quad j \in J^u, t \in T \quad (14)$$

$$\sum_{i' \in I^1} a_{i'ts} \sum_{\tau \in T_L: 1 \leq \tau \leq t} z_{i'j\tau}^1 \leq \sum_{i \in I^2} z_{ijt}^2 \quad j \in J, t \in T, s \in S \quad (15)$$

$$\sum_{s \in S} M_{ts} \sum_{i' \in I^1} a_{i'ts} \sum_{\tau \in T_L: 1 \leq \tau \leq t} z_{i'j\tau}^1 \geq \sum_{i \in I^2} z_{ijt}^2 \quad j \in J, t \in T \quad (16)$$

Constraints (11) and (12) dictate that each open facility is assigned to exactly one primary supplier in each strategic period. Observe that such a facility retains its primary supplier throughout the planning horizon due to inequalities (9) and (10). Constraints (13) and (14) ensure that, in any given period, at most a second supplier, from the backup pool, can be assigned to an open facility. Recall that the choice of backup suppliers is part of the preparedness measures and therefore the associated variables  $\mathbf{z}^2$  are not scenario-indexed. Furthermore, shipments from a backup supplier are only allowed in periods when the primary supplier assigned to a given facility faces capacity losses due to disruptions associated with a particular scenario  $s$ . This condition is imposed by constraints (15) and (16) together with inequalities (13) and (14). On the left-hand side of (16),  $M_{ts}$  denotes a sufficiently large positive constant (additional details are given below). Suppose that facility  $j$  is opened in some strategic period  $\tau \in T_L$  and thus, it will operate in periods  $\tau, \tau + 1, \dots, n_T$ . It follows that the right-hand side of either the

corresponding constraint (13) or (14) is equal to 1, depending on the facility type. According to condition (11), resp. (12), there is a primary supplier, say  $v'$ , such that  $z_{v'j\tau}^1 = 1$  (and  $z_{v'jt'}^1 = 0$  for  $t' \in T_L \setminus \{\tau\}$ ). As a result, the left-hand side of the associated constraints (15) becomes  $a_{v'ts}$  for every  $t = \tau, \tau + 1, \dots, n_T$  and  $s \in S$ . If this primary supplier is disrupted in period  $t$  of scenario  $s \in S$ , i.e.  $0 < a_{v'ts} \leq 1$ , then from (15) together with (13), resp. (14), it follows that  $\sum_{i \in I^2} z_{ijt}^2 = 1$ , indicating that exactly one backup supplier must be assigned to facility  $j$  in period  $t$ . On the other hand, if the primary supplier  $v'$  is not disrupted in period  $t$  under scenario  $s$  then  $a_{v'ts} = 0$  and thus, inequality (15) is redundant. Moreover, if  $a_{v'ts} = 0$  for all scenarios  $s$  then from (16), facility  $j$  does not need a backup supplier in time period  $t$ .

For any period  $t$  and scenario  $s$ , a suitable value for  $M_{ts}$  on the left-hand side of constraints (16) is  $(\min\{a_{its} : i \in \Upsilon_{ts}\})^{-1}$ , where  $\Upsilon_{ts}$  is the subset of primary suppliers affected by capacity losses in period  $t$  under scenario  $s$ , i.e.  $\Upsilon_{ts} = \{i \in I^1 : a_{its} > 0\}$ . Naturally, if  $\Upsilon_{ts} = \emptyset$  then we take  $M_{ts} = 1$ . Therefore,  $M_{ts} a_{v'ts} \geq 1$  when  $a_{v'ts} > 0$ .

We point out that, in order to provide flexibility in the event of disruptions, the assignment of a backup supplier to a given facility may differ from one period to the next.

### Flow conservation conditions

$$\sum_{i \in I} v_{ijts} = \sum_{k \in K} \sum_{t' = \max\{1, t - \rho_k\}}^t x_{jkt'ts} \quad j \in J, t \in T, s \in S \quad (17)$$

For each facility, time period and scenario, constraints (17) set the link between supply and customer deliveries. The left-hand side of these constraints determines the total flow into the facilities, while the right-hand side gives the total outflow. Clearly, both quantities must be equal.

### Capacity constraints

$$\sum_{j \in J} v_{ijts} \leq (1 - a_{its}) \bar{Q}_i \quad i \in I^1, t \in T, s \in S \quad (18)$$

$$\sum_{j \in J} v_{ijts} \leq \bar{Q}_i \quad i \in I^2, t \in T, s \in S \quad (19)$$

$$\sum_{s \in S} v_{ijts} \leq |S| \bar{Q}_i \sum_{\tau \in T_L: 1 \leq \tau \leq t} z_{ij\tau}^1 \quad i \in I^1, j \in J, t \in T \quad (20)$$

$$\sum_{s \in S} v_{ijts} \leq |S| \bar{Q}_i z_{ijt}^2 \quad i \in I^2, j \in J, t \in T \quad (21)$$

$$\sum_{k \in K} \sum_{t' = \max\{1, t - \rho_k\}}^t x_{jkt'ts} \leq Q_j^r \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{j\tau}^r \quad j \in J^r, t \in T, s \in S \quad (22)$$

$$\sum_{k \in K} \sum_{t'=\max\{1, t-\rho_k\}}^t x_{jkt'ts} \leq \sum_{f \in F_j} (1 - b_{jfts}) Q_{jf}^u \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{jf\tau}^u \quad j \in J^u, t \in T, s \in S \quad (23)$$

Inequalities (18)–(19) guarantee that the capacities of the suppliers are not exceeded. Capacity losses at primary suppliers as a result of disruptions are taken into account in (18). Constraints (20)–(21) ensure that if a particular supplier is not assigned to a facility, there can be no flow of goods between these two entities. Capacity conditions at the reliable and unreliable facilities are imposed by constraints (22) and (23), respectively.

### Customer requirements

$$\sum_{j \in J} \sum_{t'=t}^{\min\{t+\rho_k, n_T\}} x_{jkt't's} \leq d_{kts} \quad k \in K, t \in T, s \in S \quad (24)$$

$$\sum_{j \in J} \sum_{k \in K} \sum_{t'=t}^{\min\{t+\rho_k, n_T\}} x_{jkt't's} \geq s\ell_t \sum_{k \in K} d_{kts} \quad t \in T, s \in S \quad (25)$$

Constraints (24) require that deliveries to customers, including those with a delay, do not exceed the demand in each period. Inequalities (25) enforce a minimum demand fulfillment rate in every period and every scenario.

### Conditions on decision variables

$$y_{jt}^r \in \{0, 1\} \quad j \in J^r, t \in T_L \quad (26)$$

$$y_{jft}^u \in \{0, 1\} \quad j \in J^u, f \in F_j, t \in T_L \quad (27)$$

$$z_{ijt}^1 \in \{0, 1\} \quad i \in I^1, j \in J, t \in T_L \quad (28)$$

$$z_{ijt}^2 \in \{0, 1\} \quad i \in I^2, j \in J, t \in T \quad (29)$$

$$v_{ijts} \geq 0 \quad i \in I, j \in J, t \in T, s \in S \quad (30)$$

$$x_{jkt't's} \geq 0 \quad j \in J, k \in K, t \in T, t' = t, t+1, \dots, \min\{t + \rho_k, n_T\}, s \in S \quad (31)$$

Constraints (26)–(29) enforce integrality conditions on the binary variables, while constraints (30) and (31) impose non-negativity on the other decision variables. Finally, we note that constraints (9)–(14) represent the first-stage conditions, while constraints (15)–(25) are the second-stage conditions associated with the realisation of each scenario.

### 3.3 Additional inequalities

Recall that  $\Delta$  indicates the total number of periods between two consecutive strategic periods, i.e.  $\Delta = |T|/|T_L|$ . Moreover, let  $\rho_{\max}$  be the longest delivery lead time to a customer, i.e.  $\rho_{\max} = \max\{\rho_k : k \in K\}$ . When  $\rho_{\max} < \Delta$ , the following constraints can be added to formulation (7)–(31) to ensure that, in each scenario, the selected reliable and unreliable facilities provide sufficient capacity in each period of the planning horizon.

$$\begin{aligned} & \Delta \sum_{j \in J^r} Q_j^r \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{j\tau}^r + \\ & \sum_{j \in J^u} \sum_{f \in F_j} \sum_{t' = t}^{t + \Delta - 1} (1 - b_{jft's}) Q_{jf}^u \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{jf\tau}^u \\ & \geq \sum_{t' = t}^{t + \Delta - \rho_{\max} - 1} sl_{t'} \sum_{k \in K} d_{kt's} \quad t \in T_L, s \in S \end{aligned} \quad (32)$$

Although inequalities (32) are redundant for the linear relaxation, they may tighten formulation (7)–(31). For the particular case where no delays in deliveries to customers are allowed, i.e.  $\rho_k = 0$  for every  $k \in K$ , the previous inequalities can be simplified as follows.

$$\begin{aligned} & \sum_{j \in J^r} Q_j^r \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{j\tau}^r + \sum_{j \in J^u} \sum_{f \in F_j} (1 - b_{jfts}) Q_{jf}^u \sum_{\tau \in T_L: 1 \leq \tau \leq t} y_{jf\tau}^u \\ & \geq sl_t \sum_{k \in K} d_{kts} \quad t \in T, s \in S \end{aligned} \quad (33)$$

## 4 Solution methodology

Scalarisation-based techniques are popular approaches for solving multi-objective optimisation problems and among these, the  $\varepsilon$ -constraint method is a classic exact procedure that has been successfully applied to a wide range of problems (Ehrgott, 2005; Keller, 2017). When it comes to a bi-objective optimisation problem, as in our case, a sequence of single-objective sub-problems is solved, with each sub-problem having the original set of constraints, extended with an additional inequality that imposes a suitable bound  $\varepsilon$  on the other objective function. By varying the values for  $\varepsilon$  in an appropriate way, the Pareto front of the bi-objective optimisation problem can be generated iteratively. In the case of our problem, a preliminary computational study revealed that it is computationally less demanding to minimise the total expected cost ( $w_1$ ) than to minimise the total expected unfilled demand ( $w_2$ ). For this reason, in our application of the  $\varepsilon$ -constraint method, all sub-problems have the objective function (7), while the added constraint involves the second objective (8) bounded from above.

Let  $(\mathcal{P}_1)$  denote the minimisation sub-problem with the mean cost function (7) and con-

straints (9)–(31), along with the additional inequalities (32). In a similar way, let  $(\mathcal{P}_2)$  be the minimisation sub-problem with the mean unsatisfied demand function (8) and constraints (9)–(32). The optimal values of  $(\mathcal{P}_1)$  and  $(\mathcal{P}_2)$  are denoted by  $w_1^-$  and  $w_2^-$ , respectively, and together they represent the ideal vector. Upper bounds on the two objectives,  $w_1^+$  and  $w_2^+$ , defining the anti-ideal vector, provide additional information that is also relevant to a decision-maker. These bounds are easily determined from the optimal solutions to  $(\mathcal{P}_1)$  and  $(\mathcal{P}_2)$ . Accordingly,  $w_2^+$  is the total expected unfilled demand in the optimal solution to  $(\mathcal{P}_1)$ , while  $w_1^+$  is the total expected cost in the optimal solution to  $(\mathcal{P}_2)$ . Clearly, the objective space of  $w_2$  is bounded from below by  $w_2^-$  and bounded from above by  $w_2^+$ . This information is used to define the variation scheme for  $\varepsilon$ . To this end, the interval  $\mathcal{I} = [w_2^-, w_2^+]$  is divided into a fixed number  $\mu \in \mathbb{Z}_+$  of non-overlapping sub-intervals with equal width. The endpoint of each of these sub-intervals (except the last one,  $w_2^+$ ) defines the upper bound of an  $\varepsilon$ -constraint.

For a given value of  $\varepsilon$ , to avoid obtaining weakly efficient solutions, the  $\varepsilon$ -constraint (i.e.  $w_2 \leq \varepsilon$ ) is replaced by  $w_2 + q = \varepsilon$ , with  $q$  denoting a non-negative slack variable. In addition, the objective function of the associated  $\varepsilon$ -constraint problem is perturbed by taking

$$w_1 - \frac{\delta q}{w_2^+ - w_2^-}$$

where  $\delta$  is a sufficiently small positive constant (Mavrotas, 2009). In our numerical experiments, we set  $\delta = 0.0001$ .

Parameter  $\mu$  controls the number of compromise Pareto-optimal solutions to be identified, thus assisting a decision-maker in evaluating the different trade-offs between the two objectives. In addition, the two lexicographic solutions are also determined. Obtaining the latter proved to be computationally expensive, particularly for our large test instances. To overcome this shortcoming, the bi-objective problem is reformulated using weighted sum scalarisation, i.e.  $\lambda w_1 + (1 - \lambda)w_2$  with  $\lambda > 0$  (Miettinen, 2012), subject to constraints (9)–(32). In our numerical study, an approximation to the left lexicographic solution is identified by taking  $\lambda \approx 1$ , while an approximation to the right lexicographic solution is calculated with  $\lambda \approx 0$ . Moreover, the optimal solutions to these two problems are also used to determine the lower bounds  $w_1^-$  and  $w_2^-$ , and the upper bounds  $w_1^+$  and  $w_2^+$ . This means that the objective functions of problems  $(\mathcal{P}_1)$  and  $(\mathcal{P}_2)$  are replaced by the scalarisation presented before, giving rise to problem  $(\mathcal{P}'_1)$  for  $\lambda = 0.9999$  and to problem  $(\mathcal{P}'_2)$  for  $\lambda = 0.0001$ . These two values of  $\lambda$  were also chosen to determine the (approximate) lexicographic solutions. Algorithm 1 describes the scheme for identifying a total of  $\mu + 1$  Pareto-optimal solutions.

Observe that our problem belongs to the class of two-stage mixed-binary stochastic programmes with binary variables in the first stage (master problem) and continuous recourse variables (second-stage sub-problems). Benders decomposition (Benders, 1962; Rahmaniani et al., 2017) applied to this class of problems, also known as the L-shaped method (Van Slyke and Wets, 1969), has proved effective in solving a wide variety of problems, including those

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**Algorithm 1:** Identification of Pareto-optimal solutions with the  $\varepsilon$ -constraint method

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**Input** : All instance data;  $\mu$   
**Output:** Set of Pareto-optimal solutions,  $\Lambda$

- 1 determine the *left* (approximate) lexicographic solution  $\mathcal{S}_1^*$  by solving  $(\mathcal{P}'_1)$
- 2 obtain the projection  $w_2^+$  of  $\mathcal{S}_1^*$  on the  $w_2$ -axis
- 3 determine the *right* (approximate) lexicographic solution  $\mathcal{S}_2^*$  and its optimal objective value  $w_2^-$  by solving  $(\mathcal{P}'_2)$
- 4  $\Lambda \leftarrow \{\mathcal{S}_1^*\} \cup \{\mathcal{S}_2^*\}$  // initialisation of the efficient set
- 5 **for**  $m := 1$  **to**  $\mu - 1$  **do**
- 6      $\varepsilon \leftarrow w_2^- + m \cdot \frac{w_2^+ - w_2^-}{\mu}$
- 7     obtain optimal solution  $\mathcal{S}_\varepsilon^*$  to  $(\mathcal{P}_1)$  with the perturbed objective function  $w_1 - \delta q \cdot (w_2^+ - w_2^-)^{-1}$  and extended with constraints  $w_2 + q = \varepsilon$  and  $q \geq 0$
- 8      $\Lambda \leftarrow \Lambda \cup \{\mathcal{S}_\varepsilon^*\}$  // updating the efficient set
- 9 **end**
- 10 **return**  $\Lambda$

---

arising in the context of network design and facility location planning (e.g. Gholami-Zanjani et al. (2021); Gómez-Rocha et al. (2024); Toloie et al. (2020)). Therefore, we solve the multiple single-objective sub-problems in Algorithm 1 with this method.

Due to their combinatorial nature, it is very challenging to identify (near-)optimal solutions to the  $\varepsilon$ -constraint problems in acceptable time, particularly as the total number of time periods, scenarios, suppliers, potential facilities and customers increases. For this reason, we have developed a two-phase heuristic approach that starts by applying a procedure to construct a feasible initial solution. In the second phase, two mechanisms are employed that allow local changes to be made to the initial selection of facilities and the suppliers allocated to them, so as to obtain an improved solution. Our solution scheme belongs to the class of Mixed Integer Programming (MIP) based heuristics, an approach that has gained popularity due to significant advances in the development of general-purpose optimisation solvers (Boschetti and Maniezzo, 2024).

#### 4.1 Construction of a feasible solution

Since the structure of the distribution network is mainly driven by the choices made as to which facilities should be operating over the planning horizon, our construction scheme fixes the values of the location variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  successively, from the first to the last strategic period. Specifically, we employ a *forward* approach that involves solving a sequence of sub-problems, each of which partially covering the planning horizon. Recall that  $\Delta$  indicates the number of time periods between two consecutive strategic periods. Then, the first sub-problem

involves the first  $\Delta$  time periods, the second sub-problem considers the first  $2\Delta$  time periods and so on. Clearly, the last sub-problem spans the entire planning horizon. As a result, a total of  $|T_L|$  iterations are performed.

In the optimal solution to each sub-problem, all the location decisions are retained for the strategic periods belonging to that sub-problem. However, due to the myopic nature of our heuristic, a similar procedure is not applied to the assignment of primary and backup suppliers to operating facilities. In particular, decisions made in the first few periods regarding the choice of suppliers can prove costly and/or reduce deliveries to customers when unreliable facilities and primary suppliers are disrupted at a later stage in the planning horizon. As these decisions cannot be reversed once they have been made, we have opted not to fix them until the last sub-problem has been solved. This approach also applies to the second-stage variables,  $\boldsymbol{v}$  and  $\boldsymbol{x}$ , whose values are only definite after executing the last iteration.

Given that the fixing of binary variables evolves from one sub-problem to the next, it is necessary to keep this information up to date. For this purpose, different sets are used, the notation for which is shown in Table 2. As the number of iterations grows, the sets with the free variables gradually decrease in size, while the cardinality of the sets with the fixed binary variables rises. Even though each sub-problem increases in size as the algorithm moves forward (mainly due to an increasing number of variables  $\boldsymbol{v}$  and  $\boldsymbol{x}$ ), the computational burden does not grow disproportionately, as will be reported in Section 5, because more and more variables become fixed. Consequently, the successive decomposition of the original problem allows optimal solutions to be obtained in reasonable computing time with the L-shaped method.

Table 2: Sets that store binary variables in the construction heuristic.

description	sets of binary variables	
	free	fixed
reliable facilities / unreliable facilities	$\tilde{Y}^r / \tilde{Y}^u$	$\tilde{\tilde{Y}}^r / \tilde{\tilde{Y}}^u$
primary suppliers / backup suppliers	$\tilde{Z}^1 / \tilde{Z}^2$	$\tilde{\tilde{Z}}^1 / \tilde{\tilde{Z}}^2$

Algorithm 2 describes the general framework for constructing a feasible solution. Each iteration, which is associated with a given strategic period  $t \in T_L$ , starts with obtaining the optimal solution to a restricted  $\varepsilon$ -constraint problem (line 8). The optimal network configuration from the first time period until the end of time period  $t + \Delta - 1$  is identified. All facility location decisions associated with the strategic period  $t$  are retained (given that the location decisions for previous strategic periods were fixed in the preceding iterations). In addition, decisions made in time period  $t$  can also affect future location decisions as well as the fixing of some of the variables concerning primary and backup suppliers provided they will not restrict future choices. The associated rules are defined in procedures `ReliableFacilities` and

UnreliableFacilities. Figure 2 illustrates the sub-problems that are sequentially solved for a planning horizon with 24 periods, of which periods 1, 7, 13 and 19 are strategic periods ( $\Delta = 6$ ).

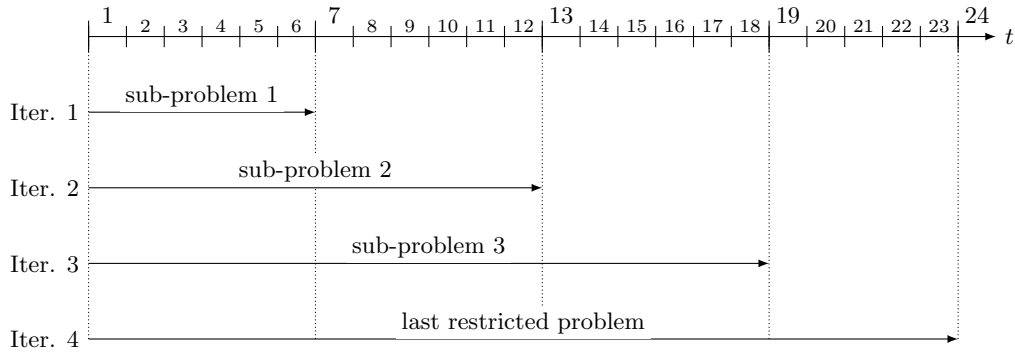


Figure 2: Construction of a feasible solution by solving four (sub-)problems sequentially when  $|T| = 24$  and  $T_L = \{1, 7, 13, 19\}$ .

Since the sub-problems to be solved (line 8), except for the last one, do not span the entire planning horizon, the current upper bound  $\varepsilon$  on the objective function  $w_2$  needs to be adjusted. This adjustment reduces  $\varepsilon$  in proportion to the number of time periods considered in the current restricted problem. This means that, at each iteration,  $w_2$  is bounded from above by  $\varepsilon \cdot (t + \Delta - 1)/n_T$ .

Procedure *ReliableFacilities* is called after a sub-problem spanning the periods  $1, \dots, t + \Delta - 1$  has been solved optimally. Accordingly,  $t$  identifies the last strategic period considered. For the reliable facilities that get the value ‘0’ in the solution to the sub-problem (i.e. they are not opened in period  $t$ ), it is clear that no primary supplier can be assigned to them in  $t$  as imposed by constraints (11) (line 5). Moreover, backup suppliers will not serve these locations in the next  $\Delta$  periods (line 7). Conversely, if a reliable facility is opened in time period  $t$ , then in all subsequent strategic periods the associated location variables take on the value 0 (line 13), thus ensuring the satisfaction of constraints (9). Although the choice of the primary supplier to be assigned to this facility from time period  $t$  onwards was made when solving the sub-problem, this decision will not be retained as it could prevent from finding a better solution. Therefore, the associated binary variables  $z_{ijt}^1$  ( $i \in I^1$ ) remain free in all subsequent sub-problems and the supplier selection only becomes definite once the last restricted problem has been solved. In addition, no primary supplier can be assigned to the facility in any strategic period after  $t$  (line 15). Regarding the selection of one or several backup suppliers, this is also made flexible, with the only requirement being that none of them be assigned to the facility before it is opened, in line with constraints (13). Note that this condition was satisfied when the sub-problem before the last was solved (line 7). A similar set of rules is also imposed for fixing

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**Algorithm 2:** Construction of a feasible solution

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**Input** : All instance data

**Output:** Feasible solution,  $\mathcal{S}$

- 1 initialize  $\tilde{Y}^r$  with all variables  $y_{jt}^r$  ( $j \in J^r, t \in T_L$ )
- 2 initialize  $\tilde{Y}^u$  with all variables  $y_{jft}^u$  ( $j \in J^u, f \in F_j, t \in T_L$ )
- 3 initialize  $\tilde{Z}^1$  with all variables  $z_{ijt}^1$  ( $i \in I^1, j \in J, t \in T_L$ )
- 4 initialize  $\tilde{Z}^2$  with all variables  $z_{ijt}^2$  ( $i \in I^2, j \in J, t \in T$ )
- 5 create empty sets  $\tilde{Y}^r, \tilde{Y}^u, \tilde{Z}^1, \tilde{Z}^2$
- 6  $\Delta \leftarrow \lceil |T|/|T_L| \rceil$  // no. of periods between two consecutive strategic periods
- 7 **for**  $t \in T_L$  **do**
- 8 solve partial  $\varepsilon$ -constraint problem for periods  $1, \dots, t + \Delta - 1$  with the free binary variables belonging to sets  $\tilde{Y}^r, \tilde{Y}^u, \tilde{Z}^1, \tilde{Z}^2$  and the already fixed binary variables belonging to sets  $\tilde{\tilde{Y}}^r, \tilde{\tilde{Y}}^u, \tilde{\tilde{Z}}^1, \tilde{\tilde{Z}}^2$
- 9 // use the optimal solution to this restricted problem for variable fixing
- 10 ReliableFacilities( $t$ )
- 11 UnreliableFacilities( $t$ )
- 12 **end**
- 13 **return** feasible solution  $\mathcal{S} = (\mathbf{y}^r, \mathbf{y}^u, \mathbf{z}^1, \mathbf{z}^2, \mathbf{v}, \mathbf{x})$

---

binary variables associated with unreliable facilities, see procedure UnreliableFacilities.

## 4.2 Improvement heuristics

In this section, we present two local search heuristics that perform a sequence of perturbations on a feasible solution in an attempt to identify a better solution. Since location decisions are at the core of our problem, both schemes allow decisions previously made about reliable and unreliable facilities to be revoked. At each iteration, a neighbourhood of the incumbent solution is explored according to a particular strategy. Whenever a better solution is identified, it becomes the new incumbent and the search proceeds by exploring the neighbourhood of the improved solution. This process is repeated until a pre-specified stopping criterion is met, which depends on the perturbation scheme employed.

### 4.2.1 Partial fixing of location decisions

Given an incumbent solution  $\mathcal{S}$  and its objective value  $\mathcal{V}(\mathcal{S})$ , the first improvement strategy retains all location decisions for a pre-specified number of consecutive strategic periods, say  $\kappa \in \mathbb{Z}_+$  ( $1 \leq \kappa \leq |T_L| - 1$ ). The restricted  $\varepsilon$ -constraint problem that results from fixing a subset of the variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  is then solved. In this way, all the remaining decisions can

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**Algorithm ReliableFacilities:** Manage variables related to *reliable* facilities in strategic period  $t$  using the solution to the last sub-problem solved

---

**Input :** Current strategic period  $t$ ; optimal values of variables regarding the choice of reliable facilities in the last sub-problem solved,  $\mathbf{y}^r$ ; current sets  $\tilde{Y}^r, \tilde{\tilde{Y}}^r, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$

**Output:** Fixing of some binary variables  $\mathbf{y}^r, \mathbf{z}^1, \mathbf{z}^2$ ; updated sets  $\tilde{Y}^r, \tilde{\tilde{Y}}^r, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$

```

1 for  $j \in J^r$  do
2   if  $y_{jt}^r$  belongs to  $\tilde{Y}^r$  and  $y_{jt}^r = 0$  then
3     move  $y_{jt}^r$  from  $\tilde{Y}^r$  to  $\tilde{\tilde{Y}}^r$  // variable fixing
4     // no assignment of a primary supplier to facility  $j$  in strategic period  $t$ 
5      $z_{ijt}^1 \leftarrow 0$  for all  $i \in I^1$  and move these variables from  $\tilde{Z}^1$  to  $\tilde{\tilde{Z}}^1$ 
6     // no assignment of backup suppliers to facility  $j$  in the next  $\Delta$  periods
7      $z_{ijt'}^2 \leftarrow 0$  for all  $i \in I^2$  and  $t' = t, \dots, t + \Delta - 1$ , move these variables from  $\tilde{Z}^2$  to  $\tilde{\tilde{Z}}^2$ 
8   end
9   if  $y_{jt}^r = 1$  then
10    // facility  $j$  is opened in strategic period  $t$ 
11    move  $y_{jt}^r$  from  $\tilde{\tilde{Y}}^r$  to  $\tilde{Y}^r$  // variable fixing
12    // no closing and reopening of this facility later in the planning horizon
13     $y_{jt'}^r \leftarrow 0$  for all  $t' \in T_L : t' > t$  and move these variables from  $\tilde{Y}^r$  to  $\tilde{\tilde{Y}}^r$ 
14    // no assignment of a primary supplier to facility  $j$  after  $t$ 
15     $z_{ijt'}^1 \leftarrow 0$  for all  $i \in I^1$  and  $t' \in T_L : t' > t$ , move these variables from  $\tilde{Z}^1$  to  $\tilde{\tilde{Z}}^1$ 
16  end
17 end
18 return  $\mathbf{y}^r, \mathbf{z}^1, \mathbf{z}^2, \tilde{Y}^r, \tilde{\tilde{Y}}^r, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$ 

```

---

be identified, i.e. the suppliers to be selected, the material flow across the network and other facilities to be opened in periods not included in the aforementioned variable fixing procedure. Depending on the value of parameter  $\kappa$ , it can be computationally demanding to find the optimal solution to the restricted problem. This hurdle is overcome by using the incumbent solution as starting solution in the L-shaped method.

The variable fixing strategy does not guarantee that the optimal solution to the restricted  $\varepsilon$ -constraint problem will be of higher quality than the incumbent. Therefore, in order to diversify the search, different choices of  $\kappa$  consecutive strategic periods are explored for which location decisions are retained from the incumbent solution. Accordingly, we visit the set of strategic periods sequentially and, at each time period  $t \in T_L$ , we examine the impact of keeping the location decisions for that period and the following  $\kappa - 1$  strategic periods. For example, for  $|T| = 24$ ,  $T_L = \{1, 7, 13, 19\}$  and  $\kappa = 2$ , this strategy requires solving three restricted problems, each of which has the binary variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  fixed as in the incumbent solution, but for a different pair of strategic periods, namely (1, 7), (7, 13) and (13, 19). If, in the course of this

---

**Algorithm UnreliableFacilities:** Manage variables related to *unreliable* facilities in strategic period  $t$  using the solution to the last sub-problem solved

---

**Input :** Current strategic period  $t$ ; optimal values of variables regarding the choice of unreliable facilities in the last sub-problem solved,  $\mathbf{y}^u$ ; current sets  $\tilde{Y}^u, \tilde{\tilde{Y}}^u, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$

**Output:** Fixing of some binary variables  $\mathbf{y}^u, \mathbf{z}^1, \mathbf{z}^2$ ; updated sets  $\tilde{Y}^u, \tilde{\tilde{Y}}^u, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$

```

1 for  $j \in J^u$  do
2   if  $y_{jft}^u$  belongs to  $\tilde{Y}^u$  for all  $f \in F_j$  and  $\sum_{f \in F_j} y_{jft}^u = 0$  then
3     // no assignment of a primary supplier to facility  $j$  in strategic period  $t$ 
4      $z_{ijt}^1 \leftarrow 0$  for all  $i \in I^1$  and move these variables from  $\tilde{Z}^1$  to  $\tilde{\tilde{Z}}^1$ 
5     // no assignment of backup suppliers to facility  $j$  in the next  $\Delta$  periods
6      $z_{ijt'}^2 \leftarrow 0$  for all  $i \in I^2$  and  $t' = t, \dots, t + \Delta - 1$ , move these variables from  $\tilde{Z}^2$  to  $\tilde{\tilde{Z}}^2$ 
7   end
8   for  $f \in F_j$  do
9     if  $y_{jft}^u$  belongs to  $\tilde{\tilde{Y}}^u$  and  $y_{jft}^u = 0$  then
10      | move  $y_{jft}^u$  from  $\tilde{Y}^u$  to  $\tilde{\tilde{Y}}^u$  // variable fixing
11    end
12    if  $y_{jft}^u = 1$  then
13      | // facility  $j$  is opened in strategic period  $t$  with fort. level  $f$ 
14      | move  $y_{jft}^u$  from  $\tilde{Y}^u$  to  $\tilde{\tilde{Y}}^u$ 
15    end
16  end
17  if  $\sum_{f \in F_j} y_{jft}^u = 1$  then
18    // no closing and reopening of this facility later in the planning horizon
19     $y_{jft'}^u \leftarrow 0$  for all  $f \in F_j$  and  $t' \in T_L : t' > t$ , move these variables from  $\tilde{Y}^u$  to  $\tilde{\tilde{Y}}^u$ 
20    // no assignment of a primary supplier to facility  $j$  after  $t$ 
21     $z_{ijt'}^1 \leftarrow 0$  for all  $i \in I^1$  and  $t' \in T_L : t' > t$ , move these variables from  $\tilde{Z}^1$  to  $\tilde{\tilde{Z}}^1$ 
22  end
23 end
24 return  $\mathbf{y}^u, \mathbf{z}^1, \mathbf{z}^2, \tilde{Y}^u, \tilde{\tilde{Y}}^u, \tilde{Z}^1, \tilde{\tilde{Z}}^1, \tilde{Z}^2, \tilde{\tilde{Z}}^2$ 

```

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procedure, an improved solution is identified, the incumbent is updated. In this case, the next restricted problem to be solved uses the new incumbent as starting solution to expedite the optimisation process. Returning to the previous example, suppose that the optimal solution to the restricted problem associated with the pair (1, 7) is better than the initial solution. Then, in the next iteration, variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  are fixed with the values of the new incumbent for periods 7 and 13.

Algorithm 3 describes this local search procedure. Let  $\phi(p)$  be a function that identifies

the  $p$ -th strategic period in  $T_L$ . Taking the previous example,  $\phi(1) = 1$  for the first strategic period,  $\phi(2) = 7$  for the second strategic period and so on. Moreover, the inverse function is also defined, e.g.  $\phi^{-1}(13) = 3$  means that time period 13 is the third strategic period. Therefore, given  $t \in T_L$ ,  $\phi^{-1}(t)$  indicates the rank of period  $t$  in  $T_L$  (line 3), which in turn is used to identify the next  $\kappa - 1$  strategic periods for which variable fixing takes place (lines 7–8). For example, for  $\kappa = 2$  and  $t = 7$ , we have  $\phi^{-1}(7) = 2$  and the pair of periods to be considered is  $(\phi(2), \phi(3)) = (7, 13)$ .

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**Algorithm 3:** Improvement heuristic 1 with partial fixing of location decisions

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**Input :** All instance data;  $\kappa$ ; feasible solution  $\bar{S} = (\bar{\mathbf{y}}^r, \bar{\mathbf{y}}^u, \bar{\mathbf{z}}^1, \bar{\mathbf{z}}^2, \bar{\mathbf{v}}, \bar{\mathbf{x}})$

**Output:** Feasible solution,  $S$

```

1  $S \leftarrow \bar{S}$ ,  $\mathcal{V}(S) \leftarrow \mathcal{V}(\bar{S})$  // incumbent solution and its objective value
2 for  $t \in T_L$  do
3    $p \leftarrow \phi^{-1}(t)$  // find the position of strategic period  $t$  in  $T_L$ 
4   if  $p + \kappa - 1 \leq |T_L|$  then
5     set all variables  $\mathbf{y}^r, \mathbf{y}^u, \mathbf{z}^1, \mathbf{z}^2, \mathbf{v}, \mathbf{x}$  as free
6     // fix decisions on reliable and unreliable facilities for  $\kappa$  consecutive
       strategic periods from incumbent solution
7      $y_{j\tau}^r \leftarrow \bar{y}_{j\tau}^r$  for all  $j \in J^r$  and  $\tau = \phi(p), \phi(p+1), \dots, \phi(p+\kappa-1)$ 
8      $y_{j\tau}^u \leftarrow \bar{y}_{j\tau}^u$  for all  $j \in J^u, f \in F_j$  and  $\tau = \phi(p), \phi(p+1), \dots, \phi(p+\kappa-1)$ 
9     find optimal solution  $S'$  to restricted  $\varepsilon$ -constraint problem with previously fixed variables
       and all other variables free, using incumbent solution as starting solution
10    if  $\mathcal{V}(S') < \mathcal{V}(S)$  then // update incumbent solution
11       $\bar{\mathbf{y}}^r \leftarrow \mathbf{y}^r, \bar{\mathbf{y}}^u \leftarrow \mathbf{y}^u, \bar{\mathbf{z}}^1 \leftarrow \mathbf{z}^1, \bar{\mathbf{z}}^2 \leftarrow \mathbf{z}^2, \bar{\mathbf{v}} \leftarrow \mathbf{v}, \bar{\mathbf{x}} \leftarrow \mathbf{x}$ 
12       $S \leftarrow S'$  and  $\mathcal{V}(S) \leftarrow \mathcal{V}(S')$ 
13    end
14  end
15 end
16 return feasible solution,  $S = (\bar{\mathbf{y}}^r, \bar{\mathbf{y}}^u, \bar{\mathbf{z}}^1, \bar{\mathbf{z}}^2, \bar{\mathbf{v}}, \bar{\mathbf{x}})$ 

```

---

For a given  $t \in T_L$ , the level of perturbation of the incumbent solution depends on the values taken by the location variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  in the  $\kappa$  periods considered, i.e. in periods  $\tau = \phi(p), \phi(p+1), \dots, \phi(p+\kappa-1)$  with  $p = \phi^{-1}(t)$ . In the case of a reliable facility  $j \in J^r$ , if it is opened in some period  $\tau$  then this decision cannot be revoked. However, it is not compulsory to assign the same primary supplier to this facility as in the incumbent. By contrast, if  $y_{j\tau}^r = 0$  in all periods  $\tau$ , one of the following three cases has necessarily occurred: (i) the facility was never opened, (ii) the facility was opened in a period prior to  $t$ , or (iii) the facility was opened in a period after  $\phi(p+\kappa-1)$ . By freeing variables  $y_{jt'}^r$  for all strategic periods  $t' < t$  and  $t' > \phi(p+\kappa-1)$ , any location decision can be made later in these periods. In cases (ii) and (iii), for example, this means bringing forward or postponing the opening of the

facility, or not operating it at all. Moreover, the choice of the primary supplier to be assigned to this facility is also free. This flexibility is even greater for an unreliable facility, since its level of fortification will be selected in addition to identifying the time period in which the facility should be established. Furthermore, there are no limitations on the choice of the primary supplier and backup supplier(s) assigned to this facility.

As mentioned earlier, there is a trade-off between selecting  $\kappa = 1$  (smallest value) and  $\kappa = |T_L| - 1$  (largest value). In the former case, a total of  $|T_L|$  iterations are performed and the associated restricted  $\varepsilon$ -constraint problems only fix facility choices for a single period, thus enabling a substantial change to the initial location decisions. While this offers added opportunities for identifying improved solutions, it is also computationally expensive. By contrast, only two iterations are performed when  $\kappa = |T_L| - 1$  and the two restricted sub-problems do not allow for significant diversification in the choice of facilities to operate, potentially reducing the chance of finding a better solution. It is therefore advisable to set  $\kappa$  at an intermediate value between 1 and  $|T_L| - 1$ . A preliminary analysis with a number of test instances revealed that the choice of  $\kappa = 2$  offers the best trade-off between quality and computing time.

#### 4.2.2 Facility deselection based on cost information

Given an initial feasible solution  $\bar{S}$ , the second improvement heuristic retains a subset of reliable and unreliable *open* facilities and deselects the rest. Since minimising the total expected cost of designing the distribution network is the objective taken in the  $\varepsilon$ -constraint method, the choice of open facilities to be unselected is controlled by the total cost of handling one unit of product at a facility.

Let  $\underline{J}^r \subseteq J^r$  and  $\underline{J}^u \subseteq J^u$  be the subsets of reliable and unreliable facilities open in solution  $\bar{S}$ , respectively. For these facilities, the corresponding variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  will become free, not all at once, but successively, thereby avoiding a major destruction of the initial solution. To this end, a pre-specified integer parameter  $N$  (with  $N < |\underline{J}^r| + |\underline{J}^u|$ ) governs the number of open facilities whose operation will be simultaneously revoked. At each iteration, the choice of the  $N$  facilities to be deselected is determined by the magnitude of their total cost of handling one unit of product. The most costly facilities will be the first to have their status freed.

The unit total expected cost incurred by a facility per period is estimated by adding up four cost components, namely the mean cost of opening the facility, the mean cost of operating the facility, the mean cost of distributing the product from suppliers to the facility and the mean cost of delivering the product to customers. For a reliable facility  $j \in \underline{J}^r$ , the unit total cost

$\underline{TC}_j$  is determined by (34).

$$\underline{TC}_j = \frac{\sum_{t \in T_L} FC_{jt}^r}{|T_L| \cdot Q_j^r} + \frac{\sum_{t \in T} OC_{jt}^r}{|T| \cdot Q_j^r} + \frac{\sum_{i \in I} \sum_{t \in T} \sum_{s \in S} \pi_s SC_{ijts}}{|I| \cdot |T|} + \frac{\sum_{k \in K} \sum_{t \in T} \sum_{s \in S} \pi_s DC_{jkts}}{|K| \cdot |T|} \quad j \in \underline{J}^r. \quad (34)$$

For an unreliable facility  $j \in \underline{J}^u$ , the calculation of  $\underline{TC}_j$  takes into account that, among the available fortification levels, the lowest level (i.e.  $f = 1$ ) incurs the least opening and operating cost. However, in the event of a disruption, the degree of failure is significant, resulting in a substantial loss of its capacity ( $Q_{j1}^u$ ), which is gradually restored over multiple consecutive periods. Therefore, both the mean fixed opening cost and the mean fixed operating cost for this facility are affected by the total capacity available over the planning horizon as shown by (35).

$$\underline{TC}_j = \frac{\sum_{t \in T_L} FC_{j1t}^u}{\frac{|T_L|}{|T|} \cdot \sum_{t \in T} \sum_{s \in S} \pi_s (1 - b_{j1ts}) Q_{j1}^u} + \frac{\sum_{t \in T} \left[ OC_{j1t}^u + \sum_{s \in S} \pi_s b_{j1ts} (AC_{j1t}^u - OC_{j1t}^u) \right]}{\sum_{t \in T} \sum_{s \in S} \pi_s (1 - b_{j1ts}) Q_{j1}^u} + \frac{\sum_{i \in I} \sum_{t \in T} \sum_{s \in S} \pi_s SC_{ijts}}{|I| \cdot |T|} + \frac{\sum_{k \in K} \sum_{t \in T} \sum_{s \in S} \pi_s DC_{jkts}}{|K| \cdot |T|} \quad j \in \underline{J}^u. \quad (35)$$

Algorithm 4 describes the rules implemented. These will be illustrated by means of an example. Suppose that  $|T| = 24$ ,  $T_L = \{1, 7, 13, 19\}$ ,  $J^r = \{1, 2, 3, 4, 5\}$ ,  $J^u = \{6, 7, 8, 9, 10\}$ ,  $\underline{J}^r = \{2, 5\}$  and  $\underline{J}^u = \{6, 8, 10\}$ . This means that in the initial feasible solution, reliable facilities were opened in locations 2 and 5, while unreliable facilities were established in locations 6, 8 and 10. At the other candidate sites (i.e. 1, 3, 4, 7, 9), no facilities are operating. Moreover, let us assume that the calculation of the cost  $\underline{TC}_j$  for  $j \in \{2, 5, 6, 8, 10\}$  yields the ordered facility set  $\underline{J} = \{5, 2, 8, 10, 6\}$  (line 4). Taking  $N = 3$ , facilities 5, 2 and 8 are the first to be deselected, meaning that the associated binary variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  will become free throughout the planning horizon. As a result, the remaining variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  are fixed for facilities 1, 3, 4, 6, 7, 9 and 10 according to the values of  $\bar{\mathbf{y}}^r$  and  $\bar{\mathbf{y}}^u$  in the initial solution, respectively. In particular, for all  $t \in T_L$ , we have  $y_{jt}^r = 0$  for  $j \in \{1, 3, 4\}$  and  $y_{jft}^u = 0$  for  $j \in \{7, 9\}$  and  $f \in F_j$ . This yields a restricted problem in which all decision variables are free (line 6) except those mentioned before (lines 10–11). Note that to diversify the search for an improved

solution, for those facilities that are retained (i.e. 6 and 10), it is not compulsory to keep the same allocation of primary and backup suppliers as in the incumbent solution. Due to these mild conditions, the incumbent is used as starting solution when solving the sub-problem, thus speeding up the process of finding the optimal solution. In the worst case, the latter is equal to the current solution; otherwise it must be a better solution, thus becoming the new incumbent. Returning to the example, suppose that in the new incumbent no facility is established at

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**Algorithm 4:** Improvement heuristic 2 with partial deselection of initially open facilities

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**Input** : All instance data;  $N$ ; feasible solution  $\bar{\mathcal{S}} = (\bar{\mathbf{y}}^r, \bar{\mathbf{y}}^u, \bar{\mathbf{z}}^1, \bar{\mathbf{z}}^2, \bar{\mathbf{v}}, \bar{\mathbf{x}})$

**Output:** Feasible solution,  $\mathcal{S}$

```

1  $\mathcal{S} \leftarrow \bar{\mathcal{S}}, \mathcal{V}(\mathcal{S}) \leftarrow \mathcal{V}(\bar{\mathcal{S}})$  // incumbent solution and its objective value
2 identify the subset  $\underline{J}^r$  of reliable open facilities in initial solution  $\bar{\mathcal{S}}$ 
3 identify the subset  $\underline{J}^u$  of unreliable open facilities in initial solution  $\bar{\mathcal{S}}$ 
4 create set  $\underline{J}$  with facilities sorted by decreasing estimated total costs,  $\underline{TC}_j$  for  $j \in \underline{J}^r \cup \underline{J}^u$ 
5 repeat
6   set all variables  $\mathbf{y}^r, \mathbf{y}^u, \mathbf{z}^1, \mathbf{z}^2, \mathbf{v}, \mathbf{x}$  as free
7   if  $N > |\underline{J}|$  then  $N \leftarrow |\underline{J}|$ 
8   consider the top  $N$  facilities in the ordered set  $\underline{J}$ , i.e.  $j'_{[1]}, \dots, j'_{[N]}$ 
9   // fix decisions on remaining reliable and unreliable facilities from
      incumbent solution
10   $y^r_{jt} \leftarrow \bar{y}^r_{jt}$  for all  $j \in \underline{J}^r \setminus (\underline{J}^r \cap \{j'_{[1]}, \dots, j'_{[N]}\})$  and  $t \in T_L$ 
11   $y^u_{jft} \leftarrow \bar{y}^u_{jft}$  for all  $j \in \underline{J}^u \setminus (\underline{J}^u \cap \{j'_{[1]}, \dots, j'_{[N]}\})$ ,  $f \in F_j$  and  $t \in T_L$ 
12  find optimal solution  $\mathcal{S}'$  to restricted  $\varepsilon$ -constraint problem with previously fixed variables
      and all other variables free, using incumbent solution as starting solution
13  if  $\mathcal{V}(\mathcal{S}') < \mathcal{V}(\mathcal{S})$  then // update incumbent solution
14     $\bar{\mathbf{y}}^r \leftarrow \mathbf{y}^r, \bar{\mathbf{y}}^u \leftarrow \mathbf{y}^u, \bar{\mathbf{z}}^1 \leftarrow \mathbf{z}^1, \bar{\mathbf{z}}^2 \leftarrow \mathbf{z}^2, \bar{\mathbf{v}} \leftarrow \mathbf{v}, \bar{\mathbf{x}} \leftarrow \mathbf{x}$ 
15     $\mathcal{S} \leftarrow \mathcal{S}'$  and  $\mathcal{V}(\mathcal{S}) \leftarrow \mathcal{V}(\mathcal{S}')$ 
16  end
17   $\underline{J} \leftarrow \underline{J} \setminus \{j'_{[1]}, \dots, j'_{[N]}\}$ 
18 until  $\underline{J} = \emptyset$ 
19 return feasible solution,  $\mathcal{S} = (\bar{\mathbf{y}}^r, \bar{\mathbf{y}}^u, \bar{\mathbf{z}}^1, \bar{\mathbf{z}}^2, \bar{\mathbf{v}}, \bar{\mathbf{x}})$ 

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location 5. This means that facilities 2 and 8 have been retained (possibly being opened at different time periods than in the initial solution and, in the case of the unreliable facility 8, a different level of fortification may have been selected). At the next iteration, the ordered set  $\underline{J}$  has only two facilities (line 17), namely 10 and 6, so it is necessary to adjust the value of parameter  $N$  from 3 to 2 (line 7). Variables  $\mathbf{y}^r$  and  $\mathbf{y}^u$  are freed for these locations 6 and 10, while they take on the values according to the incumbent solution for all the other locations. The heuristic is terminated once the associated restricted  $\varepsilon$ -constraint problem has been solved,

since no more facility choices can be revoked (i.e. set  $\underline{J}$  is now empty, line 18).

Given  $N$ , a total of  $\lceil (|\underline{J}^r| + |\underline{J}^u|) / N \rceil$  iterations are performed. Naturally, the value assigned to  $N$  has a significant impact on the outcome of this scheme, as it involves a compromise between flexibility in selecting potential facilities to open (and identifying their fortification levels if they are unreliable) and the computational effort to solve the restricted MIP problems. In our numerical experiments, taking  $N = \lceil |J| / \Delta J \rceil$  with  $\Delta J = 3$  proved to be a suitable choice.

Finally, we note that although the order in which the two improvement heuristics are used is in principle arbitrary, our computational study suggests that better performance is often achieved when Algorithm 3 is run from the feasible solution constructed in phase 1 and then Algorithm 4 is executed taking the best solution identified by Algorithm 3. Embedding the two-phase heuristic procedure in Algorithm 1 involves solving the problems in lines 1, 3 and 7 of this algorithm by executing Algorithms 2–4.

## 5 Computational study

In this section, the performance of Algorithm 1 using the MIP-based heuristic scheme is evaluated and compared with the execution of this algorithm, in which all sub-problems are solved exclusively with a general-purpose optimisation solver, for a set of test instances. The methodology developed for the generation of the latter is briefly described in Section 5.1 and the numerical results are presented in Section 5.2. Furthermore, relevant insights into the characteristics of the efficient solutions identified are provided in Section 5.3. The potential benefits of using a stochastic programming approach for our problem rather than a deterministic counterpart are examined in Section 5.4.

### 5.1 Experimental design

Since benchmark datasets are not available for our problem, we randomly generated a total of 65 test instances. The total number of suppliers ( $|I|$ ), candidate facility locations ( $|J|$ ), customers ( $|K|$ ), time periods ( $|T|$ ) and scenarios ( $|S|$ ) selected for our instances are given in columns 2–6 of Table 3. We believe this is a satisfactory range compared to similar scenario-based studies (e.g. Azad and Hassini (2019)). Each row of Table 3 is associated with a group of five instances, the groups being labelled  $G1, \dots, G13$ .

The probability  $\pi_s$  of scenario  $s$  is  $1/|S|$ . The number of primary and backup suppliers is half of  $|I|$ , respectively, in all instances. In addition, the number of candidate locations for opening reliable, resp. unreliable, facilities is  $|J|/2$ . Three fortification levels are considered for each unreliable facility, representing low ( $f = 1$ ), medium ( $f = 2$ ) and high ( $f = 3$ ) levels (i.e.  $|F_j| = 3$ ,  $j \in J^u$ ). The number of strategic periods  $|T_L|$  is  $|T|/6$  and these are equally spaced, i.e.  $\Delta = 6$ . Furthermore, in each scenario, at least 25 percent of total demand must be met even if with a delay, i.e.  $sl_t = 0.25$  for  $t \in T$  (recall constraints (25)). Customers tolerate

Table 3: Size of the test instances.

Inst.	$ I $	$ J $	$ K $	$ T $	$ S $	# bin. var.	# cont. var.	# constr.
G1	4	8	16	18	30	384	224,640	25,214
G2	4	8	40	18	30	384	535,680	38,174
G3	4	8	64	18	30	384	846,720	51,134
G4	4	8	16	18	60	384	449,280	49,604
G5	4	8	40	18	60	384	1,071,360	75,524
G6	4	8	64	18	60	384	1,693,440	101,444
G7	6	10	20	24	30	920	475,200	43,010
G8	6	10	50	24	30	920	1,123,200	64,610
G9	6	10	80	24	30	920	1,771,200	86,210
G10	6	10	20	24	60	920	950,400	84,170
G11	6	10	50	24	60	920	2,246,400	127,370
G12	6	10	80	24	60	920	3,542,400	170,570
G13	6	10	80	24	180	920	10,627,200	508,010

lateness in demand fulfilment up to two periods ( $\rho_k = 2$ ,  $k \in K$ ). This parameter, together with the values for  $|K|$ ,  $|T|$  and  $|S|$ , significantly affects the total number of variables  $x$  which in turn accounts for the very large number of continuous variables, as shown in column 8 of Table 3 for some instances. The table also gives the total number of binary variables (column 7) and the total number of constraints (column 9), whereby the latter also include the additional inequalities (32). As can be seen, our testbed comprises medium-sized and large instances.

The locations of the suppliers, candidate sites and customers are generated in the square  $[0, 100] \times [0, 100]$  as displayed in Figure 3. An inner square  $[10, 90] \times [10, 90]$  is also considered, which is divided into a ‘safe’ region where the reliable facilities are located (blue shaded area) and an ‘unsafe’ region where the primary suppliers and unreliable facilities are located (red shaded area). Customers are distributed across the large square, while the locations of the backup suppliers are outside the inner square, so that distribution costs are high in the event of the latter being selected. In this way, geographical dispersion is ensured and, at the same time, safe zones are also included.

The scenarios associated with an instance are divided into three subsets, each with a specific demand pattern. Thus, in one third of the scenarios, all customer demands are monotone decreasing over the planning horizon. In another third of the scenarios, the opposite demand pattern is generated. In the last subset of scenarios, customer demand follows an irregular pattern. Appendix A specifies the demand generation scheme. The appendix also includes detailed information on the random generation of additional parameters, namely costs, capacities of suppliers and facilities, and disruptive events. The start and end periods of disruptions for each primary supplier and unreliable facility are selected at random in any scenario. These facilities

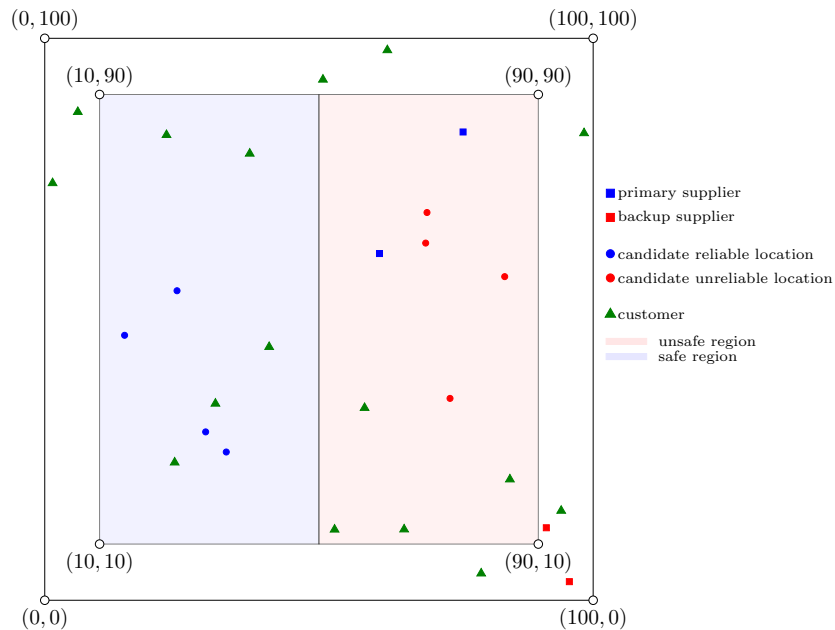


Figure 3: Example of locations of suppliers, potential facilities and customers.

fail independently of each other. The same applies to the primary suppliers. An unreliable facility is disturbed three times longer at the lowest fortification level than at the highest level. In the first period of a disruptive event, the amount of capacity loss of a facility is randomly generated, also taking into account its level of fortification. Moreover, a gradual recovery of capacity is considered, in line with Sawik (2019). A complete disruption of the network never occurs, and product flows are redirected through operating facilities. All fixed costs concerning facilities reflect economies of scale. Finally, Euclidean distances are included in the random generation of distribution costs in the network. However, only arcs whose length does not exceed a given maximum distance are considered. Therefore, each test instance represents an incomplete network, which is a realistic assumption. Our data generation scheme also ensures that shipments from unreliable facilities to customers are more expensive during periods when the operation of these facilities is disturbed, as it may be necessary to use alternative transport modes or routes that incur additional costs.

## 5.2 Numerical results

Algorithm 1 and the heuristics were coded in Julia (version 1.6.7). The implementation of the L-shaped method available in IBM ILOG CPLEX (version 22.1) was used to solve the single-objective sub-problems in the  $\varepsilon$ -constraint method. All experiments were performed on a computer with a 2.1 GHz Intel Core i7-12700 processor, 16 GB RAM and running a 64-bit operating system. For each test instance, Algorithm 1 was executed twice. In the first run, the

$\varepsilon$ -constraint problems were solved exclusively by CPLEX, with a limit of 7,200 seconds set for the total CPU time and 0.01 percent for the optimality gap. The second run was carried out without a time limit and involved solving the individual  $\varepsilon$ -constraint problems with the proposed two-phase heuristic procedure. In this case, CPLEX was only used to solve sub-problems in which the location and supplier selection decisions had previously been fixed according to the rules specified by the construction and improvement heuristics. For the largest group of instances, G13, which proved to be the most challenging, we deviated from this setting by allowing the first run of Algorithm 1 with CPLEX to be as long as that of the heuristics, when the latter required more than 7,200 seconds in total.

In the  $\varepsilon$ -constraint method, the objective space for the total expected unmet demand ( $w_2$ ) can be divided into  $\mu$  sub-intervals, yielding  $\mu - 1$  compromise Pareto-optimal solutions for each test instance, in addition to the two lexicographic solutions. For time and practical reasons, we have opted for  $\mu = 3$ , and as will be shown, the four solutions thus obtained are good representatives of the conflicting nature of objectives  $w_1$  and  $w_2$ . Furthermore, they provide the decision-maker with relevant information about the trade-offs achieved by each alternative network configuration. This approach also avoids overloading the decision-maker with a large number of solutions and a lengthy comparative analysis.

The characteristics of the different efficient solutions returned by the two executions of Algorithm 1 are summarised in Table 4. Each row in this table shows the averages of five test instances. 'Sol. 1' represents the lexicographic solution that gives the greatest preference to minimising the total expected amount of unsatisfied demand (*right* lex. sol.), while 'Sol. 4' is the lexicographic solution in which minimising the total expected cost is the most relevant objective (*left* lex. sol.). 'Sol. 2' and 'Sol. 3' present a compromise with regard to the two objectives, with 'Sol. 2' having a higher total cost than 'Sol. 3', but a lower total amount of unmet demand. Columns 3–6 give, for the (approximate) Pareto-optimal solutions identified by CPLEX, the total cost (in monetary units, m.u.), the percentage of demand covered, the optimality gap upon termination and the total CPU time. Columns 7–9 report the performance of the two-phase heuristic scheme compared to executing Algorithm 1 exclusively with CPLEX. To this end, the gap between the total cost of a solution returned by the heuristic procedure ( $w_1^H$ ) and the total cost associated with the solution found by CPLEX ( $w_1^{CPLEX}$ ) is determined as follows:  $100 \cdot (w_1^H - w_1^{CPLEX}) / w_1^{CPLEX}$ . The average values are shown under '% cost dev. to CPLEX'. Moreover, the total CPU time required by the heuristics is divided by the total time reported by CPLEX. The average values are given in the column '% CPU to CPLEX'. A value lower than 100 percent in this column indicates that the heuristic scheme was faster than CPLEX.

As expected, we are dealing with a challenging problem for which the chance of achieving optimality within the pre-specified time limit by solving the  $\varepsilon$ -constraint problems with the L-shaped method provided by CPLEX gradually declines as the number of scenarios and the length of the planning horizon increase. This feature is particularly noticeable in the largest

Table 4: Characteristics of the solutions identified.

Inst.	Sol.	Algorithm 1 with CPLEX				Algorithm 1 with two-phase heuristic		
		Total cost (m.u.)	Demand met (%)	MIP gap (%)	CPU time (s)	% cost dev. to CPLEX	Demand met (%)	% CPU to CPLEX
G1	1	10,042,088.2	100.0	0.00	57.6	0.59	100.0	180.57
	2	6,495,876.9	75.1	0.00	205.5	0.60	75.1	128.15
	3	4,426,578.0	50.1	0.00	127.8	0.01	50.1	147.78
	4	2,598,873.0	25.2	0.00	15.0	0.00	25.2	261.29
G2	1	20,640,021.9	100.0	0.00	323.6	0.53	100.0	65.29
	2	13,321,700.4	75.0	0.00	599.8	0.08	75.0	104.78
	3	8,288,292.9	50.0	0.00	465.6	0.07	50.0	61.68
	4	4,581,312.0	30.0	0.00	61.0	0.62	30.5	120.98
G3	1	31,087,536.2	100.0	0.55	5,878.4	0.35	100.0	8.32
	2	20,425,145.2	74.8	3.27	7,200.0	0.07	74.8	18.03
	3	12,826,938.5	49.6	1.57	7,200.0	0.00	49.6	11.79
	4	6,831,416.6	25.0	0.00	116.8	0.99	25.0	147.44
G4	1	10,371,304.8	100.0	0.00	195.3	0.72	100.0	168.63
	2	6,454,770.5	74.9	0.00	859.3	0.84	74.9	76.73
	3	4,164,258.2	49.9	0.00	301.5	0.00	49.9	150.46
	4	2,591,849.6	25.0	0.00	44.8	0.49	25.4	311.08
G5	1	21,690,670.2	100.0	0.14	2,056.4	1.00	100.0	21.89
	2	14,314,411.1	75.2	8.86	7,200.0	0.02	75.2	17.81
	3	8,902,281.0	50.3	0.88	4,658.7	0.00	50.3	20.99
	4	5,066,871.3	25.5	0.00	143.6	0.18	25.5	120.76
G6	1	30,571,062.7	100.0	0.82	7,200.0	0.64	100.0	16.77
	2	19,922,482.2	75.1	4.69	7,200.0	0.02	75.1	27.85
	3	12,595,073.0	50.3	1.54	7,200.0	0.00	50.3	30.96
	4	7,029,470.1	25.4	0.00	392.5	0.57	25.5	90.14
G7	1	17,195,454.2	100.0	0.00	1,242.8	0.55	100.0	21.56
	2	10,287,900.0	75.3	0.00	1,080.8	1.25	75.3	72.93
	3	6,627,875.0	50.5	0.00	568.9	0.00	50.5	59.74
	4	3,891,913.0	25.8	0.00	47.6	1.21	26.0	183.79
G8	1	35,902,878.0	100.0	0.79	7,200.0	0.21	100.0	12.80
	2	22,835,712.4	75.0	4.28	7,200.0	1.83	75.0	23.14
	3	14,237,440.1	50.0	3.03	7,200.0	0.00	50.0	24.00
	4	7,777,304.3	25.0	0.00	303.8	0.78	25.6	87.60
G9	1	52,976,165.8	100.0	1.14	7,200.0	0.06	100.0	24.41
	2	33,843,960.3	75.1	3.33	7,200.0	0.00	75.1	38.57
	3	20,196,957.6	50.1	1.77	7,200.0	0.00	50.1	39.15
	4	10,744,296.7	25.2	0.00	748.0	1.15	25.5	84.87
G10	1	17,759,847.0	100.0	0.37	4,768.2	0.82	100.0	13.54
	2	11,109,251.5	74.9	4.47	7,200.0	1.05	74.9	16.64
	3	7,017,759.6	49.9	0.77	4,768.4	1.63	49.9	18.56
	4	4,163,280.2	25.0	0.00	155.9	0.56	25.4	161.49
G11	1	36,220,018.8	100.0	1.23	7,200.0	1.32	100.0	17.55
	2	23,168,643.4	74.9	4.77	7,200.0	0.05	74.9	45.19
	3	13,888,179.3	49.8	2.02	7,200.0	0.00	49.8	35.07
	4	7,482,881.2	25.0	0.00	685.5	0.59	25.2	67.12
G12	1	55,465,656.2	100.0	3.08	7,200.0	0.62	100.0	46.89
	2	36,351,339.5	75.1	7.81	7,200.0	0.03	75.1	75.96
	3	22,332,050.5	50.2	6.71	7,200.0	0.00	50.2	56.90
	4	11,565,837.6	25.4	0.25	4,968.0	2.37	25.4	28.07
G13	1	-	-	-	7,380.0	-	100.0	95.44
	2	-	-	-	8,214.0	-	74.7	100.00
	3	-	-	-	7,200.0	-	48.5	62.48
	4	14,056,196.4	25.0	22.44	7,200.0	-2.31	25.4	35.69

– no feasible solution found.

group G13 with 180 scenarios, in which CPLEX is unable to identify feasible solutions for all but one instance and, in this case, the only lexicographic solution found (sol. 4) has poor quality, as evidenced by an optimality gap of 22.44 percent. By contrast, the two-phase heuristic procedure finds all 20 solutions to the 5 instances in this group, and achieves higher solution quality in terms of total expected cost, compared to the single solution identified by CPLEX. For reasons of fairness, when the heuristic scheme required more than 7,200 seconds in G13, Algorithm 1 was run with CPLEX for the same amount of time as the heuristic. Furthermore, the heuristic used significantly less than 7,200 seconds to generate the (approximate) efficient solutions 3 and 4, which is remarkable given that the instances in G13 have more than 10 million decision variables.

Table 4 also reveals that it is computationally more expensive for CPLEX to identify Pareto-optimal solutions the greater the relevance given to satisfying customer demand as opposed to minimising the total expected cost. When costs do not play a prominent role, there are multiple options for deploying facilities and selecting suppliers, as well as for deciding on which customer orders to serve (some of which possibly with delay), thus explaining the additional difficulty in obtaining the best network configuration in this case.

For the smallest instances (i.e. G1, G2, G4, G7), CPLEX finds the four Pareto-optimal solutions within the time limit. For the remaining instances, with the exception of group G13, the mean optimality gap is relatively low (less than 5 percent) and only occasionally increases, but on average does not exceed 9 percent. These good results provide a valid basis for evaluating the performance of the two-phase heuristic procedure. Columns 7 and 8 show that the quality of the solutions returned by the latter is only slightly inferior than that achieved when CPLEX is used exclusively. In particular, the level of demand fulfilment is, on average, very similar for both solution approaches (cf. columns 4 and 8). Interestingly, we noticed that this observation also applies to the average percentage of demand met for each individual customer. Although for some of the smaller instances the total CPU time required by the heuristic scheme exceeds that of CPLEX (e.g. G1), for most instances, and especially when their size increases significantly (e.g. G8–G13), the computational effort is considerably lower (cf. column 9). This indicates that the proposed construction and improvement mechanisms are effective, especially when it comes to obtaining good approximations to Pareto-optimal solutions for very large problems within reasonable computing time.

To further compare the performance of the two executions of Algorithm 1, we selected a large instance of G8 and set  $\mu = 9$ , thus obtaining a total of 10 solutions in each run. Figure 4 depicts the corresponding objective values. Since the optimality gaps associated with the values marked with blue dots are all 0, these allow the Pareto front to be constructed. The extreme points (top left blue dot and bottom right blue dot) are the left and right lexicographic solutions, respectively (i.e. sol. 4 and sol. 1 according to the notation in Table 4). Interestingly, the 10 blue dots are evenly distributed with respect to objective  $w_2$ . However, Pareto-optimal solutions with a relatively small amount of unmet demand (blue dots on the right) have noticeably different

costs, while the opposite is visible for the blue dots on the left side. From a practical viewpoint, this suggests that our choice of calculating two Pareto-optimal solutions between the two lexicographic solutions yields a representative sample for our problem. It should be noted that sol. 3 and sol. 2 (using the notation in Table 4) correspond to the fourth and seventh blue dots, from left to right, respectively. More importantly, the red circles, which are the images of the heuristic solutions, provide a very good approximation of the Pareto front. Observe that the latter is not convex which further substantiates our choice of the  $\varepsilon$ -constraint method for solving our problem (Mavrotas, 2009; Mavrotas and Florios, 2013).

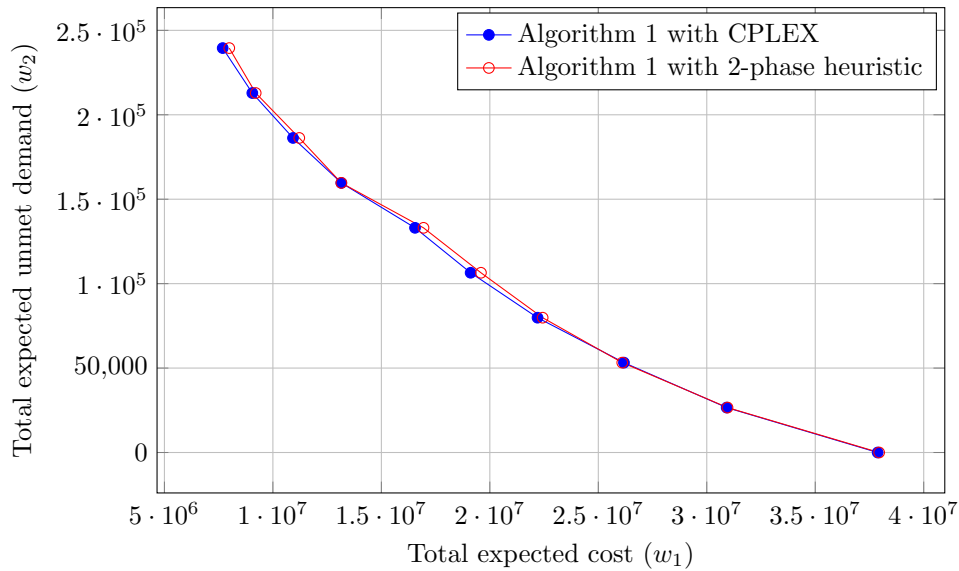


Figure 4: Pareto front (blue line) and its approximation (red line) for an instance with 50 customers, 6 suppliers, 10 candidate locations, 24 time periods and 30 scenarios.

### 5.3 Managerial insights

To gain a deeper insight into the trade-offs between the two conflicting objectives, this section provides a further analysis of various features of the (approximate) Pareto-optimal solutions identified for the instances in our testbed. The aim is to assist the decision-maker in understanding how the deployment of (reliable/unreliable) facilities and other decisions affect the design of the distribution network.

We start by examining the facility location decisions associated with the 10 Pareto-optimal solutions whose images in the objective space are depicted in Figure 4 (blue dots). This information is shown in Figure 5. When the design of the distribution network is governed mainly by the expected fixed and variable costs, two unreliable facilities are opened (sol. 10, left lexicographic solution), one in period 1 and the other in period 19, both with the highest level of fortification ( $f = 3$ ) so that disruptive events have little impact on their capacities.

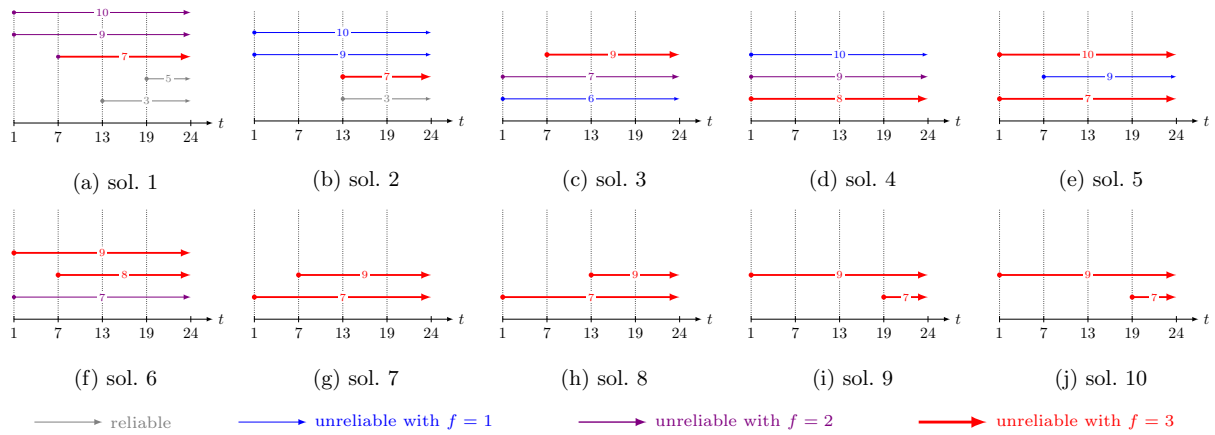


Figure 5: Facility location decisions in the 10 Pareto-optimal solutions obtained for an instance with  $|K| = 50$ ,  $|T| = 24$ ,  $|S| = 30$ ,  $|I^1| = |I^2| = 3$  and  $|J^r| = |J^u| = 5$ .

These decisions guarantee that exactly 25% of the total expected demand is satisfied, possibly resorting to some late deliveries to customers. As the relevance of the cost objective decreases and the importance of demand fulfilment increases, the deployment of some unreliable facilities is gradually brought forward (see, e.g., sol. 7 and sol. 8) for the purpose of providing additional capacity to meet more customer demand. Eventually, additional facilities will have to be opened, as illustrated by solutions 1–6. In this case, not only does the choice of unreliable facilities and their fortification levels change progressively, but reliable facilities are also selected, even though they are more costly. However, their capacities are always available. In the extreme case of solution 1 (right lexicographic solution), where all demand is met and the total expected cost is the highest, half of the candidate sites are selected (i.e. 5), with two reliable and three unreliable facilities being opened. The latter operate with medium and high levels of fortification. The deployment periods for these five facilities differ over the planning horizon due to the different demand patterns in the scenarios generated. The interested reader is referred to Figure 9 in Appendix B, which illustrates the changes in the network configuration over the planning horizon for a specific scenario in this solution.

Figure 6 provides similar information, but for the four solutions identified for all test instances, except G13. The average number of facilities deployed gradually decreases when moving from solution 1 to solution 4 (with the latter giving greater importance to reducing total cost,  $w_1$ ), thus confirming the analysis of Figure 5. This property also affects the material flow in the network, which, on average, is 1.5-2 times higher for solution 1 than for solution 4 per unit of distance travelled. In addition, compared to CPLEX, the heuristic rules tend to choose a slightly higher number of unreliable facilities on average, especially those with the lowest level of fortification. With regard to reliable facilities, there appears to be no significant differences.

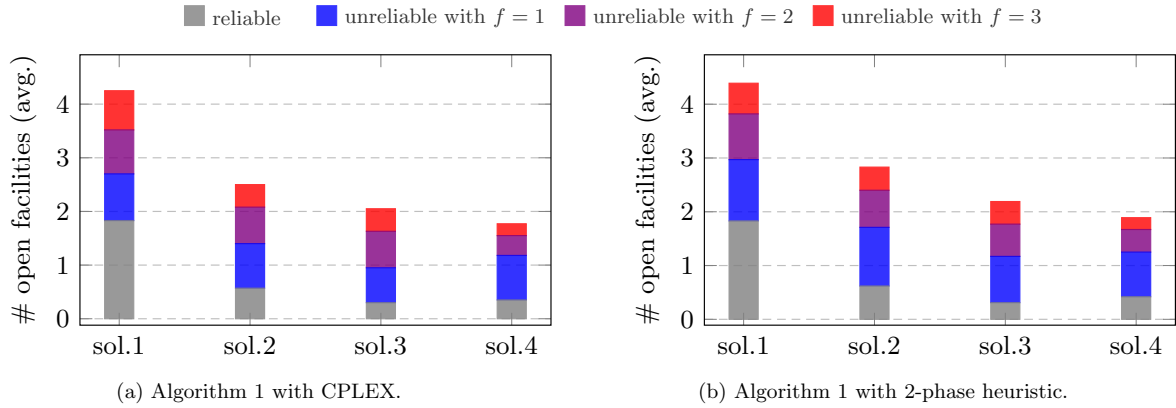


Figure 6: Average number and type of facilities selected in the solutions obtained for the test instances (G1–G12).

A particular characteristic of our problem is that late delivery of customer orders is permitted (recall column 9 in Table 1). Figure 7 shows how the average demand met with a delay evolves across the four Pareto solutions. While solutions 1, 2 and 4 have relatively low percentages of late deliveries, the opposite is true for solutions 3 in both runs of Algorithm 1. One possible explanation for this feature is that the tardiness penalty costs for customer deliveries are relatively low. Thus, the transition from solution 4 (minimisation of total cost) to solution 3 avoids opening many more facilities, which is a costly measure, and succeeds in covering about 50 percent of the total expected demand on average through postponing shipments to customers (see columns 4 and 8 of Table 4 for solution 3). This situation changes with solutions 1 and 2, where only the deployment of new facilities makes it possible to increase the customer service level. As a result of more capacity being available in the network, the amount of delayed demand decreases.

To study the impact on the design of the network of not allowing late deliveries to customers, thereby reducing the network’s responsiveness to disruptions and demand fluctuations, we have also considered all test instances with  $\rho_k = 0$  for every customer  $k \in K$ . This particular case results in a 64.6 percent decrease in the number of continuous variables in the two-stage stochastic formulation, while the total number of constraints slightly increases due to inequalities (33). As shown in Table 6 in Appendix C, a significantly larger number of instances can be solved to optimality by Algorithm 1 run exclusively with CPLEX within the time limit (G1–G8, G10). Nevertheless, the very large instances with 180 scenarios remain challenging and CPLEX is again unable to even find feasible solutions to all of them. Compared to CPLEX, the two-phase heuristic is extremely fast, taking only 5 percent of the CPU time. Furthermore, it returns high-quality solutions for all instances, including G13, with average gaps ranging from -2 percent to 1.7 percent. Hence, in some instances the heuristic outperforms CPLEX.

The analysis of the (approximate) efficient solutions reveals that the expected total cost of

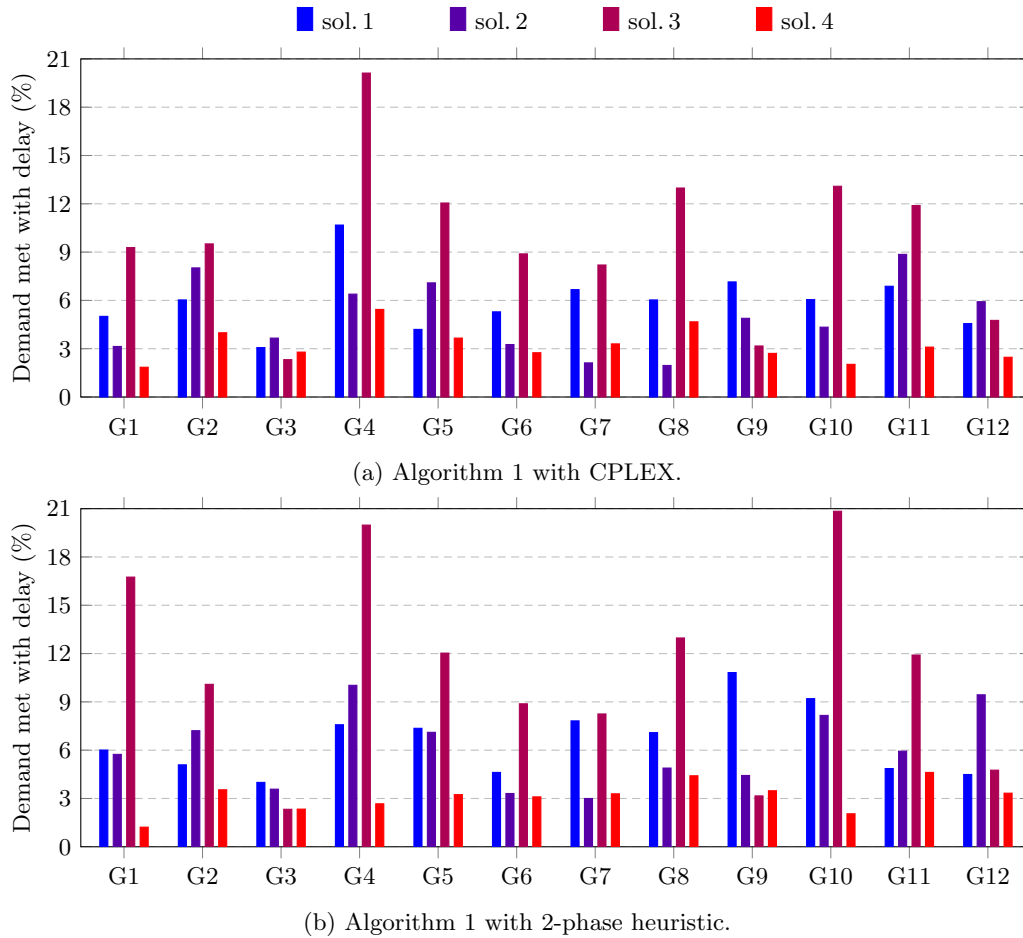


Figure 7: Average percentage of demand met with delay.

designing the network increases when considering this particular case (cf. column 3 in Table 4 and Table 6). For example, the left lexicographic solutions (sol. 4), which meet the minimum service level (i.e. exactly 25 percent of customer demand is satisfied), have on average 3.6 percent higher total cost than when delays in demand fulfilment are tolerated. This is due to the fact that the facilities have to be opened early in the planning horizon so that sufficient capacity is available for meeting customer demand in time. Therefore, these facilities are operated for longer periods, resulting in higher total costs. This feature is illustrated in Figure 8, which shows the average number of time periods during which a facility is operating for the original instances with  $\rho_k = 2$  and the instances with  $\rho_k = 0$  ( $k \in K$ ) in the four (approximate) Pareto-optimal solutions returned by Algorithm 1 executed with the two-phase heuristic. While solutions 3 and 4 clearly show longer operating times for the different types of facilities, this is to a lesser extent the case for solutions 1 and 2. Similar findings are obtained when comparing the solutions identified by CPLEX for  $\rho_k = 2$  and  $\rho_k = 0$  (see Figure 10 in Appendix C).

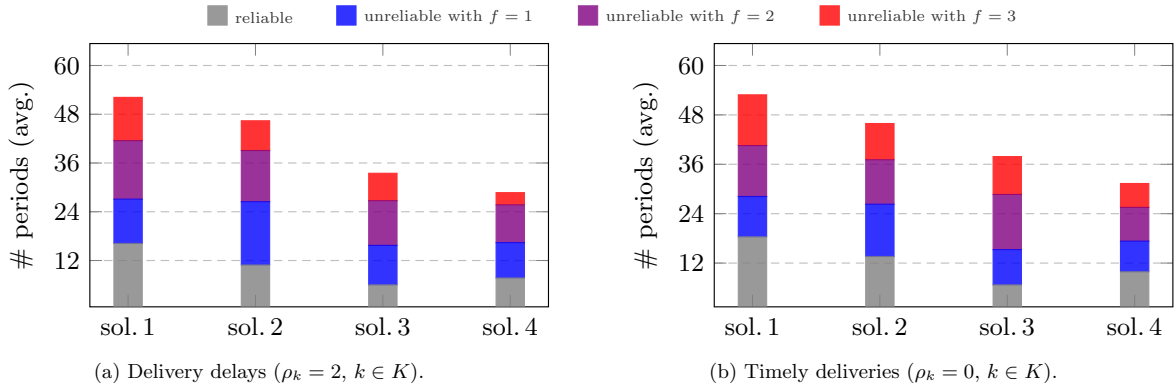


Figure 8: Average number of periods that a facility is in operation for solutions returned by Algorithm 1 with two-phase heuristic (instances G1–G12).

## 5.4 Relevance of the stochastic setting

The additional computational complexity that a stochastic programming approach involves, as opposed to a deterministic counterpart, justifies the need to examine the potential advantages of the former compared to the latter. When probabilities are associated with the scenarios, as in our problem, two measures are often used to assess the relevance of a stochastic (single-objective) model, namely the *expected value of perfect information* (EVPI) and the *value of the stochastic solution* (VSS), see Birge and Louveaux (2011). The EVPI is the maximum economic value that a decision-maker would be willing to pay in exchange for complete information about future conditions. It is defined as the difference between the optimal value of the stochastic programme ( $z_{sp}$ ) and the optimal value of the so-called *wait-and-see* (WS) problem. The WS objective value is given by the weighted sum of the optimal objective values associated with the individual scenarios (i.e. the original problem is solved separately for each scenario). The weights correspond to the probabilities of the scenarios. If, for example, the aim is to minimise the total cost, a low EVPI indicates a small cost reduction in the presence of perfect information, thus revealing that a stochastic approach has limited benefits.

The VSS is a distinct metric, that gauges the difference between the optimal objective value of the so-called *expected value problem* (EEV),  $z_{eev}$ , and the optimal value of the stochastic programme. To obtain  $z_{eev}$ , a single scenario is created by replacing all random variables with their expected values. In our case, this average scenario is constructed by taking the average demand of each customer and the average distribution cost from each supplier to a facility and from that facility to a customer in each time period. Unfortunately, taking the average demand per customer and period can result in a first-stage solution for which no feasible completion exists for some scenarios (i.e. the stochastic problem is infeasible). To overcome this issue, a ‘reference scenario’ is often used (Birge and Louveaux, 2011). Due to the specific characteristics of our data (see Appendix A), such scenario is created with the largest demand

$d_{kt} = \max_{s \in S} \{d_{kts}\}$  for every  $k \in K$  and  $t \in T$ . As with EVPI, a small VSS does not justify the use of a stochastic programming approach.

Table 5 reports the average values of the two metrics with respect to the minimisation of total cost ( $w_1$ ) for the solutions identified in the instances belonging to G1–G12 when Algorithm 1 is run exclusively with CPLEX. The values given in columns 2–5 correspond to  $100 \cdot EVPI/z_{sp}$ , while columns 6–9 indicate  $100 \cdot VSS/z_{sp}$ . With regard to EVPI, the results show that it would be valuable for the decision-maker to have access to perfect information about future customer demand, future distribution costs and future disruptions, regardless of the compromise solution they choose. In particular, when minimising the total cost is the most relevant criterion (solution 4), the EVPI is, on average, the highest. Considering the VSS and the rather large values in columns 6–9, it is even more apparent that it is beneficial to adopt a stochastic programming approach instead of a deterministic one. Interestingly, on average the highest VSS is obtained when greater importance is given to maximising the network's responsiveness (i.e. minimising total expected unmet demand, solution 1).

Table 5: Average EVPI and VSS values for objective  $w_1$ , obtained by running Algorithm 1 exclusively with CPLEX.

Inst.	EVPI (%)				VSS (%)			
	Sol. 1	Sol. 2	Sol. 3	Sol. 4	Sol. 1	Sol. 2	Sol. 3	Sol. 4
G1	11.0	4.4	9.8	15.7	26.4	20.6	7.8	20.1
G2	7.6	4.9	7.4	13.6	22.3	15.6	13.1	19.7
G3	6.5	4.7	8.2	14.8	16.7	13.6	11.7	9.8
G4	12.6	5.9	8.3	17.3	30.4	26.0	27.6	21.2
G5	8.9	7.1	8.1	15.1	24.1	18.2	19.6	21.2
G6	7.9	6.7	9.9	19.3	18.7	14.3	16.8	17.5
G7	14.1	6.7	11.1	19.0	33.8	32.3	24.9	34.2
G8	9.3	6.9	10.0	16.5	26.0	18.9	19.8	20.9
G9	7.0	6.0	5.8	10.1	21.9	17.6	19.2	23.8
G10	12.0	9.4	14.2	21.5	40.8	33.1	29.8	30.8
G11	9.8	8.5	9.6	16.1	24.5	25.3	21.7	29.0
G12	7.7	8.0	9.8	14.1	22.2	17.2	14.5	25.7
Avg	9.5	6.6	9.4	16.1	25.9	21.1	18.9	22.8

## 6 Conclusions

In this paper, we studied the problem of designing a resilient and flexible three-echelon supply chain network that includes suppliers, facilities and customers. To mitigate the adverse impact of disruptive events, different resilience strategies and contingency plans were deployed on both the

supply and demand sides of the supply chain, namely contingency procurement (i.e. switching to backup suppliers in the event of supply shortfalls), facility fortification (i.e. investing in protection measures) and demand deferral (i.e. delaying the fulfilment of customer orders). We considered disruptions at primary suppliers and unreliable facilities to be stochastic with respect to the onset, duration and impact of their occurrence, and incorporated two other sources of uncertainty involving customer demand and distribution costs. A bi-objective two-stage stochastic formulation was developed that minimises the total expected cost and maximises the network's responsiveness by minimising the total expected unmet demand. The first-stage decisions specify a schedule over a multi-period planning horizon for the opening of (reliable and unreliable) facilities, the choice of the fortification level at unreliable locations, and the assignment of primary and backup suppliers to facilities. Due to the strategic nature of these decisions, they can only be made in a subset of the periods. The second-stage decisions identify the optimal product flows across the network, possibly with delays in meeting some customer requirements, and determine the activation of backup suppliers according to the first-stage information and the realised uncertainty scenario.

To find Pareto-optimal solutions to this challenging problem, we used the  $\varepsilon$ -constraint method and developed a specially tailored two-phase MIP-based heuristic to solve the individual sub-problems. Numerical experiments with randomly generated instances, some of which of very large size, revealed that the heuristic approach is effective in terms of both solution quality and computational performance. In fact, the heuristic was able to identify good feasible solutions to all of the instances, unlike a general-purpose optimisation solver such as CPLEX. In particular, for the medium and large instances, the heuristic required significantly less computing time and even outperformed CPLEX in some cases. By analysing a representative subset of Pareto-optimal solutions, we examined the trade-offs between the two conflicting objectives, which is a critical step in helping decision-makers arrive at informed decisions. When customer service is given the highest importance, all customer orders are met, albeit sometimes with some delay. However, the associated total cost is 3.6 times higher than when the relevance of the two objectives is reversed. This is because additional capacity has to be made available, which not only requires resorting to backup sourcing and opening more facilities, but also investing in expensive fortification measures. In the case of the compromise solutions, on the other hand, the expected total cost for designing the network is 0.8 to 2 times higher than the cost incurred by a network configuration that exactly provides the minimum service level. This finding also applies when delays in shipments to customers are not allowed, thereby reducing flexibility in responding to disruptions and other uncertain events, such as changes in demand. However, in this case, the number of facilities to be opened is slightly lower, but their operating time is longer. Finally, we also showed the advantages of tackling our problem in a stochastic setting, despite the significant increase in complexity.

A future research venue could be the consideration of additional resilient measures to further hedge against disruption risks, such as planning for alternative transport modes and lateral

shipments at the intermediate network level. The latter would enable reliable facilities to share their inventory with unreliable facilities experiencing failures. Another interesting research direction would be to consider multiple types of products. Naturally, incorporating all these features would further increase the complexity of an already very challenging problem. Therefore, the development of an effective heuristic methodology would be all the more meaningful, particularly to assist practitioners in understanding and quantifying the trade-offs of different resilience and flexibility strategies on the configuration of their supply chains.

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## Appendix A Generation of problem instances

Since the number and type of fortification levels are assumed to be the same at all candidate locations for opening unreliable facilities, we denote this set by  $F$  in the remainder. Moreover, let  $\mathcal{U}[a, b]$  indicate the generation of random numbers over the range  $[a, b]$  according to a continuous uniform distribution.

### Customer demand

In the first time period, we consider  $d_{k1s} = \lceil \mathcal{U}[100, 300] \rceil$  for every  $k \in K$  and  $s \in S$ , with  $\lceil x \rceil$  denoting the smallest integer greater than or equal to  $x$ . In each of the following periods  $t \in T \setminus \{1\}$ , we set  $d_{kts} = \lceil \beta_{ks} d_{k(t-1)s} \rceil$  with  $k \in K$ ,  $s \in S$ , and  $\beta_{ks}$  is a random parameter that defines the demand pattern for each customer and scenario. Accordingly, we divide the set of scenarios  $S$  into three subsets, where the demand for all customers in each subset follows a specific pattern. For  $s = 1, \dots, |S|/3$ , we define  $\beta_{ks} \in \mathcal{U}[0.9, 1]$ , which indicates that demand is monotone decreasing over the planning horizon. For the subset of scenarios  $s = |S|/3 + 1, \dots, 2|S|/3$ , demand is monotone increasing since  $\beta_{ks} \in \mathcal{U}[1, 1.1]$ . In the last subset of scenarios, the demand pattern is irregular by taking  $\beta_{ks} \in \mathcal{U}[0.9, 1.1]$  for  $s = 2|S|/3 + 1, \dots, |S|$ . In this case, customer demand may fall by a maximum of 10% or rise by up to 10% from one period to the next. Recall that in our testbed,  $|S|$  is a multiple of three (see Table 3 in Section 5.1).

### Capacity of suppliers and facilities

The regular capacity of each primary supplier  $i \in I^1$  is defined as follows:

$$\bar{Q}_i = \left\lceil \bar{\delta}_i \frac{D}{|I^1|} \right\rceil \quad \text{with} \quad \bar{\delta}_i \in \mathcal{U}[1.5, 2.0]$$

and  $D$  provides an estimate of the total customer demand per period, i.e.

$$D = \frac{\sum_{k \in K} \sum_{t \in T} \max_{s \in S} \{d_{kts}\}}{|T|}.$$

The calculation of  $\bar{Q}_i$  for each backup supplier  $i \in I^2$  is performed in a similar way as above but we take  $|I^2|$  in the denominator instead of  $|I^1|$ .

A slightly different scheme is adopted for generating the capacity of the reliable facilities, namely  $Q_j^r = \lceil \delta_j D / |J| \rceil$ , with  $\delta_j \in \mathcal{U}[3, 4]$ , for every  $j \in J^r$ . Regarding the capacities of the unreliable facilities  $j \in J^u$ , these are generated in the same way, but the originally determined quantities are additionally multiplied by 0.5. Moreover, the amount of capacity is the same for each level of fortification. These settings are in line with those used by other authors, e.g. Nazemi and Parragh (2022).

### Capacity loss due to disruptions

To generate disruptions in the network, we first select the earliest time period  $t_s^0$  in which a disaster may occur in scenario  $s \in S$ , with  $t_s^0$  chosen at random from  $\{2, \dots, n_T\}$ . For a given unreliable location  $j \in J^u$  and scenario  $s \in S$ , we randomly determine the first period  $t_{js}^0$  in which this facility gets disrupted, with  $t_{js}^0 \in \{t_s^0, \dots, n_T\}$ . This choice applies to all fortification levels  $f \in F$ . On the other hand, the duration of the disturbance depends on  $f$  in such a way that the lower the level of fortification, the longer it takes for the facility to recover. Hence, the last period in which facility  $j$  with fortification level  $f$  is disturbed in scenario  $s$  is  $t_{jfs}^1 = \min \{t_{js}^0 + \Gamma \cdot (|F| + 1 - f) - 1, n_T\}$ , with  $\Gamma$  denoting a pre-specified parameter. We set  $\Gamma = 2$ , indicating that a facility with a high level of fortification (i.e.  $f = 3$ ) will be disturbed in two consecutive periods, while a facility with a low level of fortification (i.e.  $f = 1$ ) will be disturbed three times as long. The relative capacity loss faced by the unreliable facility  $j \in J^u$  in the first period with a disturbance is determined by

$$b_{jft^*s} = \begin{cases} 0.80 + 0.05 k_j & \text{if } f = 1 \\ 0.45 + 0.05 k_j & \text{if } f = 2 \\ 0.10 + 0.05 k_j & \text{if } f = 3 \end{cases} \quad \text{for } t^* = t_{js}^0$$

and  $k_j$  selected at random from the set  $\{0, 1, 2, 3\}$ . Therefore, a facility with a low fortification level ( $f = 1$ ) may lose between 80% ( $k_j = 0$ ) and 95% ( $k_j = 3$ ) of its capacity, whereas the capacity loss faced by a facility with a high fortification level ( $f = 3$ ) varies between 10% and 25%. For the medium fortification level  $f = 2$ , the capacity loss lies between 45% and 60%. Moreover, we assume that there will be a gradual capacity recovery until period  $t_{jfs}^1$ . Without loss of generality, the latter is represented by a fixed proportion  $q$  applied to all periods up to  $t_{jfs}^1$  such that

$$b_{jft's} = b_{jft^{(t-1)}s} - q \quad \text{for } t' = t_{js}^0 + 1, \dots, t_{jfs}^1$$

and  $q = \frac{b_{jft^*s}}{t_{jfs}^1 - t_{js}^0 + 1}$  with  $t^* = t_{js}^0$ . In all time periods before  $t_{js}^0$  and after  $t_{jfs}^1$ , the unreliable facility can be operated at its full capacity, i.e.  $b_{jfts} = 0$  for  $t \in \{1, \dots, t_{js}^0 - 1\} \cup \{t_{jfs}^1 + 1, \dots, n_T\}$ .

For the sake of illustration, let us assume that  $n_T = 12$ ,  $t_{js}^0 = 5$  in some scenario  $s$ ,  $f = 2$  and  $k_j = 1$ . In this case, the unreliable facility  $j$  with a medium level of fortification loses half of its original capacity in period 5 of scenario  $s$ . In the following periods and until the end of period 8 ( $t_{j2s}^1 = 5 + 2 \cdot (3 + 1 - 2) - 1$ ), the capacity recovery amounts to 12.5% per period ( $q = 0.5/(8 - 5 + 1)$ ). Hence, the capacity loss in periods 6, 7 and 8 is 37.5%, 25% and 12.5%, respectively.

Disruptions faced by primary suppliers  $i \in I^1$  in scenario  $s \in S$  start in period  $t_{is}^0$ , with  $t_{is}^0$  selected at random from  $\{t_s^0, \dots, n_T\}$ , and end in period  $t_{is}^1$ . The latter parameter is given by  $\min \{t_{is}^0 + \Gamma k_i - 1, n_T\}$ , with  $\Gamma = 2$  and  $k_i$  is chosen at random from the set  $\{0, 1, 2, 3\}$ . In all other periods  $t \in \{1, \dots, t_{is}^0 - 1\} \cup \{t_{is}^1 + 1, \dots, n_T\}$ , no disturbances occur, so that  $a_{its} = 0$ .

A similar scheme as for unreliable facilities is used to specify the capacity losses from period  $t_{is}^0$  to period  $t_{is}^1$ :

$$\begin{aligned} a_{it^*s} &= \mathcal{U}[0.3, 0.5] && \text{for } t^* = t_{is}^0, \\ a_{it's} &= a_{i(t'-1)s} - \frac{a_{it^*s}}{t_{is}^1 - t_{is}^0 + 1} && \text{for } t' = t_{is}^0 + 1, \dots, t_{is}^1. \end{aligned}$$

### Fixed facility costs

The fixed costs of opening reliable facilities at candidate sites  $j \in J^r$  reflect economies of scale as follows:

$$FC_{j1}^r = \lambda_j + \gamma_j \sqrt{Q_j^r}, \quad \lambda_j \in \mathcal{U}[5000, 10000], \quad \gamma_j \in \mathcal{U}[10000, 15000].$$

In the remaining strategic periods  $t \in T_L \setminus \{1\}$ , we take  $FC_{jt}^r = \bar{\lambda}_{jt} FC_{j(t-1)}^r$  with  $\bar{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$ . This scheme is also used for defining the fixed opening costs of unreliable facilities  $j \in J^u$  with high fortification level ( $f = 3$ ). A medium ( $f = 2$ ), resp. low ( $f = 1$ ), level of fortification incurs a fixed cost that is 80%, resp. 70%, of the cost of the high level. Furthermore, the fixed cost of operating a facility in a strategic period  $t \in T_L$  corresponds to 5% of the associated opening cost, i.e.,  $OC_{jt}^r = 0.05 FC_{jt}^r$  ( $j \in J^r$ ) and  $OC_{jft}^u = 0.05 FC_{jft}^u$  ( $j \in J^u$ ,  $f \in F$ ). In the remaining time periods  $t \in T \setminus T_L$ , we consider  $OC_{jt}^r = \bar{\lambda}_{jt} OC_{j(t-1)}^r$  ( $j \in J^r$ ) and  $OC_{jft}^u = \bar{\lambda}_{jt} OC_{jft(t-1)}^u$  ( $j \in J^u$ ,  $f \in F$ ), with  $\bar{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$ .

The fixed cost arising from the total damage of an unreliable facility is assumed to be 150% of its opening cost, i.e.,  $AC_{jft}^u = 1.5 FC_{jft}^u$  ( $j \in J^u$ ,  $f \in F$ ,  $t \in T_L$ ). In non-strategic periods, these fixed costs are defined in a similar way to the fixed operating costs, by multiplying the cost associated to period  $t - 1$  by  $\bar{\lambda}_{jt} \in \mathcal{U}[1.01, 1.03]$ .

### Distribution costs

In the first time period, the unit distribution cost from a reliable facility  $j \in J^r$  to a customer  $k \in K$  in scenario  $s \in S$  is defined as  $DC_{jk1s} = 1.5 \tilde{d}_{jk}$ , with  $\tilde{d}_{jk}$  denoting the Euclidean distance between the two locations  $j$  and  $k$ . In the remaining periods  $t \in T \setminus \{1\}$ , we take  $DC_{jkts} = \mu_{jkts} DC_{jk(t-1)s}$  with  $\mu_{jkts} \in \mathcal{U}[1.01, 1.03]$ . The costs of distributing one unit of the product from unreliable facilities  $j \in J^u$  to customers are determined in the same way in periods  $t \in \{1, \dots, t_{js}^0 - 1\} \cup \{t_{jfs}^1 + 1, \dots, n_T\}$ , i.e. periods without disturbances ( $f \in F$ ). In the remaining periods  $t = t_{js}^0, \dots, t_{jfs}^1$ , the distribution cost is more expensive, i.e.,  $DC_{jkts} = 1.1 \mu_{jkts} DC_{jk(t-1)s}$ .

Distribution costs from suppliers to facilities reflect the fact that transport from backup suppliers is more expensive than from primary suppliers, i.e.,  $SC_{i'jts} > SC_{ijts}$  for  $i' \in I^2$  and  $i \in I^1$ . To ensure this, we first define for  $i \in I^1$ ,  $j \in J$  and  $s \in S$ :

$$SC_{ij1s} = \tilde{d}_{ij}, \quad SC_{ijts} = \bar{\mu}_{ijts} SC_{ij(t-1)s}, \quad \bar{\mu}_{ijts} \in \mathcal{U}[1.01, 1.03], \quad t \in T \setminus \{1\}.$$

Next, we determine the smallest Euclidean distance from all backup suppliers to a given facility  $j \in J$ :  $d_j^{min} = \min_{i' \in I^2} \{\tilde{d}_{i'j}\}$ . In addition, we also identify the largest unit distribution cost to facility  $j \in J$  among the primary suppliers, for period  $t = 1$  and under scenario  $s$ :  $\overline{SC}_{j1s} = \max_{i \in I^1} \{SC_{ij1s}\}$ . We then set

$$SC_{i'j1s} = \frac{\overline{SC}_{j1s} + 1}{d_j^{min}} \cdot \tilde{d}_{i'j}, \quad i' \in I^2, j \in J, s \in S.$$

A monotone increasing cost sequence is generated in the following periods as follows:

$$SC_{i'jts} = \bar{\mu}_{i'jts} SC_{i'j(t-1)s} \quad i' \in I^2, j \in J, t \in \{2, \dots, n_T\}, s \in S.$$

Finally, only arcs whose length does not exceed a certain limit are considered. The latter corresponds to a pre-specified percentage  $p$  of the maximum distance generated,  $d^{max} = \max_{j \in J} \{\max_{i \in I} \tilde{d}_{ij}, \max_{k \in K} \tilde{d}_{jk}\}$ . In other words, variables  $x_{jktts}$  are only defined if  $\tilde{d}_{jk} \leq p d^{max}$ . Likewise, variables  $v_{ijts}$  are only considered for the arcs  $(i, j)$  such that  $\tilde{d}_{ij} \leq p d^{max}$ . In all the instances generated, we have taken  $p = 0.6$ .

### Tardiness penalty costs

Inspired by Correia and Melo (2021), the unit penalty cost for customer orders delivered with a delay is defined by

$$TC_{ktt'} = 0.01 \theta_k^t (t' - t)^2, \quad t \in T, t' = t, t + 1, \dots, \min\{t + \rho_k, n_T\}$$

with

$$\theta_k^t = \frac{\sum_{j \in J^r} OC_{jt}^r + \sum_{j \in J^u} \sum_{f \in F} OC_{jft}^u}{TD |J|} + \frac{\sum_{j \in J} \sum_{s \in S} DC_{jks}}{|J| |S|}$$

and  $TD$  denoting the total maximum demand across all scenarios. Accordingly,  $TD = \sum_{k \in K} \sum_{t \in T} \max_{s \in S} d_{kts}$ . We note that the unit tardiness cost is a function of the average fixed facility operating costs and the average variable distribution costs to the customers.

## Appendix B Optimal network configuration

To illustrate how the network configuration evolves over the strategic periods, we have selected the right lexicographic solution (minimisation of the total expected unmet demand) of an instance with 50 customers, 24 time periods, 3 primary suppliers, 3 backup suppliers, 5 potential locations for reliable facilities and 5 potential locations for unreliable facilities. Recall that the corresponding optimal (first-stage) location decisions are depicted in Figure 5 (a) in Section 5.3. Figure 9 displays the allocation of primary suppliers to the operating facilities and the flows

across the network for a particular scenario exhibiting monotone increasing demand between any two consecutive periods.

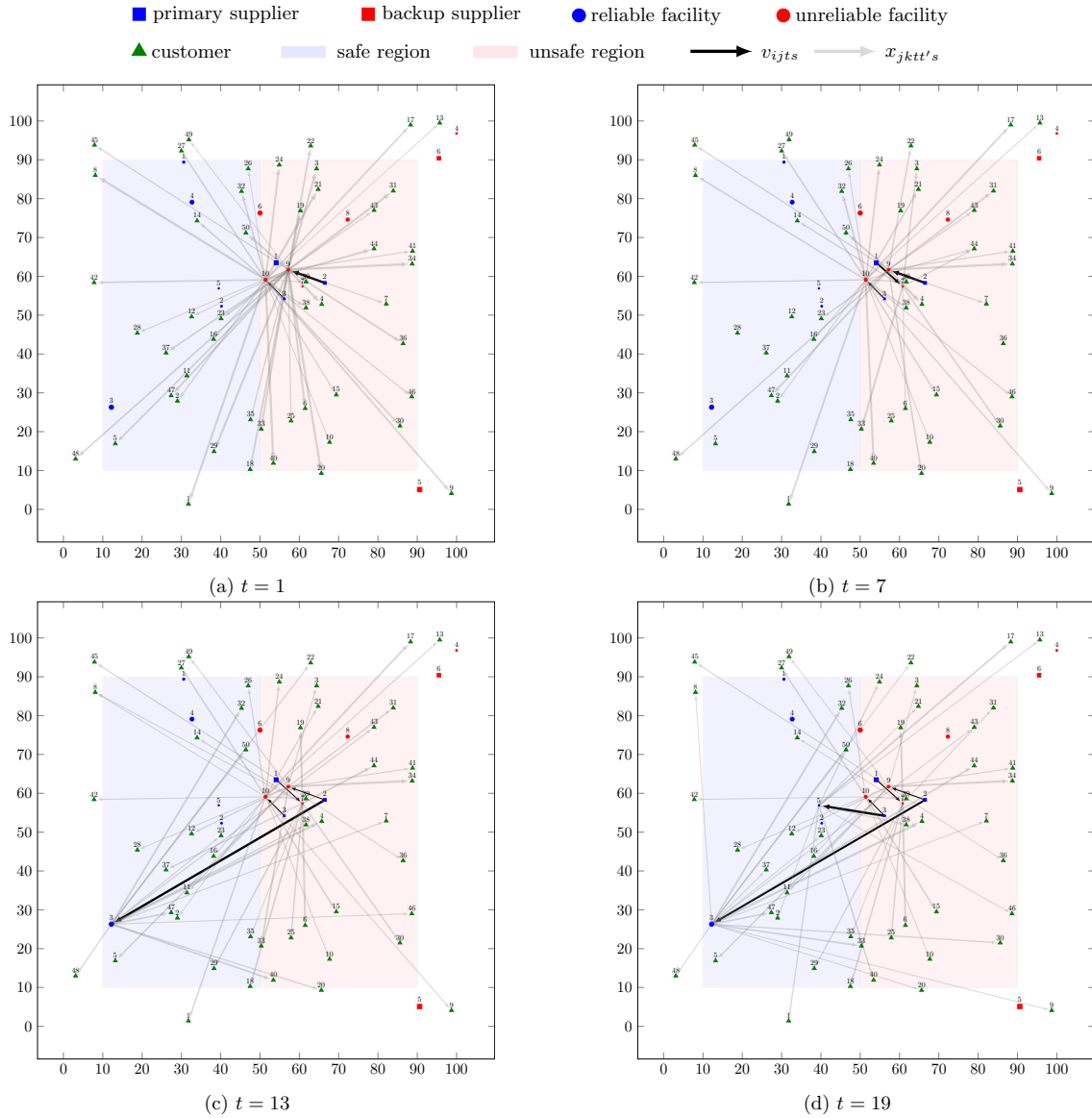


Figure 9: Optimal network configuration in the right lexicographic solution (sol. 1) and scenario  $s = 15$  for an instance with  $|K| = 50$ ,  $|T| = 24$ ,  $|S| = 30$ ,  $|I^1| = |I^2| = 3$  and  $|J^r| = |J^u| = 5$ .

Initially ( $t = 1$ ), two unreliable facilities are opened at sites 9 and 10, both with a medium level of fortification. The largest quantity of product is delivered by primary supplier 2 to facility 9, while primary supplier 3 serves facility 10. In the next strategic period ( $t = 7$ ), a third unreliable facility is opened in location 7, but with the highest fortification level, and primary supplier 1 is assigned to it, a choice possibly justified by its proximity to this location. In the

strategic period  $t = 13$ , the deployment of the reliable facility 3 takes place. Primary supplier 2 has sufficient capacity to serve this facility (in addition to location 9), although it is further away than the other primary suppliers. Note that the allocation of suppliers to facilities does not depend on a specific scenario. Finally, in the last strategic period ( $t = 19$ ), the reliable facility 5 starts operating and is served by primary supplier 3.

In this particular scenario, the unreliable facilities 7, 9 and 10 experience capacity losses in periods 14–17, 17–20 and 17–18, respectively. It is therefore not surprising that the reliable facilities 3 and 5 start operating in the second half of the planning horizon ( $t \geq 13$ ). Moreover, the backup supplier 5 is activated in periods 23 and 24 to compensate for the partial unavailability of the primary supplier 3 in these periods. Accordingly, the reliable facility 3 is serviced by suppliers 3 and 5. Since in this Pareto-optimal solution all customer demand is satisfied, deliveries occur for all customers not only in the strategic periods depicted in figures (a)–(d), but also in all other (tactical) periods.

## Appendix C Supply chain network design with limited responsiveness

Table 6 summarises the characteristics of the efficient solutions returned by the two executions of Algorithm 1 when late deliveries to customers are not permitted, i.e.  $\rho_k = 0$  for all  $k \in K$ . Note that this special case limits the responsiveness of the network to disruptions and fluctuations in demand. The structure of this table is the same as that of Table 4 in Section 5.2.

Since the index  $t'$  can now be eliminated from the flow variables  $x_{jkt't's}$  defined in (6), the objective functions (7) and (8) as well as constraints (17), (22)–(25) and (31) are simplified. Therefore, compared to the case  $\rho_k = 2$  ( $k \in K$ ), the two-stage stochastic formulation has on average 64.6 percent fewer continuous variables, but about 1 percent more constraints due to inequalities (33).

As expected, a significantly larger number of instances can be solved to optimality within the time limit of 7,200 seconds by Algorithm 1 run exclusively with CPLEX, namely G1–G8 and G10. For the remaining instances G9, G11 and G12, high-quality solutions are achieved with an optimality gap smaller than 2 percent. However, the G13 instances with 180 scenarios remain very challenging, and CPLEX is once again unable to even find feasible solutions to all of them. Specifically, only the approximate lexicographic solutions (sol. 1 and sol. 4) are available for the five instances in this group. In contrast, Algorithm 1 with the two-phase heuristic requires on average 95 percent less CPU time than CPLEX and identifies feasible solutions to all instances, including G13. Furthermore, the quality of the heuristic solutions is very impressive, with an optimality gap not exceeding 1.74 percent. Finally, the heuristic outperforms CPLEX for some of the larger instances, namely in G9, G12 and G13.

Figure 10 (a) reports the average number of time periods a facility is operated when Algo-

Table 6: Characteristics of the solutions identified when delivery delays to customers are not permitted ( $\rho_k = 0, k \in K$ ).

Inst.	Sol.	Algorithm 1 with CPLEX				Algorithm 1 with two-phase heuristic		
		Total cost (m.u.)	Demand met (%)	MIP gap (%)	CPU time (s)	% cost dev. to CPLEX	Demand met (%)	% CPU to CPLEX
G1	1	10,237,223.7	100.0	0.00	15.6	0.15	100.0	14.10
	2	6,625,043.1	75.0	0.00	28.1	0.41	75.0	8.47
	3	4,501,614.9	50.0	0.00	21.9	0.82	50.0	5.75
	4	2,827,742.4	25.0	0.00	6.3	0.00	25.0	4.13
G2	1	21,398,390.1	100.0	0.00	55.7	0.73	100.0	12.93
	2	13,443,138.6	75.0	0.00	172.6	0.85	75.0	4.73
	3	8,565,923.1	50.0	0.00	119.0	0.92	50.0	2.91
	4	4,832,311.2	25.0	0.00	13.8	0.85	25.0	6.38
G3	1	32,151,960.8	100.0	0.00	133.3	0.43	100.0	7.17
	2	20,651,823.6	75.0	0.00	404.7	0.97	75.0	5.87
	3	12,890,635.8	50.0	0.00	276.2	0.42	50.0	2.32
	4	7,192,008.6	25.0	0.00	25.2	0.79	25.0	7.14
G4	1	10,725,824.5	100.0	0.00	46.6	0.90	100.0	12.66
	2	6,590,065.7	75.0	0.00	122.4	1.14	75.0	7.83
	3	4,248,102.9	50.0	0.00	68.0	0.64	50.0	4.97
	4	2,756,980.7	25.0	0.00	15.1	0.00	25.0	5.30
G5	1	22,297,469.4	100.0	0.00	176.9	0.37	100.0	10.21
	2	14,203,377.6	75.0	0.00	588.2	0.63	75.0	4.44
	3	8,851,517.3	50.0	0.00	336.3	1.01	50.0	5.26
	4	5,346,748.9	25.0	0.00	40.8	0.77	25.0	8.09
G6	1	31,514,352.2	100.0	0.00	369.2	1.49	100.0	8.35
	2	19,433,285.9	75.0	0.00	1,877.5	1.26	75.0	3.14
	3	12,806,659.8	50.0	0.00	1,241.2	1.18	50.0	4.08
	4	7,331,246.6	25.0	0.00	72.8	1.05	25.0	6.95
G7	1	17,722,727.9	100.0	0.00	1,852.7	0.66	100.0	0.49
	2	10,326,481.0	75.0	0.00	237.1	0.53	75.0	2.87
	3	6,834,607.6	50.0	0.00	143.5	1.57	50.0	3.60
	4	4,042,580.3	25.0	0.00	17.1	0.91	25.0	7.02
G8	1	37,155,835.3	100.0	0.00	1,338.7	1.41	100.0	1.75
	2	22,979,329.4	75.0	0.00	2,101.6	0.32	75.0	2.45
	3	14,564,967.4	50.0	0.00	1,074.5	0.30	50.0	2.60
	4	8,212,232.5	25.0	0.00	57.4	0.42	25.0	4.67
G9	1	53,387,439.5	100.0	0.00	2,475.1	1.07	100.0	1.30
	2	33,562,143.7	75.0	0.85	5,093.9	1.61	75.0	2.21
	3	20,721,062.4	50.0	0.88	4,570.9	-0.64	50.0	0.69
	4	11,263,568.9	25.0	0.00	101.9	0.00	25.0	4.53
G10	1	18,441,486.9	100.0	0.00	997.7	0.68	100.0	1.32
	2	11,339,602.0	75.0	0.00	1,154.9	0.93	75.0	2.77
	3	7,182,100.6	50.0	0.00	567.6	1.44	50.0	2.58
	4	4,388,481.8	25.0	0.00	55.1	0.00	25.0	2.65
G11	1	36,548,618.2	100.0	0.29	3,933.9	1.57	100.0	1.56
	2	23,021,511.0	75.0	1.38	7,200.2	0.70	75.0	1.75
	3	14,214,956.7	50.0	0.60	4,188.1	0.62	50.0	1.81
	4	8,155,230.7	25.0	0.00	143.9	0.75	25.0	5.32
G12	1	56,791,349.4	100.0	0.65	6,493.4	1.74	100.0	2.31
	2	36,973,150.7	75.0	1.68	6,387.3	-0.43	75.0	6.37
	3	22,724,806.5	50.0	1.93	6,578.9	-1.65	50.0	3.55
	4	11,988,338.1	25.0	0.00	305.6	0.90	25.0	2.98
G13	1	63,694,523.4	100.0	3.63	2,641.4	-1.09	98.4	27.37
	2	-	-	-	7,200.0	-	74.8	18.76
	3	-	-	-	7,200.2	-	49.8	16.67
	4	12,801,782.0	25.0	2.99	1,508.5	-2.01	25.1	8.68

– no feasible solution found.

rithm 1 is executed with CPLEX and delivery delays to customers are allowed. In Figure 10 (b) the opposite case is depicted. Recall that a similar pattern is also observed for the heuristic solutions (see Figure 8 in Section 5.3).

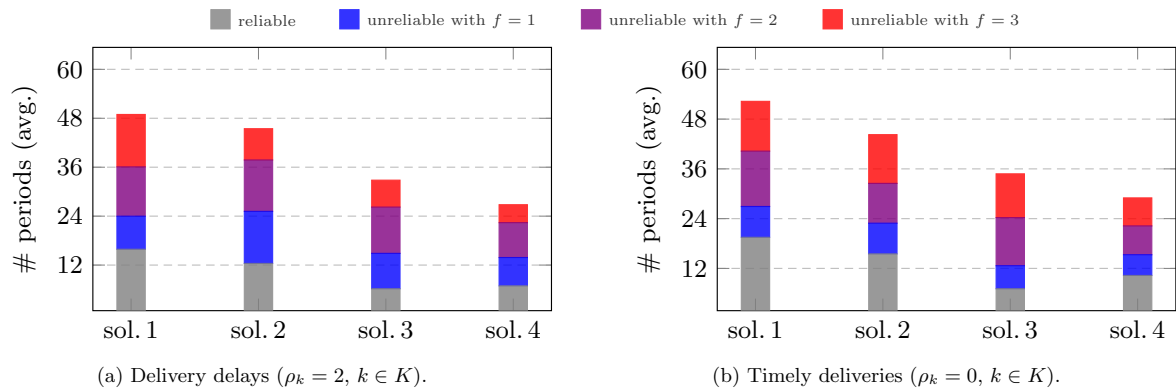


Figure 10: Average number of periods that a facility is in operation for solutions returned by Algorithm 1 with CPLEX (instances G1–G12).

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