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# Redesigning a three-echelon logistics network over multiple time periods with transportation mode selection and outsourcing opportunities

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**Abstract**

We address the problem of designing/redesigning a multi-echelon logistics network over a multi-period planning horizon. Strategic decisions comprise opening new plants and warehouses at candidate sites and selecting their capacities from a set of available discrete sizes. Capacity expansion may occur more than once over the time horizon both at new locations and at existing facilities. Capacity contraction is a viable option as well that involves closing existing plants and/or warehouses. The operation of the network is also subject to logistics decisions involving supplier selection in conjunction with procurement, production, and distribution of multiple products. Distribution channels are to be identified in each time period as well as the modes of transportation for raw materials and end products. Finally, a strategic choice between in-house manufacturing and a mixed approach with product outsourcing is to be taken. We propose a mixed-integer linear programming model and develop several valid inequalities to enhance the original formulation. To gain insight into the complexity of the problem at hand, an extensive computational study is performed with randomly generated instances that are solved with standard mathematical optimization software. Useful managerial insights are derived from varying several parameters and analyzing the impact of different business strategies on various segments of the logistics network.

**Keywords:** logistics network design/re-design, multiple periods, transportation mode selection, product outsourcing, mixed-integer linear programming

# Contents

# 1 Introduction

Logistics network design (LND) is the strategic planning process for optimizing the configuration of a supply chain. In broad terms, LND involves determining the optimal number and location of facilities (e.g., manufacturing plants and warehouses), allocating capacity and technology requirements to facilities, and deciding on the flow of products throughout the supply chain such that customer demands are satisfied at minimum cost or maximum profit.

Depending on the actual business requirements, a company may consider either redesigning its supply chain or designing a new chain in order to align its logistics network with new business conditions or to meet new strategic objectives. Logistics network re-design (LNRD) is typically prompted by changing market and business conditions, frequently in conjunction with increased cost pressure and service requirements. These factors compel companies, for example, to expand or restructure their supply chain operations. If a company grows through external acquisitions, network re-design addresses the integration of acquired operations to fully exploit all benefits and synergies at supply chain level. In contrast, the need for designing a new network arises when a company enters new geographical markets or grows into new product segments. So-called "greenfield" approaches are less frequent compared with re-design projects. However, a company may wish to evaluate how far its existing logistics network deviates from an optimal configuration.

The role of LND and LNRD has become even more prominent in today's business environment, as companies have to cope with a variety of challenges in order to deliver outstanding supply chain performance. Strategic network decisions affect all levels of supply chain management and provide the framework for successful tactical and operational supply chain processes. As highlighted by Ballou [4] and Harrison [13], a network re-design project can result in a 5 to 15 percent reduction of the overall logistics costs, with 10 percent being often achieved.

In this paper, an integrated and comprehensive view of the supply chain is taken by considering raw material suppliers, manufacturing facilities, warehouses, transportation channels, and customer zones as shown in Figure 1. In an LNRD approach, a network is already in place with a number of plants and warehouses being operated at fixed locations (these are highlighted by the dashed lines in the figure). A variety of decisions have to be made regarding facility location and logistics functions along the supply chain. The former concern opening new plants and/or warehouses at potential sites (the facilities without dashed lines in Figure 1) and selecting their capacity levels from a set of available discrete sizes. This is motivated by the fact that capacity

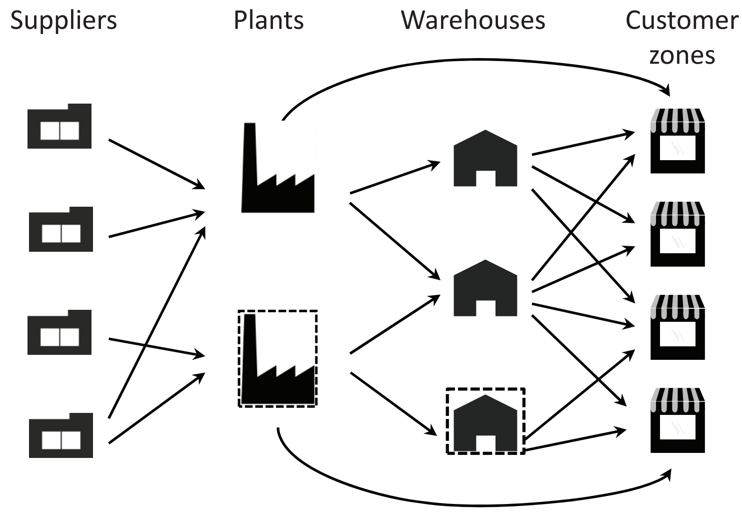


Figure 1: Possible configuration of a multi-echelon logistics network

is often purchased in the form of equipment which is only available at a few discrete sizes. As strategic planning for multiple time periods is considered, capacity can be acquired more than once over the time horizon both in new and existing locations. Capacity contraction is also possible by closing existing plants and/or warehouses. These options are attractive when adjustments in the network configuration of a company are required to enable future growth in new markets or to meet increasing product demand in current markets. In an LND approach, by contrast, the scope of the location decisions is limited to deciding on the optimal size, number, and location of new facilities.

Logistics decisions, the second group of key business decisions, involve supplier selection in conjunction with procurement as well as production and distribution decisions. Furthermore, a strategic choice between in-house manufacturing, outsourcing or a mixed approach is to be taken.

In the network depicted in Figure 1, multiple types of products are manufactured at plants by processing sub-assemblies and components, hereafter called raw materials. The latter can be procured from various suppliers taking into account their availability and cost. Finished products can be delivered to warehouses or shipped directly to customer zones. The flow of goods throughout the network and the use of transportation modes are to be determined in each time period. In addition, end products can also be purchased from external sources and consolidated at the warehouses. The objective is to determine the optimal network configuration over a planning horizon so as to minimize the total cost.

The contribution of this paper is threefold. First, we propose a new mathematical model that significantly generalizes several existing network design models. This is accomplished through the integration of various strategic features of practical relevance into a single model. The new model can be used both for redesigning a logistics network that is already in place and for designing a new supply chain. Applications can be found in a number of industrial contexts, e.g. consumer goods industry. Second, we perform a computational study on a large set of randomly generated instances and assess the impact of various problem characteristics on the ability of state-of-the-art optimization software to solve problem instances within a reasonable time limit. This analysis is performed using the proposed model strengthened with additional valid inequalities. Third, valuable managerial insights are derived that illustrate the far-reaching implications of strategic network design on different supply chain segments (location, supply, manufacturing, distribution, outsourcing). Without the support of the model developed in this paper it would otherwise be difficult to obtain most of these insights. Given the typically high investment volumes and the limited reversibility of strategic decisions, it is essential for stakeholders to perceive the impact of (re-)design and logistics decisions on supply chain performance.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature dedicated to LND/LNRD and describe its relation to our new model. Section 3 introduces a mixed-integer linear programming formulation for logistics network design and re-design. Section 4 presents various ways of tightening some of the constraints of the model. Valid inequalities are proposed in Section 5 to enhance the original formulation in an attempt to strengthen its linear relaxation bound. In Section 6, a methodology to randomly generate test instances reflecting real-world situations is presented. Section 7 reports on the computational experiments carried out and the managerial insights gained from analyzing various scenarios involving the reconfiguration of existing logistics networks as well as the design of new networks. Finally, in Section 8, conclusions are provided and directions for future research are identified.

## 2 Literature review

Beginning with the pioneering work of Geoffrion and Graves [11] on multi-commodity distribution network design in 1974, a large number of optimization-based approaches have been proposed for the design of logistics networks as shown by the recent surveys of Melo et al. [17] and Mula et al. [20]. These works have resulted in significant improvements in the modeling of these problems as well as in algorithmic and computational efficiency. One of the reasons that contributes to such a large number of literature references is the variety of characteristics that can be taken into account in LND problems: type of planning horizon (single or multi-period), facility location and sizing, number of echelons and type of distribution levels, multi-stage production taking the bill of materials (BOM) into account, and transportation mode selection, among others.

Although the timing of facility locations and expansions over an extended time horizon is of major importance to decision-makers in strategic network design, the majority of the literature addresses single-period problems, e.g., Babazadeh et al. [2], Cordeau et al. [8], Elhedhli and Gzara [9], Eskigun et al. [10], Olivares et al. [21], and Sadjady and Davoudpour [22]. Our research is different in that a multi-period planning horizon is considered. Unlike our work, in some multi-period LND problems facility sizing is static, meaning that facilities cannot have their capacities expanded or contracted over the planning horizon. The model proposed by Gourdin and Klopfenstein [12] falls into this category.

We will focus next on multi-period LND and LNRD problems with dynamic facility sizing decisions. In particular, we will discuss the extent to which the features of the model to be detailed in Section 3 differ from those reported so far in the literature.

To re-design a two-layer network, Antunes and Peeters [1] suggest a modeling framework that allows opening new facilities and closing existing locations, as well as expanding and contracting capacity. Budget constraints are taken into account over the time horizon. Simulated annealing is used to find feasible solutions.

Melo et al. [16] study the re-design of a multi-echelon network considering facility expansion and contraction. This feature is modeled through moving capacity from existing facilities to new facilities over the planning horizon. Network re-design decisions (opening, closing, and relocating facilities) are subject to budget constraints in each time period. General purpose optimization software is used to solve small and medium-sized problem instances. Melo et al. [18, 19] also developed heuristic procedures for this special form of network re-design. The numerical experiments indicate that good solutions can be obtained for large-sized instances within acceptable computational time.

Hinojosa et al. [14] address a multi-echelon, multi-commodity network re-design problem with inventory strategic decisions at warehouses and outsourcing of demand. Commodities flow from manufacturing plants to customers via warehouses. Outsourced commodities are delivered directly to customers. An initial network configuration is considered that gradually changes over a multi-period horizon through opening new facilities and closing existing facilities. A lower bound is imposed on the number of plants and warehouses operating in the first and last time periods. The problem is solved using a Lagrangian-based procedure through decomposition into simpler subproblems. A heuristic method is then applied to obtain a feasible solution.

Thanh et al. [23] consider a multi-period, multi-product logistics network comprising suppliers, plants, warehouses (public and private), and customers. Strategic decisions include facility location and capacity acquisition as well as supplier selection, production, distribution, and inventory planning. In particular, plants and warehouses can have their capacities expanded (but not contracted) over the time horizon. Production decisions take into account the BOM and intermediate components can be sub-contracted to an external plant. Furthermore, products flow downstream not only between adjacent supply chain layers but also directly from plants to a selected subset of customers. In addition, plants may exchange components. To identify the least-cost network configuration, Thanh et al. [24] propose an LP-rounding heuristic. This method is later improved by combining it with DC programming (cf. Thanh et al. [25]).

Bashiri and Badri [5] address the problem of designing a new supply chain network with a similar topology to that considered in [23]. The objective is to find the network configuration that maximizes the total net profit subject to a given budget in each time period. In this work, demand requirements may not be completely satisfied. Strategic decisions include opening and expanding new plants and private (company-owned) warehouses. In addition, public warehouses can be hired for a pre-specified number of time periods and variable costs are charged for their operation. The proposed model is extended in Bashiri et al. [6] through introducing different time scales for strategic and tactical decisions. In particular, the latter are made in each time period, whereas network design decisions are only made over a subset of periods of the planning horizon. The models presented in [5, 6] are solved with a general-purpose optimization solver. Later, Badri et al. [3] develop a Lagrangian-based approach. Feasible solutions are obtained by dualizing the budget constraints for opening new facilities or expanding the capacities of existing facilities and adding some constraints to the subproblems to guarantee feasibility.

In order to relate the existing literature to the LND and LNRD problems that are studied in this paper, a classification of the aforementioned works is given in Table 1. This table is not intended to provide an exhaustive list of all the features described in this section but rather to illustrate the extent to which our research generalizes the existing literature.

The characteristics of the surveyed LND and LNRD problems are classified according to five categories. The category "Network" comprises the number of layers in the supply chain, including the customer level (column 2) and the number of layers involving location decisions (column 3). Furthermore, it is specified whether products can be distributed directly from higher level facilities to customer locations (column 4). The second category (column 5) refers to the type of planning horizon. The category "Facility sizing" summarizes the type of capacity decisions that can be made. To this end, column 6 indicates if the size of each facility is limited (G: global capacities) and if capacity levels are selected from a set of available discrete sizes (M: modular capacities). In column 7, the type of capacity planning is described through expansion (E) and/or downsizing (D) over the time horizon. Moreover, to distinguish network design from network re-design problems, the latter are highlighted with the letter C, meaning that existing facilities can be closed. The category in column 8 refers to the selection of transportation modes. Finally, the category "Products" outlines characteristics related to the number of products (column 9), multi-stage production taking into account the BOM (column 10), satisfaction of demand requirements (column 11), and the possibility of outsourcing components or end products as an alternative to in-house manufacturing (column 11).

The last row of Table 1 highlights the main features of the model to be detailed in Section 3. It can be seen that in our model various features are considered simultaneously in a multi-period setting. To the best of our knowledge, this integration of practically relevant features into a single model has not been addressed in the related literature so far.

| | Network | | | Multiple | Facility sizing | | Transportation | Products | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | Layers | Location | Direct | time | Capacity | Capacity | Mode selection | Multiple | BOM | Demand | Outsourcing |
| References | | levels | shipments | periods | type | changes | | | | satisfaction | |
| Antunes and Peeters [1] | 2 | 1 | | ✓ | M,G | E,D,C | | | | ✓ | |
| Babazadeh et al. [2] | 3 | 2 | ✓ | | M | | ✓ | | | | ✓ |
| Badri et al. [3] | 4 | 2 | | ✓ | M,G | E | | ✓ | ✓ | | |
| Bashiri and Badri [5] | 4 | 2 | | ✓ | M,G | E | | ✓ | ✓ | | |
| Bashiri et al. [6] | 4 | 2 | | ✓ | M,G | E | | ✓ | ✓ | | |
| Cordeau et al. [8] | 4 | 2 | | | G | | ✓ | ✓ | ✓ | ✓ | |
| Elhedhli and Gzara [9] | 3 | 2 | | | M | | | ✓ | | ✓ | |
| Eskigun et al. [10] | 3 | 1 | ✓ | | G | | ✓ | | | ✓ | |
| Gourdin and Klopfenstein [12] | 2 | 1 | | ✓ | M,G | E | | | | ✓ | |
| Hinojosa et al. [14] | 3 | 2 | | ✓ | G | C | | ✓ | | ✓ | ✓ |
| Melo et al. [16, 18, 19] | N | N | ✓ | ✓ | G | E,C | | ✓ | ✓ | ✓ | |
| Olivares et al. [21] | 3 | 1 | | | G | | ✓ | | | ✓ | |
| Sadjady and Davoudpour [22] | 3 | 2 | | | M | | ✓ | ✓ | | ✓ | |
| Thanh et al. [23, 24, 25] | 4 | 2 | ✓ | ✓ | M,G | E,C | | ✓ | ✓ | ✓ | ✓ |
| **New model** (cf. Section 3) | **4** | **2** | ✓ | ✓ | **M,G** | **E,C** | ✓ | ✓ | ✓ | ✓ | ✓ |

C: Closing existing facilities
D: Downsizing (capacity contraction)
E: Capacity expansion
G: Global capacities
M: Modular capacities
N: No limit on the number of network layers and location levels

Table 1: Classification of the existing literature

# 3 Mathematical model

In this section, we introduce a mathematical model for a comprehensive LNRD problem. The formulation integrates location and capacity choices for plants and warehouses with supplier and transportation mode selection as well as outsourcing options over multiple time periods. As it will be seen, the model also can be used for designing a new logistics network.

## 3.1 Index sets

Table 2 describes the index sets that are used.

| Symbol | Description |
|---|---|
| $T$ | Set of time periods |
| $S$ | Set of suppliers |
| $L^e$ | Set of existing plants at the beginning of the time horizon |
| $L^n$ | Set of potential sites for locating new plants |
| $L = L^e \cup L^n$ | Set of all plant locations |
| $K_L$ | Set of discrete capacity levels that can be installed in plant locations |
| $W^e$ | Set of existing warehouses at the beginning of the time horizon |
| $W^n$ | Set of potential sites for locating new warehouses |
| $W = W^e \cup W^n$ | Set of all warehouse locations |
| $K_W$ | Set of discrete capacity levels that can be installed in warehouse locations |
| $C$ | Set of customer zones |
| $R$ | Set of raw materials |
| $P$ | Set of end products |
| $M$ | Set of transportation modes |
| $OD$ | Set of origin-destination pairs in the logistics network |

Table 2: Index sets

For notational convenience, we introduce the set $OD$ with all origin-destination pairs in the logistics network (recall Figure 1):

$$\begin{aligned} OD &= \{(s,\,\ell) : s \in S,\, \ell \in L\} \cup \{(\ell,\,w) : \ell \in L,\, w \in W\} \cup \\ &\quad \{(\ell,\,c) : \ell \in L,\, c \in C\} \cup \{(w,\,c) : w \in W,\, c \in C\} \end{aligned}$$

The links $(s,\,\ell)$ are used to transport raw materials $r \in R$, while all other origin-destination pairs concern links to move end products, $p \in P$.

## 3.2 Model parameters

Table 3 summarizes all costs. The parameters $FC_{t,j}$ and $SC_{t,j}$ reflect fixed costs associated with location decisions. The first term comprises the fixed costs of setting up an infrastructure for a new plant or warehouse (e.g., property acquisition) in time period $t$. Facility closing costs (the second term) are incurred when an existing plant or warehouse is closed in period $t$. Other facility costs concern the acquisition of capacity in both new and existing locations and the operation of that capacity. To this end, $IC_{t,j,k} = I_{t,j,k} + \sum_{\tau=t}^{|T|} OC^n_{\tau,j,k}$, where $I_{t,j,k}$ denotes the fixed installation cost of capacity level $k \in K_L \cup K_W$ in location $j \in L \cup W$ in period $t \in T$, and $OC^n_{\tau,j,k}$ is the fixed operating cost in period $\tau \geq t$. The installation costs $IC_{t,j,k}$ reflect economies of scale. Observe that it may be necessary to expand the logistics network to respond to increasing demands over the time horizon. Expansion plans may result in extending the capacity of existing facilities and/or establishing new facilities with given capacity.

| Symbol | Description |
|---|---|
| $FC_{t,j}$ | Fixed cost of establishing a new facility in location $j \in L^n \cup W^n$ in period $t \in T$ |
| $SC_{t,j}$ | Fixed cost of closing an existing facility in location $j \in L^e \cup W^e$ in period $t \in T$ |
| $IC_{t,j,k}$ | Fixed cost of installing capacity level $k \in K_L \cup K_W$ in location $j \in L \cup W$ in period $t \in T$ and operating it until the end of the time horizon |
| $OC_{t,j}$ | Fixed cost of operating existing facility $j \in L^e \cup W^e$ in period $t \in T$ |
| $PC_{t,s,r}$ | Cost of procuring one unit of raw material $r \in R$ from supplier $s \in S$ in period $t \in T$ |
| $MC_{t,\ell,p}$ | Cost of manufacturing one unit of product $p \in P$ at plant $\ell \in L$ in period $t \in T$ |
| $TC_{t,o,d,i,m}$ | Cost of shipping of one unit of item $i \in R \cup P$ using transportation mode $m \in M$ from origin $o$ to destination $d$, $(o,d) \in OD$, in period $t \in T$ |
| $EC_{t,w,p}$ | Cost of purchasing one unit of product $p \in P$ from an external source to be processed at warehouse $w \in W$ in period $t \in T$ |

Table 3: Fixed and variable costs

Logistics costs include procurement, production, outsourcing, and distribution costs. The latter depend on the choice of transportation modes for moving raw materials and end products throughout the network. The available transportation modes differ with respect to their variable costs and capacities. For example, rail and road freight transport may be viable options with known trade-offs (cost, service time, environmental impact). External costs concern the purchase of end products from other companies. The consolidation of outsourced products takes place at the warehouses. A pre-specified percentage of the total customer demand for each end product sets an upper bound on the total quantity that can be acquired from an external source. This option may be attractive when the cost of setting up a new facility to process given items is higher than the cost of outsourcing them.

Table 4 introduces additional input parameters. We assume that the available capacity levels are sorted in non-decreasing order of their sizes. For plant locations $\ell \in L$ this means that $Q_{\ell,1} < Q_{\ell,2} < \ldots < Q_{\ell,|K_L|}$. Similar conditions hold for warehouse locations. As distinct technologies may be used by different plants to manufacture a given product $p \in P$, we consider a production consumption factor $\mu_{\ell,p}$ for every plant location $\ell \in L$. Moreover, the quantity of raw materials required to manufacture one unit of a specific product may also differ among the various plants. Plant-dependent BOMs are specified by the parameters $a_{\ell,r,p}$. In contrast, the usage of warehouse capacity by an end product does not depend on the warehouse location, thus a consumption factor $\gamma_p$ is assumed for every $p \in P$.

| Symbol | Description |
|---|---|
| $a_{\ell,r,p}$ | Number of units of raw material $r \in R$ required to manufacture one unit of product $p \in P$ in plant $\ell \in L$ |
| $QS_{t,s,r}$ | Capacity of supplier $s \in S$ for raw material $r \in R$ in period $t \in T$ |
| $\mu_{\ell,p}$ | Production capacity usage by one unit of product $p \in P$ in plant $\ell \in L$ |
| $\gamma_p$ | Handling capacity usage by one unit of product $p \in P$ in a warehouse |
| $Q_j^e$ | Capacity of existing facility $j \in L^e \cup W^e$ at the beginning of the time horizon |
| $Q_{j,k}$ | Capacity of level $k \in K_L \cup K_W$ that can be installed in facility $j \in L \cup W$ |
| $\overline{Q}_j$ | Maximum overall capacity of facility $j \in L \cup W$ in each time period |
| $QM_{t,o,d,m}$ | Capacity of transportation mode $m \in M$ from origin $o$ to destination $d$, $(o,d) \in OD$, in period $t \in T$ |
| $\sigma_{i,m}$ | Capacity utilization factor by one unit of item $i \in R \cup P$ in transportation mode $m \in M$ |
| $d_{t,c,p}$ | Demand of customer zone $c \in C$ for product $p \in P$ in period $t \in T$ |
| $\lambda_{t,p}$ | Fraction of the total demand for product $p \in P$ in period $t \in T$ that can be directly shipped from plants to customers ($0 \leq \lambda_{t,p} < 1$) |
| $\beta_{t,p}$ | Fraction of the total demand for product $p \in P$ in period $t \in T$ that can be supplied by an external source ($0 \leq \beta_{t,p} < 1$) |
| $\alpha_j$ | Factor used to set a minimum capacity usage at facility $j \in L \cup W$ ($0 \leq \alpha_j < 1$) |

Table 4: Additional input parameters

Constant $\alpha_j$ is used to set a minimum throughput level at facility $j$, the latter being defined by the capacity installed in location $j \in L \cup W$ multiplied by $\alpha_j$. In this way, it will be guaranteed that facilities are operated at least at a meaningful level.

## 3.3 Decision variables

All decisions are implemented at the beginning of each time period. As indicated in Table 5, strategic decisions on facility location and capacity acquisition are ruled by binary variables, while tactical logistics decisions are described by continuous variables. The statuses of *new* facilities (i.e. plants, warehouses) over the time horizon are controlled by the variables $y^n_{t,j}$. If a new plant or warehouse is established at the beginning of period $t$ in site $j \in L^n \cup W^n$ then $y^n_{t,j} = 1$ and $y^n_{\tau,j} = 0$ for all other periods $\tau \in T, \tau \neq t$. Regarding the *existing* facilities, if facility $j \in L^e \cup W^e$ ceases to operate at the beginning of period $t$ then $y^e_{t,j} = 1$ and $y^e_{\tau,j} = 0$ for all periods $\tau \in T, \tau \neq t$. Observe that if a *new* facility $j$ is available in period $t$ then $\sum_{\tau=1}^{t} y^n_{\tau,j} = 1$. Similarly, if an *existing* facility $j$ is operated in period $t$ then $\sum_{\tau=1}^{t} y^e_{\tau,j} = 0$.

| Symbol | Description |
|---|---|
| $y^n_{t,j}$ | 1 if a new facility is established in location $j \in L^n \cup W^n$ at the beginning of period $t \in T$, 0 otherwise |
| $y^e_{t,j}$ | 1 if an existing facility in location $j \in L^e \cup W^e$ is closed at the beginning of period $t \in T$, 0 otherwise |
| $u_{t,j,k}$ | 1 if capacity level $k \in K_L \cup K_W$ is installed in location $j \in L \cup W$ in period $t \in T$, 0 otherwise |
| $x_{t,o,d,i,m}$ | Quantity of item $i \in R \cup P$ shipped in period $t \in T$ from origin $o$ to destination $d$, $(o,\, d) \in OD$, using transportation mode $m \in M$ |
| $z_{t,w,p}$ | Quantity of product $p \in P$ provided by an external source to warehouse $w \in W$ in period $t \in T$ |

Table 5: Decision variables

## 3.4 Network re-design constraints

In this section, we describe in detail the specific constraints that compose our LNRD problem.

### 3.4.1 Supplier-related constraints

The following constraints impose the required conditions for the selection of suppliers in each time period and for the quantities of raw materials to be procured.

$$\sum_{\ell \in L} \sum_{m \in M} x_{t,s,\ell,r,m} \leq QS_{t,s,r} \qquad t \in T,\ s \in S,\ r \in R \tag{1}$$

$$\sum_{r \in R} \sigma_{r,m}\, x_{t,s,\ell,r,m} \leq QM_{t,s,\ell,m} \qquad t \in T,\ s \in S,\ \ell \in L,\ m \in M \tag{2}$$

Constraints (1) limit the quantity of each raw material provided by every supplier, while constraints (2) ensure that the capacity of the selected modes for transporting raw materials from suppliers to plants is not exceeded.

### 3.4.2 Plant-related constraints

The following constraints impose the required conditions for establishing new plants and closing existing plants over the planning horizon. In addition, they also rule the acquisition of additional capacity as well as its utilization both to manufacture products and to transport these using the selected transportation modes. End products can be delivered directly to customer zones or shipped to warehouses.

$$\sum_{t \in T} y^n_{t,\ell} \leq 1 \qquad \ell \in L^n \tag{3}$$

$$y^n_{t,\ell} \leq \sum_{k \in K_L} u_{t,\ell,k} \leq \sum_{\tau=1}^{t} y^n_{\tau,\ell} \qquad t \in T,\ \ell \in L^n \tag{4}$$

$$\sum_{k \in K_L} u_{t,\ell,k} \leq 1 - \sum_{\tau=1}^{|T|} y^e_{\tau,\ell} \qquad t \in T,\ \ell \in L^e \tag{5}$$

$$\sum_{s \in S} \sum_{m \in M} x_{t,s,\ell,r,m} = \sum_{p \in P} a_{\ell,r,p} \left( \sum_{w \in W} \sum_{m \in M} x_{t,\ell,w,p,m} + \sum_{c \in C} \sum_{m \in M} x_{t,\ell,c,p,m} \right) \qquad t \in T,\ \ell \in L,\ r \in R \tag{6}$$

$$\alpha_\ell \sum_{k \in K_L} Q_{\ell,k} \sum_{\tau=1}^{t} u_{\tau,\ell,k} \leq \sum_{p \in P} \mu_{\ell,p} \left( \sum_{w \in W} \sum_{m \in M} x_{t,\ell,w,p,m} + \sum_{c \in C} \sum_{m \in M} x_{t,\ell,c,p,m} \right) \leq \sum_{k \in K_L} Q_{\ell,k} \sum_{\tau=1}^{t} u_{\tau,\ell,k} \qquad t \in T,\ \ell \in L^n \tag{7}$$

$$\alpha_\ell \left( Q^e_\ell \left(1 - \sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) + \sum_{k \in K_L} Q_{\ell,k} \sum_{\tau=1}^{t} u_{\tau,\ell,k} \right) \leq$$

$$\sum_{p \in P} \mu_{\ell,p} \left( \sum_{w \in W} \sum_{m \in M} x_{t,\ell,w,p,m} + \sum_{c \in C} \sum_{m \in M} x_{t,\ell,c,p,m} \right) \leq$$

$$Q^e_\ell \left(1 - \sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) + \sum_{k \in K_L} Q_{\ell,k} \sum_{\tau=1}^{t} u_{\tau,\ell,k} \qquad t \in T,\ \ell \in L^e \tag{8}$$

$$\sum_{k \in K_L} Q_{\ell,k} \sum_{t \in T} u_{t,\ell,k} \leq \overline{Q}_\ell \sum_{t \in T} y^n_{t,\ell} \qquad \ell \in L^n \tag{9}$$

$$Q^e_\ell \left(1 - \sum_{t \in T} y^e_{t,\ell}\right) + \sum_{k \in K_L} Q_{\ell,k} \sum_{t \in T} u_{t,\ell,k} \leq$$

$$\overline{Q}_\ell \left(1 - \sum_{t \in T} y^e_{t,\ell}\right) \qquad \ell \in L^e \tag{10}$$

$$\sum_{\ell \in L} \sum_{c \in C} \sum_{m \in M} x_{t,\ell,c,p,m} \leq \lambda_{t,p} \sum_{c \in C} d_{t,c,p} \qquad t \in T,\ p \in P \tag{11}$$

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \leq QM_{t,\ell,w,m} \qquad t \in T,\ \ell \in L,\ w \in W,\ m \in M \tag{12}$$

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,\ell,c,p,m} \leq QM_{t,\ell,c,m} \qquad t \in T,\ \ell \in L,\ c \in C,\ m \in M \tag{13}$$

Constraints (3) guarantee that at most one new plant can be established in a potential location over the time horizon. Moreover, once open, new plants cannot be closed. Constraints (4), resp. (5), rule the installation of capacity in new, resp. existing, plant locations. In each time period, at most one capacity level can be selected provided that a plant is already operating in that site. On the other hand, if a new plant is established in a given time period then a capacity level must also be acquired in the same period. In addition, the expansion and closing of existing facilities are also ruled by inequalities (5). Clearly, both terms involving the binary variables $u_{t,j,k}$ and $y^e_{\tau,j}$ cannot be equal to one. As a result, if an existing facility has its capacity expanded it cannot be closed. Furthermore, constraints (5) ensure that an existing facility is closed at most once over the time horizon.

Equalities (6) ensure that the required quantity of each raw material is purchased in order to be able to manufacture the end products in a plant. Constraints (7) state that the total quantity of products manufactured by a new plant must be within pre-defined lower and upper limits in each time period. Observe that the lower capacity utilization limit refers to a minimum throughput corresponding to a given percentage of the available capacity. The middle term gives the quantities of products shipped to warehouses and to customer zones. Constraints $(8)$ are also minimum throughput and capacity constraints but for existing plants. In this case, the capacity available at the beginning of the time horizon may have been extended. Naturally, only plants that are still operated in period $t$ are considered. Constraints $(9)$, resp. $(10)$, guarantee that the overall size of a new, resp. existing, plant does not exceed the maximum allowed capacity. Inequalities $(11)$ restrict the quantity of direct shipments from plants to customers for every product. This type of constraints is motivated by the fact that the resources available for direct product distribution are often limited (e.g., storage space at plants, size of fleet, manpower for order processing and consignment, etc.). Furthermore, response times to customer orders are typically longer with direct shipping and so this option is not offered extensively throughout the network.

Finally, constraints $(12)$, resp. $(13)$, rule the transportation of end products from plants to warehouses, resp. to customer zones, given the available capacities of the selected transportation modes.

### 3.4.3 Warehouse-related constraints

The following constraints impose the required conditions for establishing new warehouses and closing existing warehouses over the planning horizon. Moreover, capacity constraints ruling product handling and the selection of transportation modes are also introduced. As warehouses may purchase products from external sources, additional constraints are defined to limit the total quantity that can be outsourced per product type.

$$\sum_{t \in T} y^n_{t,w} \leq 1 \qquad w \in W^n \tag{14}$$

$$y^n_{t,w} \leq \sum_{k \in K_W} u_{t,w,k} \leq \sum_{\tau=1}^{t} y^n_{\tau,w} \qquad t \in T,\ w \in W^n \tag{15}$$

$$\sum_{k \in K_W} u_{t,w,k} \leq 1 - \sum_{\tau=1}^{|T|} y^e_{\tau,w} \qquad t \in T,\ w \in W^e \tag{16}$$

$$\alpha_w \sum_{k \in K_W} Q_{w,k} \sum_{\tau=1}^{t} u_{\tau,w,k} \leq \sum_{p \in P} \gamma_p \sum_{c \in C} \sum_{m \in M} x_{t,w,c,p,m}$$
$$\leq \sum_{k \in K_W} Q_{w,k} \sum_{\tau=1}^{t} u_{\tau,w,k} \qquad t \in T,\ w \in W^n \tag{17}$$

$$\alpha_w \left( Q^e_w \left( 1 - \sum_{\tau=1}^{t} y^e_{\tau,w} \right) + \sum_{k \in K_W} Q_{w,k} \sum_{\tau=1}^{t} u_{\tau,w,k} \right) \leq$$

$$\sum_{p \in P} \gamma_p \sum_{c \in C} \sum_{m \in M} x_{t,w,c,p,m} \leq Q^e_w \left( 1 - \sum_{\tau=1}^{t} y^e_{\tau,w} \right) +$$

$$\sum_{k \in K_W} Q_{w,k} \sum_{\tau=1}^{t} u_{\tau,w,k} \qquad t \in T,\ w \in W^e \qquad (18)$$

$$\sum_{k \in K_W} Q_{w,k} \sum_{t \in T} u_{t,w,k} \leq \overline{Q}_w \sum_{t \in T} y^n_{t,w} \qquad w \in W^n \qquad (19)$$

$$Q^e_w \left( 1 - \sum_{t \in T} y^e_{t,w} \right) + \sum_{k \in K_W} Q_{w,k} \sum_{t \in T} u_{t,w,k} \leq$$

$$\overline{Q}_w \left( 1 - \sum_{t \in T} y^e_{t,w} \right) \qquad w \in W^e \qquad (20)$$

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,w,c,p,m} \leq QM_{t,w,c,m} \qquad t \in T,\ w \in W,\ c \in C,\ m \in M \qquad (21)$$

$$\sum_{c \in C} \sum_{m \in M} x_{t,w,c,p,m} = \sum_{\ell \in L} \sum_{m \in M} x_{t,\ell,w,p,m} + z_{t,w,p} \qquad t \in T,\ w \in W,\ p \in P \qquad (22)$$

$$\sum_{w \in W} z_{t,w,p} \leq \beta_{t,p} \sum_{c \in C} d_{t,c,p} \qquad t \in T,\ p \in P \qquad (23)$$

Constraints (14) guarantee that at most one new warehouse can be established in a potential location over the time horizon. Moreover, once new warehouses are established, they must remain in operation until the end of the time horizon. Constraints (15), resp. (16), rule the acquisition of capacity in new, resp. existing, warehouses. In each time period, at most one capacity level can be selected provided that a warehouse is already operating in that site. On the other hand, if a new warehouse is established in a given time period then a capacity level must be installed in that period. Moreover, constraints (16) guarantee that if an existing warehouse has its capacity extended then it cannot be closed.

Constraints (17) impose that the total quantity handled by a new warehouse must achieve at least a given minimum throughput and not exceed the maximum handling capacity. Constraints (18) are also minimum throughput and capacity constraints but for existing warehouses. In this case, only warehouses that are still operated in period $t$ are considered. Observe that the initial capacity of existing warehouses may have been extended. Constraints (19), resp. (20), guarantee that the overall size of a new, resp. existing, warehouse is not larger than the maximum allowed capacity.

Constraints (21) ensure that the quantities of end products shipped from warehouses to customers do not exceed the available capacities of the selected transportation modes. Constraints (22) guarantee the conservation of product flows for all operated warehouses in each time period. These constraints along with inequalities (17) and (18) state that outsourced products also use the handling capacity available at warehouses. Finally, constraints (23) impose an upper limit on the total outsourced quantity per product type.

### 3.4.4 Demand satisfaction constraints

The following constraints ensure the satisfaction of all customer demands over the time horizon. The left-hand side of equalities (24) gives the product flow from plants and warehouses to each customer zone.

$$\sum_{\ell \in L} \sum_{m \in M} x_{t,\ell,c,p,m} + \sum_{w \in W} \sum_{m \in M} x_{t,w,c,p,m} = d_{t,c,p} \qquad t \in T,\ c \in C,\ p \in P \tag{24}$$

### 3.4.5 Domains of variables

The following constraints (25)-(29) represent non-negativity and binary conditions.

$$x_{t,o,d,i,m} \geq 0 \qquad t \in T,\ (o,d) \in OD,\ i \in R \cup P,\ m \in M \tag{25}$$

$$z_{t,w,p} \geq 0 \qquad t \in T,\ w \in W,\ p \in P \tag{26}$$

$$y^n_{t,j} \in \{0,1\} \qquad t \in T,\ j \in L^n \cup W^n \tag{27}$$

$$y^e_{t,j} \in \{0,1\} \qquad t \in T,\ j \in L^e \cup W^e \tag{28}$$

$$u_{t,j,k} \in \{0,1\} \qquad t \in T,\ j \in L \cup W,\ k \in K_L \cup K_W \tag{29}$$

## 3.5 Objective function

The objective function (30)–(38) minimizes the sum of all fixed and variable costs. Fixed opening, resp. closing, costs are given by (30), resp. (31). Fixed capacity acquisition costs at new and existing locations are determined by (32). In addition, fixed costs for operating existing facilities are described by (33). The total cost of providing plants with the required raw materials are given by (34) and include procurement as well as distribution costs. The terms (35) and (36) represent the total production and transportation costs incurred by plants. The distribution of end products from warehouses to customer zones are determined by (37). Finally, the total cost of purchasing end products from external sources is given by (38).

$$\text{Min } z = \sum_{t\in T}\sum_{\ell\in L^n} FC_{t,\ell}\, y^n_{t,\ell} + \sum_{t\in T}\sum_{w\in W^n} FC_{t,w}\, y^n_{t,w} + \tag{30}$$

$$\sum_{t\in T}\sum_{\ell\in L^e} SC_{t,\ell}\, y^e_{t,\ell} + \sum_{t\in T}\sum_{w\in W^e} SC_{t,w}\, y^e_{t,w} + \tag{31}$$

$$\sum_{t\in T}\sum_{\ell\in L}\sum_{k\in K_L} IC_{t,\ell,k}\, u_{t,\ell,k} + \sum_{t\in T}\sum_{w\in W}\sum_{k\in K_W} IC_{t,w,k}\, u_{t,w,k} + \tag{32}$$

$$\sum_{t\in T}\sum_{\ell\in L^e} OC_{t,\ell}\left(1-\sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) + \sum_{t\in T}\sum_{w\in W^e} OC_{t,w}\left(1-\sum_{\tau=1}^{t} y^e_{\tau,w}\right) + \tag{33}$$

$$\sum_{t\in T}\sum_{s\in S}\sum_{\ell\in L}\sum_{r\in R}\sum_{m\in M} \left(PC_{t,s,r} + TC_{t,s,\ell,r,m}\right) x_{t,s,\ell,r,m} + \tag{34}$$

$$\sum_{t\in T}\sum_{\ell\in L}\sum_{w\in W}\sum_{p\in P}\sum_{m\in M} \left(MC_{t,\ell,p} + TC_{t,\ell,w,p,m}\right) x_{t,\ell,w,p,m} + \tag{35}$$

$$\sum_{t\in T}\sum_{\ell\in L}\sum_{c\in C}\sum_{p\in P}\sum_{m\in M} \left(MC_{t,\ell,p} + TC_{t,\ell,c,p,m}\right) x_{t,\ell,c,p,m} + \tag{36}$$

$$\sum_{t\in T}\sum_{w\in W}\sum_{c\in C}\sum_{p\in P}\sum_{m\in M} TC_{t,w,c,p,m}\, x_{t,w,c,p,m} + \tag{37}$$

$$\sum_{t\in T}\sum_{w\in W}\sum_{p\in P} EC_{t,w,p}\, z_{t,w,p} \tag{38}$$

The problem of redesigning an existing logistics network is modeled by the above objective function subject to the constraints (1)–(29). This problem is NP-hard as it generalizes the multi-period uncapacitated facility location problem. We remark that for $L^e = \emptyset$ and $W^e = \emptyset$, the above mathematical formulation reduces to the special case of designing a new network. Finally, we highlight that the proposed model is flexible since it can capture different types of network structures and tailored distribution strategies. Furthermore, applications can be found in a number of industrial contexts, e.g. consumer goods industry (see Cintron et al. [7], Manzini and Bindi [15], Melo et al. [17] and references therein).

## 4 Tightening constraints on transportation

In this section, we propose several procedures to tighten constraints involving the distribution of raw materials and end products. Recall that in constraints (2), (12), (13), and (21) the quantity shipped from a location is limited by the capacity of the selected transportation mode. However, other capacities and other parameters may also limit the quantities shipped from a facility. For instance, the quantity of raw material moved from a given supplier to a certain plant depends on the availability of that raw material at the supplier, on the transportation capacity from the supplier to the plant, and on the production capacity at the plant. Thus, in some cases, transportation capacities can be replaced by tighter values.

## 4.1 Transportation from suppliers to plants

As mentioned before, the quantity shipped from a supplier to a plant depends not only on the capacity of the transportation mode but also on the amount of raw materials available at the supplier and on the production capacity at the plant. An upper bound on the utilization of transportation mode $m$ from supplier $s$ to plant $\ell$ can be obtained by considering the total availability at $s$ that could be shipped to $\ell$. Hence, we calculate

$$\overline{Q}_{t,s,m} = \sum_{r \in R} \sigma_{r,m}\, QS_{t,s,r} \qquad t \in T,\ s \in S,\ m \in M \tag{39}$$

Another upper bound on the same quantity can be obtained by considering the plant capacity. For each product, the maximum quantity that can be produced in a plant is given by the ratio between the production capacity and the unit capacity consumption factor. The capacity utilization of transportation mode $m$, associated with the raw materials required for each level of production, is obtained by multiplying the capacity size by its associated consumption of raw materials and its capacity utilization in this transportation mode. The maximum of these usages is an upper bound on the transportation capacity utilization. Therefore, it follows that

$$\overline{QQ}_{\ell,m} = \max_{p \in P} \left\{ \sum_{r \in R} \sigma_{r,m}\, a_{\ell,r,p}\, \frac{\overline{Q}_\ell}{\mu_{\ell,p}} \right\} \qquad \ell \in L,\ m \in M \tag{40}$$

As a result, constraints $(2)$ can be replaced by

$$\sum_{r \in R} \sigma_{r,m}\, x_{t,s,\ell,r,m} \leq QQM_{t,s,\ell,m} \qquad t \in T,\ s \in S,\ \ell \in L,\ m \in M \tag{41}$$

where

$$QQM_{t,s,\ell,m} = \min\left\{QM_{t,s,\ell,m},\ \overline{Q}_{t,s,m},\ \overline{QQ}_{\ell,m}\right\} \tag{42}$$

Observe that the production capacity can also depend on the period considered. For instance, if the maximum production capacity cannot be reached in the first or in the second periods then $\overline{Q}_\ell$ can be replaced in $(40)$ by a lower coefficient for some periods.

## 4.2 Transportation from plants to warehouses

The quantity shipped from a plant to a warehouse depends not only on the capacity of the selected transportation mode but also on the production capacity of the plant and on the handling capacity of the warehouse. The determination of the maximum quantities produced or handled is similar to (40). However, end products, instead of raw materials, are now considered. Constraints (12) can be replaced by

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \leq QQM_{t,\ell,w,m} \qquad t \in T,\ \ell \in L,\ w \in W,\ m \in M \tag{43}$$

where

$$QQM_{t,\ell,w,m} \;=\; \min\left\{QM_{t,\ell,w,m},\ \max_{p \in P}\left\{\sigma_{p,m}\,\frac{\overline{Q}_\ell}{\mu_{\ell,p}}\right\},\ \max_{p \in P}\left\{\sigma_{p,m}\,\frac{\overline{Q}_w}{\gamma_p}\right\}\right\} \tag{44}$$

Once again, in the first periods, the production and handling capacities can be lower than the maximum capacities, thereby allowing to set tighter upper bounds.

## 4.3 Transportation to customers

The quantity of products shipped to a customer depends on the capacity of the transportation mode used, on the production or handling capacity at the shipping origin, and on the customer demand. In order to take in account the demand of each customer zone, we need to convert it into a required capacity for the transportation mode used, that is,

$$\sum_{p \in P} \sigma_{p,m}\, d_{t,c,p} \qquad t \in T,\ c \in C,\ m \in M$$

It follows that constraints (13) and (21) can be replaced by, respectively,

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,\ell,c,p,m} \leq QQM_{t,\ell,c,m} \qquad t \in T,\ \ell \in L,\ c \in C,\ m \in M \tag{45}$$

$$\sum_{p \in P} \sigma_{p,m}\, x_{t,w,c,p,m} \leq QQM_{t,w,c,m} \qquad t \in T,\ w \in W,\ c \in C,\ m \in M \tag{46}$$

with

$$QQM_{t,\ell,c,m} \;=\; \min\left\{QM_{t,\ell,c,m},\ \max_{p \in P}\left\{\sigma_{p,m}\,\frac{\overline{Q}_\ell}{\mu_{\ell,p}}\right\},\ \sum_{p \in P} \sigma_{p,m}\, d_{t,c,p}\right\} \tag{47}$$

and

$$QQM_{t,w,c,m} \;=\; \min\left\{QM_{t,w,c,m},\; \max_{p\in P}\left\{\sigma_{p,m}\,\frac{\overline{Q}_w}{\gamma_p}\right\},\; \sum_{p\in P}\sigma_{p,m}\,d_{t,c,p}\right\} \tag{48}$$

As mentioned in the previous subsections, better upper bounds on production and handling capacities might be considered for the first periods.

Finally, we remark that if a given transportation mode has unlimited capacity then the new constraints (41), (43), (45), and (46) may be rather effective.

# 5 Model enhancements

There are various ways of enhancing the mathematical formulation in an attempt to improve the lower bound of the linear programming relaxation. In addition, the chance of obtaining good feasible solutions in the course of a branch-and-bound algorithm may increase. In this section, we propose several groups of valid inequalities tying different sets of binary variables. This strategy is applied to those constraints involving the distribution of raw materials and end products throughout the logistics network. Furthermore, aggregated demand inequalities are developed. Finally, inequalities are also derived that set lower bounds on the number of capacity levels and operating facilities.

## 5.1 Inequalities involving capacity constraints

A simple strategy is to multiply the right-hand side of the capacity constraints by appropriate sets of facility location variables. For the flow of raw materials from suppliers to plants, this corresponds to replacing inequalities (2) by

$$\sum_{r\in R}\sigma_{r,m}\,x_{t,s,\ell,r,m} \le QQM_{t,s,\ell,m}\sum_{\tau=1}^{t} y^{n}_{\tau,\ell} \qquad t\in T,\ s\in S,\ \ell\in L^{n},\ m\in M \tag{2a}$$

$$\sum_{r\in R}\sigma_{r,m}\,x_{t,s,\ell,r,m} \le QQM_{t,s,\ell,m}\left(1-\sum_{\tau=1}^{t} y^{e}_{\tau,\ell}\right) \qquad t\in T,\ s\in S,\ \ell\in L^{e},\ m\in M \tag{2b}$$

with $QQM_{t,s,\ell,m}$ given by (42).

Furthermore, end products can only be moved from plants to warehouses when facilities are operated both at the origin and destination locations. Hence, instead of constraints (12) we can use the following inequalities:

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \le QQM_{t,\ell,w,m} \sum_{\tau=1}^{t} y^n_{\tau,\ell} \qquad t\in T,\ \ell\in L^n,\ w\in W,\ m\in M \tag{12a}$$

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \le QQM_{t,\ell,w,m} \left(1-\sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) \qquad t\in T,\ \ell\in L^e,\ w\in W,\ m\in M \tag{12b}$$

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \le QQM_{t,\ell,w,m} \sum_{\tau=1}^{t} y^n_{\tau,w} \qquad t\in T,\ \ell\in L,\ w\in W^n,\ m\in M \tag{12c}$$

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,w,p,m} \le QQM_{t,\ell,w,m} \left(1-\sum_{\tau=1}^{t} y^e_{\tau,w}\right) \qquad t\in T,\ \ell\in L,\ w\in W^e,\ m\in M \tag{12d}$$

with $QQM_{t,\ell,w,m}$ defined by $(44)$. We note that the size of the model increases considerably when all these inequalities (12a)–(12d) are considered in the LNRD formulation. In this case, $|T|\cdot|L|\cdot|W|\cdot|M|$ new constraints are introduced. In our computational experiments (cf. Section 7), we opted not to include inequalities (12a) and (12b), thus keeping the original number of constraints.

Similar constraints to (12a)–(12d) can also be formulated for direct product flows from plants to customers. In this case, constraints $(13)$ are replaced by the following conditions:

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,c,p,m} \le QQM_{t,\ell,c,m} \sum_{\tau=1}^{t} y^n_{\tau,\ell} \qquad t\in T,\ \ell\in L^n,\ c\in C,\ m\in M \tag{13a}$$

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,\ell,c,p,m} \le QQM_{t,\ell,c,m} \left(1-\sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) \qquad t\in T,\ \ell\in L^e,\ c\in C,\ m\in M \tag{13b}$$

with $QQM_{t,\ell,c,m}$ given by $(47)$.

Finally, constraints $(21)$ that rule the flows of end products from warehouses to customers can be replaced by the following two sets of inequalities:

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,w,c,p,m} \le QQM_{t,w,c,m} \sum_{\tau=1}^{t} y^n_{\tau,w} \qquad t\in T,\ w\in W^n,\ c\in C,\ m\in M \tag{21a}$$

$$\sum_{p\in P} \sigma_{p,m}\, x_{t,w,c,p,m} \le QQM_{t,w,c,m} \left(1-\sum_{\tau=1}^{t} y^e_{\tau,w}\right) \qquad t\in T,\ w\in W^e,\ c\in C,\ m\in M \tag{21b}$$

with $QQM_{t,w,c,m}$ defined by $(48)$.

## 5.2 Inequalities involving customer demands

The next set of valid inequalities is referred to as *Aggregated Demand Constraints*. Although they are redundant for the linear programming relaxation, they help general-purpose optimization software at generating cover cuts.

$$\sum_{\ell \in L} \sum_{k \in K_L} Q_{\ell,k} \sum_{\tau=1}^{t} u_{\tau,\ell,k} + \sum_{\ell \in L^e} Q^e_\ell \left(1 - \sum_{\tau=1}^{t} y^e_{\tau,\ell}\right) \geq D^L_t \qquad t \in T \tag{49}$$

$$\sum_{w \in W} \sum_{k \in K_W} Q_{w,k} \sum_{\tau=1}^{t} u_{\tau,w,k} + \sum_{w \in W^e} Q^e_w \left(1 - \sum_{\tau=1}^{t} y^e_{\tau,w}\right) \geq D^W_t \qquad t \in T \tag{50}$$

Constraints $(49)$, resp. $(50)$, state that in each time period the total capacity of open plants, resp. warehouses, should cover the overall demand requirements. The latter are converted into the same capacity units used in plants and warehouses as follows:

$$D^L_t = \sum_{c \in C} \sum_{p \in P} (1 - \beta_{t,p}) \min_{\ell \in L} \{\mu_{\ell,p}\}\, d_{t,c,p} \qquad t \in T \tag{51}$$

$$D^W_t = \sum_{c \in C} \sum_{p \in P} (1 - \lambda_{t,p})\, \gamma_p\, d_{t,c,p} \qquad t \in T \tag{52}$$

## 5.3 Additional valid inequalities

We also derived a new set of valid inequalities, $(53)$–$(54)$, that impose a lower bound on the total number of capacity levels that should be available in plant and warehouse locations in each time period, resp. $N^L_t$ and $N^W_t$.

$$\sum_{\tau=1}^{t} \sum_{\ell \in L} \sum_{k \in K_L} u_{\tau,\ell,k} \geq N^L_t \qquad t \in T \tag{53}$$

$$\sum_{\tau=1}^{t} \sum_{w \in W} \sum_{k \in K_W} u_{\tau,w,k} \geq N^W_t \qquad t \in T \tag{54}$$

The calculation of an appropriate value for the lower bound $N^L_t$, resp. $N^W_t$, relies on the identification of the largest capacity levels that could possibly be installed until period $t$ in order to cover the total demand requirements $D^L_t$, resp. $D^W_t$. For this purpose, let us introduce $AC_{t,j}$ as the total capacity that is possibly available in location $j \in L \cup W$ in time period

$t = 0, \ldots, |T|$. At the beginning of the time horizon ($t = 0$), we set $AC_{0,j} = Q^e_j$ for every existing facility $j \in L^e \cup W^e$ and $AC_{0,j} = 0$ for every new facility $j \in L^n \cup W^n$.

We now illustrate the calculation of $N^L_t$, similar steps are performed to obtain $N^W_t$. For every $t \in T$ we proceed as follows: if the inequality $\sum_{\ell \in L} AC_{t-1,\ell} \geq D^L_t$ holds then this means that there is enough plant capacity in period $t$ and therefore, $N^L_t = N^L_{t-1}$ (with $N^L_0 = 0$). In this case, the total capacity available in each plant location remains unchanged in period $t$, that is, $AC_{t,\ell} = AC_{t-1,\ell}$ for $\ell \in L$. If $\sum_{\ell \in L} AC_{t-1,\ell} < D^L_t$ then it is clear that additional capacity is required in period $t$. For each plant location $\ell \in L$, we identify the largest possible capacity level that could be installed in period $t$. This entails taking into account the global capacity limit of the plant. If $AC_{t-1,\ell} + Q_{\ell,|K_L|} \leq \overline{Q}_\ell$ then the largest capacity level $|K_L|$ is considered. Otherwise, the capacity size $k$ satisfying the following conditions is selected $(1 \leq k \leq |K_L| - 1)$: $AC_{t-1,\ell} + Q_{\ell,k} \leq \overline{Q}_\ell$ and $AC_{t-1,\ell} + Q_{\ell,k+1} > \overline{Q}_\ell$. In case no size $k$ satisfies these conditions, plant location $\ell$ is excluded from further consideration. After identifying all possible capacity levels for plant expansion (let us denote these by $\widetilde{Q}_\ell$), we arrange them in non-increasing order of their sizes. To this end, we create a sequence $\widetilde{Q}_{[1]}, \widetilde{Q}_{[2]}, \ldots$ such that $\widetilde{Q}_{[1]} \geq \widetilde{Q}_{[2]} \geq \ldots$. Each capacity size in this list corresponds to a different plant location and this information is used later on in the procedure. The total number $j$ $(j \geq 0)$ of additional plant capacity levels required in period $t$ must satisfy the following inequalities:

$$\sum_{i=1}^{j-1} \widetilde{Q}_{[i]} < D^L_t - \sum_{\ell \in L} AC_{t-1,\ell} \leq \sum_{i=1}^{j} \widetilde{Q}_{[i]} \tag{55}$$

It follows that $N^L_t = N^L_{t-1} + j$. Finally, for each one of the $j$ plant locations involved in (55), it is necessary to update the corresponding total capacity possibly acquired up to period $t$, that is, the corresponding size $\widetilde{Q}_\ell$ is added to $AC_{t,\ell}$. For all other plant locations whose capacities are not expanded at this stage we set $AC_{t,\ell} = AC_{t-1,\ell}$. This way of determining the value of $N^L_t$ relies on constraints (4)–(5) that allow the same type of capacity level to be installed in different periods in the same location, provided that the overall capacity does not exceed the global size. In practice, smaller capacity levels than the ones selected according to (55) can be installed. Thus, $N^L_t$ sets indeed a lower bound on the total number of required plant capacity levels that should be in operation in period $t$.

For the LND variant of the problem (i.e. $L^e = \emptyset$, $W^e = \emptyset$), the lower bounds $N^L_1$ and $N^W_1$ are also used to set the minimum number of plants and warehouses that should operate in the first time period as follows:

$$\sum_{\ell \in L^n} y^n_{1,\ell} \geq N^L_1 \tag{56}$$

$$\sum_{w \in W^n} y^n_{1,w} \geq N^W_1 \tag{57}$$

If a network is already in place with a number of plants and warehouses being operated at fixed locations then the corresponding lower bounds are calculated differently in $t = 1$. In this case, we have to consider the largest possible size of each facility in the first time period. Regarding an existing facility, this entails taking its existing capacity and extending it as much as possible without exceeding its global size. For each potential location the corresponding largest capacity level is considered. In the plant layer, the capacities thus formed are sorted in non-increasing order. This sequence is then used to identify the minimum number of plant locations $\overline{N}^L_1$ that cover the total demand requirements in $t = 1$. For the warehouse layer, a lower bound $\overline{N}^W_1$ is calculated in a similar way. Therefore, in the LNRD variant of the problem the following inequalities are imposed:

$$\sum_{\ell \in L^e} \left(1 - y^e_{1,\ell}\right) + \sum_{\ell \in L^n} y^n_{1,\ell} \geq \overline{N}^L_1 \tag{58}$$

$$\sum_{w \in W^e} \left(1 - y^e_{1,w}\right) + \sum_{w \in W^n} y^n_{1,w} \geq \overline{N}^W_1 \tag{59}$$

Along with constraints $(49)$ and $(50)$, the new valid inequalities are computationally inexpensive since in total at most $4 \cdot |T| + 2$ new constraints are added to the original formulation. Finally, we remark that the new inequalities $(53)$–$(54)$ and $(56)$–$(59)$ may be very useful, in particular in the LND variant of the problem, when the right-hand side terms are significantly larger than one, since this will restrict the number of possible combinations of new locations and new capacity levels that should be considered. In addition, the formulation with all valid inequalities provides a stronger LP relaxation bound than the original formulation.

# 6 Data generation

In this section, the methodology for obtaining test instances for both problem classes, LNRD and LND, is presented in detail. Various realistic features were considered in the generation scheme such as fixed facility costs reflecting economies of scale.

In what follows, we denote by $U[a, b]$ the generation of random numbers over the range $[a, b]$ according to a uniform distribution. The generation of random integer values in the same interval is denoted by $I[a, b]$.

## 6.1 Sets

Table 6 describes the index sets. The size of each instance is mainly dictated by the number of time periods and the number of customer zones. Test instances with $|T| = 4$ span four years, while instances with $|T| = 6$ correspond to three years with each time period representing six months. In all planning horizons, instances with 50, 75, and 100 customer zones are considered. These choices also determine the number of suppliers, plants, warehouses, raw materials, and end products.

| Symbol | Description | Value |
|---|---|---|
| $\|T\|$ | Number of time periods | $4, 6$ |
| $\|C\| = n$ | Number of customer zones | $50, 75, 100$ |
| $\|L^e\|$ | Number of existing plants | 1 if $n = 50$, 2 otherwise |
| $\|L^n\|$ | Number of candidate sites for new plants | $\lceil \frac{n}{10} \rceil$ |
| $\|W^e\|$ | Number of existing warehouses | 2 if $n = 50$, 3 otherwise |
| $\|W^n\|$ | Number of candidate sites for new warehouses | $\lceil \frac{n}{5} \rceil$ |
| $\|K_L\| = \|K_W\|$ | Number of capacity levels for plants, resp. warehouses | 3 |
| $\|R\| = \|P\|$ | Number of raw materials, resp. end products | $\lceil \frac{n}{5} \rceil$ |
| $\|R^p\|$ | Number of raw materials used in the production of product $p$ | $I\left[\frac{\|R\|}{2},\ \lfloor\frac{3}{4}\|R\|\rfloor\right]$, $p \in P$ |
| $\|S\|$ | Number of suppliers | $\lceil \frac{n}{10} \rceil$ |
| $\|S^r\|$ | Number of suppliers providing raw material $r$ | $I\left[\frac{\|R\|}{\|S\|},\ \|S\|\right]$, $r \in R$ |
| $\|M\|$ | Number of transportation modes | 2 |

Table 6: Generation of index sets

Each end product is manufactured with a randomly selected subset of raw materials and each raw material is provided by a subset of suppliers. Two different transportation modes are assumed, namely rail and road freight transport. Although road transports are not subject to capacity limitations, they are more expensive compared to rail. Regarding the capacity acquisition options for facilities (plants and warehouses), three capacity levels are considered representing small, medium, and large sizes.

## 6.2 Customer demands

In the first time period, the demand of each customer zone for a given product is randomly generated according to a uniform distribution in the interval $[1, 10]$. In each subsequent time period, the demand requirements may increase up to a certain limit compared to the previous period as follows:

$$
\begin{aligned}
d_{1,c,p} &= U[1,\ 10] \\
d_{t,c,p} &= U[1,\ g]\, d_{t-1,c,p}, \quad t = 2, \ldots, |T|
\end{aligned}
$$

with $g$ denoting the largest demand growth rate ($g > 1$). As described in Section 6.1, a time period covers a certain length of time (e.g. six months, one year) depending on $|T|$. In order to have similar demand growths, the parameter $g$ is adjusted accordingly. For $|T| = 4$ years, we set $g = 1.05$, meaning that demand in the last year can be at most 15.76% ($1.05^3 = 1.1576$) larger than in the first year. For $|T| = 6$, we take $g = 1.025$, which corresponds to a maximum demand growth of 13.14%.

Regarding the maximum proportion of the total demand that may be outsourced, three scenarios were considered, namely $\beta_{t,p} \in \{0,\ 0.1,\ 0.2\}$, $t \in T$, $p \in P$. Therefore, in one scenario product outsourcing is not available, while in the other two scenarios at most 10%, resp. 20%, of the total demand can be satisfied by purchasing commodities from an external source. For the maximum fraction of demand delivered from plants to customer zones, we set $\lambda_{t,p} \in \{0,\ 0.1\}$ for every $t \in T$ and $p \in P$.

## 6.3 Production and storage capacities

A simple bill-of-materials is considered for product manufacturing. In particular, the same production technology is available in each plant (either in an existing location or in a candidate site). Furthermore, for any raw material $r$ that is required to manufacture product $p$, we set

$$a_{\ell,r,p} = 1, \quad \ell \in L,\ r \in R^p,\ p \in P$$

In the following Table 7, the generation of parameters related to capacities is described. Regarding the capacity levels that can be installed in a given location, we first define the largest level. Any other size is equal to 70% of the size of the subsequent capacity level. Since we previously set $|K_L| = |K_W| = 3$ (recall Table 6), it follows that the smallest, resp. medium, level corresponds to 49%, resp. 70%, of the largest size.

| Symbol | Description | Value |
|---|---|---|
| $QS_{t,s,r}$ | Capacity of supplier $s \in S$ for raw material $r \in R$ in period $t \in T$ | 0 if $s \notin S^r$, $\frac{U[1,3] \sum_{p\in P}\sum_{c\in C} d_{t,c,p}}{\lvert S^r\rvert}$ if $s \in S^r$ |
| $\widetilde{\mu}_p$ | Production capacity usage by one unit of product $p \in P$ in any plant | $I[1,\ 10]$ |
| $\mu_{\ell,p}$ | Production capacity usage by one unit of product $p \in P$ in plant $\ell \in L$ | $\widetilde{\mu}_p$ |
| $\gamma_p$ | Handling capacity usage by one unit of product $p \in P$ in any warehouse | $I[1,\ 10]$ |
| $Q^e_\ell$ | Capacity of existing plant $\ell \in L^e$ at the beginning of the time horizon | $0.9\,\frac{\sum_{p\in P}\mu_{\ell,p}\sum_{c\in C} d_{1,c,p}}{\lvert L^e\rvert}$ |
| $Q^e_w$ | Capacity of existing warehouse $w \in W^e$ at the beginning of the time horizon | $0.6\,\frac{\sum_{p\in P}\gamma_p\sum_{c\in C} d_{1,c,p}}{\lvert W^e\rvert}$ |
| $Q_{\ell,\lvert K_L\rvert}$ | Largest capacity level that can be installed in plant $\ell \in L$ | $\frac{U[4,6] \sum_{p\in P}\mu_{\ell,p}\sum_{c\in C} d_{\lvert T\rvert,c,p}}{\lvert L\rvert}$ |
| $Q_{\ell,k}$ | Size of capacity level $k$ that can be installed in plant $\ell \in L$, $k = 1, ..., \lvert K_L\rvert - 1$ | $0.7\,Q_{\ell,k+1}$ |
| $Q_{w,\lvert K_W\rvert}$ | Largest capacity level that can be installed in warehouse $w \in W$ | $\frac{U[1,3] \sum_{p\in P}\gamma_p\sum_{c\in C} d_{\lvert T\rvert,c,p}}{\lvert W\rvert}$ |
| $Q_{w,k}$ | Size of capacity level $k$ that can be installed in warehouse $w \in W$, $k = 1, ..., \lvert K_W\rvert - 1$ | $0.7\,Q_{w,k+1}$ |
| $\overline{Q}_\ell$ | Maximum total capacity of *new* plant $\ell \in L^n$ | $\sum_{k\in K_L} Q_{\ell,k}$ |
| $\overline{Q}_w$ | Maximum total capacity of *new* warehouse $w \in W^n$ | $\sum_{k\in K_W} Q_{w,k}$ |
| $\overline{Q}_\ell$ | Maximum total capacity of *existing* plant $\ell \in L^e$ | $Q^e_\ell$ or $Q^e_\ell + Q_{\ell,1}$ |
| $\overline{Q}_w$ | Maximum total capacity of *existing* warehouse $w \in W^n$ | $Q^e_w$ or $Q^e_w + Q_{w,1}$ |
| $\alpha_j$ | Capacity usage factor to set a minimum throughput level on facility $j \in L \cup W$ | 0.2 |

Table 7: Generation of parameters related to capacities

In each candidate location, the global capacity of a facility is limited by the sum of the capacities of the three randomly generated sizes. Regarding the existing facilities, only a subset of these may have their capacities expanded. In this case, capacity expansion is limited to installing the smallest capacity level. In instances with $\lvert L^e\rvert = 2$, one of the two existing plants is randomly chosen for capacity expansion. Regarding warehouse locations, in all instances one of the existing warehouses is randomly selected and we set $\overline{Q}_w = Q^e_w + Q_{w,1}$. For all other non-selected existing facilities $j$, we define $\overline{Q}_j = Q^e_j$.

## 6.4 Transportation modes

Table 8 defines the parameters related to the capacities of the transportation modes as well as the corresponding utilization factors. As mentioned in Section 6.1, distribution channels from suppliers to plants and from plants to warehouses are available for both rail and road freight transports. Transportation links to customer zones only use road transportation. It is assumed that this mode of transportation has unlimited capacity.

| Symbol | Description | Value |
|---|---|---|
| $\sigma_{i,m}$ | Unit capacity utilization factor of item $i \in R \cup P$ in transportation mode $m \in M$ | $I[1,\ 10]$ |
| $QM_{t,s,\ell,1}$ | Capacity of transportation mode 1 (rail) from supplier $s \in S$ to plant $\ell \in L$ in period $t \in T$ | $\frac{\sum_{r \in R} \sigma_{r,m} \sum_{p \in P} \sum_{c \in C} \max_{\ell \in L}\{a_{\ell,r,p}\}\, d_{t,c,p}}{\lvert S \rvert \times \lvert L \rvert}$ |
| $QM_{t,\ell,w,1}$ | Capacity of transportation mode 1 (rail) from plant $\ell \in L$ to warehouse $w \in W$ in period $t \in T$ | $\frac{\sum_{p \in P} \sigma_{p,m} \sum_{c \in C} d_{t,c,p}}{\lvert L \rvert \times \lvert W \rvert}$ |
| $QM_{t,j,c,1}$ | Capacity of transportation mode 1 (rail) from facility $j \in L \cup W$ to customer zone $c \in C$ in period $t \in T$ | 0 |
| $QM_{t,o,d,2}$ | Capacity of transportation mode 2 (road) from origin $o$ to destination $d$, $(o,d) \in OD$, in period $t \in T$ | $+\infty$ |

Table 8: Generation of parameters related to transportation modes

## 6.5 Facility costs

The generation of the various costs relies on the following parameters:

- Average utilization of the production capacity of a plant by one unit of product:

$$\overline{\mu}_\ell = \frac{\sum_{p \in P} \mu_{\ell,p}}{\lvert P \rvert}, \quad \ell \in L$$

- Average utilization of the storage capacity of any warehouse by one unit of product:

$$\overline{\gamma} = \frac{\sum_{p \in P} \gamma_p}{|P|}$$

We describe next the procedures employed to generate all fixed costs considered in the model.

- The fixed costs of installing capacity in facility locations (plants/warehouses) are generated in order to reflect economies of scale.

$$\begin{aligned} IC_{1,\ell,k} &= 100\sqrt{\frac{Q_{\ell,k}}{\overline{\mu}_\ell}} && \ell \in L,\ k \in K_L \\ IC_{1,w,k} &= 100\sqrt{\frac{Q_{w,k}}{\overline{\gamma}}} && w \in W,\ k \in K_W \\ IC_{t,j,k} &= U[1.02,\ 1.05]\, IC_{t-1,j,k} && j \in L \cup W,\ t = 2, \ldots, |T|-1 \end{aligned}$$

- The fixed costs of opening a new facility (plant/warehouse) are set according to

$$\begin{aligned} FC_{1,\ell} &= \sum_{k \in K_L} IC_{1,\ell,k} && \ell \in L^n \\ FC_{1,w} &= \sum_{k \in K_W} IC_{1,w,k} && w \in W^n \\ FC_{t,j} &= U[1.02,\ 1.05]\, FC_{t-1,j} && j \in L^n \cup W^n,\ t = 2, \ldots, |T|-1 \end{aligned}$$

  Observe that the setup cost of a new facility is determined by the sum of the fixed costs incurred by installing all capacity levels in that location.

- The fixed cost of closing an existing facility may be related to disposal activities and/or cover additional expenditures due to, for example, employee re-training.

$$\begin{aligned} SC_{1,\ell} &= 20\sqrt{\frac{Q^e_\ell}{\overline{\mu}_\ell}} && \ell \in L^e \\ SC_{1,w} &= 20\sqrt{\frac{Q^e_w}{\overline{\gamma}}} && w \in W^e \\ SC_{t,j} &= U[1.02,\ 1.05]\, SC_{t-1,j} && j \in L^e \cup W^e,\ t = 2, \ldots, |T|-1 \end{aligned}$$

  As shown above, economies of scale are considered in setting the closing costs. Moreover, any of these costs is considerably lower than the cost of establishing a new facility.

- The fixed costs of operating existing facilities follow the same line of reasoning as above.

$$\begin{aligned} OC_{1,\ell} &= 2\sqrt{\frac{Q^e_\ell}{\overline{\mu}_\ell}} && \ell \in L^e \\ OC_{1,w} &= 2\sqrt{\frac{Q^e_w}{\overline{\gamma}}} && w \in W^e \\ OC_{t,j} &= U[1.02,\ 1.05]\, OC_{t-1,j} && j \in L^e \cup W^e,\ t = 2, \ldots, |T|-1 \end{aligned}$$

## 6.6 Logistics costs

We now focus on the generation of logistics costs related to procurement, production, and distribution functions.

- The variable cost of purchasing one unit of raw material $r \in R$ from supplier $s \in S^r$ is fixed as follows:

$$\begin{aligned} PC_{1,s,r} &= U[0.75\,\phi_r,\ 1.25\,\phi_r] \\ PC_{t,s,r} &= U[1.02,\ 1.05]\,PC_{t-1,s,r} \qquad t = 2, \ldots, |T|-1 \end{aligned}$$

  with $\phi_r = U[0.01,\ 0.05]$.

- Regarding the variable cost of manufacturing one unit of product $p \in P$ at plant $\ell \in L$, we consider the following procedure:

$$\begin{aligned} MC_{1,\ell,p} &= U[0.75\,\phi_p,\ 1.25\,\phi_p] \\ MC_{t,\ell,p} &= U[1.02,\ 1.05]\,MC_{t-1,\ell,p} \qquad t = 2, \ldots, |T|-1 \end{aligned}$$

  with $\phi_p = U[0.5,\ 1]$.

- The cost of transporting one unit of an item (raw material or end product) from an origin $o$ to a destination $d$ relies on the Euclidean distance $dist_{o,d}$ between the two locations. For $o \in L \cup W$, $d \in L \cup W \cup C$, $i \in R \cup P$, and $m \in M$ we set

$$\begin{aligned} TC_{1,o,d,i,m} &= \phi_i\, dist_{o,d}\, \xi_m \\ TC_{t,o,d,i,m} &= U[1.02,\ 1.05]\,TC_{t-1,o,d,i,m} \qquad t = 2, \ldots, |T|-1 \end{aligned}$$

  with $\phi_i = U[0.1,\ 0.5]$, $\xi_1 = 0.7$, and $\xi_2 = 1$. Observe that the transportation of goods by train ($m = 1$) incurs a lower cost than by truck ($m = 2$). The coordinates of all locations are chosen randomly in the square $[0,\ 10] \times [0,\ 10]$.

  In the first time period, the unit costs for moving raw materials $r \in R$ from suppliers $s \in S$ to plants $\ell \in L$ using mode $m \in M$ are specified according to a slightly different scheme, namely

$$TC_{1,s,\ell,r,m} = \frac{\phi_i\, dist_{s,\ell}\, \xi_m}{\Delta}$$

  where $\Delta$ is defined by

$$\Delta = \frac{\sum_{p \in P} \left( \sum_{c \in C} d_{|T|,c,p} \sum_{r \in R} a_{r,p} \right)}{\sum_{c \in C} \sum_{p \in P} d_{|T|,c,p}}$$

We note that $a_{r,p} = a_{\ell,r,p}$ for every $\ell \in L$ as a result of the definition of the production structure (see Section 6.3). The above generation procedure ensures that the cost of transporting one unit of raw material is lower than the cost of moving one unit of an end product. This choice is due to the fact that in each plant a commodity is obtained by processing several raw materials. Therefore, the quantity of raw materials moved from the suppliers to the plants is considerably greater than the total produced quantity of end products. The parameter $\Delta$ adjusts the magnitude of the unit transport costs of raw materials compared to that of end products.

## 6.7 External costs

Regarding the variable cost of purchasing one unit of product from an external source, we consider that it should reflect the network costs. Therefore, the following scheme is employed:

- Average cost of acquiring raw materials to manufacture one unit of product $p \in P$ in time period $t \in T$:

$$\overline{PC}_{t,p} = \frac{\sum_{\ell \in L} \sum_{r \in R^p} \sum_{s \in S^r} a_{\ell,r,p} \, PC_{t,s,r}}{|S^r| \, |R^p| \, |L|}$$

- Average cost of manufacturing one unit of product $p \in P$ in time period $t \in T$:

$$\overline{MC}_{t,p} = \frac{\sum_{\ell \in L} MC_{t,\ell,p}}{|L|}$$

- Average cost of transporting one unit of product $p \in P$ in time period $t \in T$ from a plant:

$$\overline{TC}^1_{t,p} = \frac{\sum_{\ell \in L} \sum_{d \in W \cup C} \sum_{m \in M} TC_{t,\ell,d,p,m}}{|M| \, |L| \, (|W| + |C|)}$$

- Average cost of transporting raw materials to manufacture one unit of product $p \in P$ in time period $t \in T$:

$$\overline{TC}^2_{t,p} = \frac{\sum_{\ell \in L} \sum_{r \in R^p} \sum_{s \in S^r} \sum_{m \in M} TC_{t,s,\ell,r,m} a_{\ell,r,p}}{|L| \, |S^r| \, |R^p| \, |M|}$$

- Average cost of opening a new facility per unit of product $p \in P$ in time period $t \in T$:

$$\overline{FC}_{t,p} = \frac{\sum_{\ell \in L^n} FC_{t,\ell}}{\sum_{\ell \in L^n} \overline{Q}_\ell} \times \frac{\sum_{\ell \in L^n} \mu_{\ell,p}}{|L^n|}$$

- Average cost of closing an existing facility per unit of product $p \in P$ in time period $t \in T$:

$$\overline{SC}_{t,p} = \frac{\sum_{\ell \in L^e} SC_{t,\ell}}{\sum_{\ell \in L^e} \overline{Q}^e_\ell} \times \frac{\sum_{\ell \in L^e} \mu_{\ell,p}}{|L^e|}$$

- Average cost of operating an existing facility per unit of product $p \in P$ in time period $t \in T$:

$$\overline{OC}_{t,p} = \frac{\sum_{\ell \in L^e} OC_{t,\ell}}{\sum_{\ell \in L^e} \overline{Q}^e_\ell} \times \frac{\sum_{\ell \in L^e} \mu_{\ell,p}}{|L^e|}$$

- Average cost of installing a capacity level per unit of product $p \in P$ in time period $t \in T$:

$$\overline{IC}_{t,p} = \frac{\sum_{\ell \in L} \sum_{k \in K_L} IC_{t,\ell,k}}{\sum_{\ell \in L} \overline{Q}_\ell} \times \frac{\sum_{\ell \in L} \mu_{\ell,p}}{|L|}$$

Finally, for every $t \in T$, $w \in W$, and $p \in P$ we set $EC_{t,w,p} = U[0.7A_{t,p},\ 0.9A_{t,p}]$ with $A_{t,p}$ given by

$$A_{t,p} = \overline{PC}_{t,p} + \overline{MC}_{t,p} + \overline{TC}^1_{t,p} + \overline{TC}^2_{t,p} + \overline{FC}_{t,p} + \overline{SC}_{t,p} + \overline{OC}_{t,p} + \overline{IC}_{t,p}$$

# 7 Computational study

In order to evaluate the tractability of the model presented in Section 3, numerical experiments were performed on a large set of randomly generated test instances. The latter were generated according to the procedure described in the previous section. A summary of the computational results is presented in Section 7.1 and a discussion of various managerial implications is provided in Section 7.2 for a number of scenarios.

For each combination of the values of $|T|$ and $|C|$, five instances were randomly generated. In total, 60 instances were obtained, half of them belong to the LNRD class and the other half to the LND class. Each instance was solved with six different combinations of the parameters $\lambda_{t,p}$ and $\beta_{t,p}$ according to Section 6.2, thus yielding a total of 360 runs. These choices impact the number of variables and constraints of formulation (1)–(38). For example, when outsourcing is not an option ($\beta_{t,p} = 0$), and so all end products must be in-house manufactured, variables $z_{t,w,p}$ and constraints (23) can be dropped from the formulation. Moreover, the flow conservation constraints (22) and the objective function have less terms. When direct shipments from plants to customers are not considered ($\lambda_{t,p} = 0$), constraints (6)–(8) and (24) can be simplified. Furthermore, constraints (11) can be removed from the formulation. The largest decrease in the number of variables and constraints is obtained when both parameters $\beta_{t,p}$ and $\lambda_{t,p}$ are equal to zero. Table 9 summarizes the sizes of the test instances. In particular, the total number of continuous variables is significantly impacted by variables $x_{t,o,d,i,m}$. Since test instances belonging to the LND class do not include existing facilities, they have on average 17% less variables and 7% less constraints compared to LNRD instances. Finally, extending the length of the planning horizon from 4 to 6 time periods increases the instance size by a factor of 1.5, both with respect to the number of variables and constraints.

| Problem class | Number of | $\|T\| = 4$ Avg | Min | Max | $\|T\| = 6$ Avg | Min | Max |
|---|---|---|---|---|---|---|---|
| LNRD | Binary variables | 432 | 288 | 560 | 648 | 432 | 840 |
| | Continuous variables | 156830 | 32161 | 345201 | 235244 | 48241 | 517801 |
| | Constraints | 8605 | 3637 | 13949 | 12883 | 5439 | 20891 |
| LND | Binary variables | 363 | 240 | 480 | 544 | 360 | 720 |
| | Continuous variables | 129505 | 26001 | 289601 | 194258 | 39001 | 434401 |
| | Constraints | 8010 | 3370 | 13100 | 11992 | 5040 | 19620 |

Table 9: Size of the test instances

## 7.1 Summary of results

The model including its enhancements was implemented in C++ using IBM ILOG Concert Technology and solved with IBM ILOG CPLEX 12.6.0. All experiments were conducted on a PC with a 3.5 GHz Intel Core i7-3770K processor, 8 GB RAM and running Windows 7 (64-bit). A limit of 8 h of CPU time was set for each instance. CPLEX was used with default settings, as it is typically the case in practice, making full use of its MIP heuristics in an attempt to find high quality solutions at an early stage of the branch-and-cut algorithm.

Table 10 displays the average, minimum, and maximum CPU times for the two problem classes. The formulation (1)–(38) with the simple enhancements described in Section 5.1 is named *base model*. Recall that these enhancements do not impact the original number of constraints. The formulation with all enhancements and valid inequalities introduced in Section 5 is named *enhanced model*. Although the LND class includes smaller instances, these require larger computational effort. This is due to the fact that when a new network is to be designed, location and capacity acquisition decisions are harder to take. In contrast, in the LNRD class some facilities are already in place at the beginning of the planning horizon. Since strategic location decisions are very costly, most of these facilities tend to remain in operation and only a few new facilities need to be established. In addition, adjustments in the network configuration take often the form of capacity expansion at (some of the) existing facilities over the time horizon.

| Problem class | Model type | $\|T\| = 4$ Avg | Min | Max | $\|T\| = 6$ Avg | Min | Max |
|---|---|---|---|---|---|---|---|
| LNRD | Base | 162.5 | 0.5 | 480 | 217.1 | 1.4 | 480 |
| | Enhanced | 99.8 | 0.3 | 480 | 192.9 | 0.7 | 480 |
| LND | Base | 230.9 | 0.4 | 480 | 309.0 | 2.4 | 480 |
| | Enhanced | 236.7 | 0.5 | 480 | 290.8 | 2.4 | 480 |

Table 10: CPU time (minutes)

As expected, increasing the number of time periods results in larger computational times. While all instances with 50 customers were solved to optimality, for many of the instances with 75 customer zones and for almost all instances with 100 customer zones the time limit was attained without achieving optimality (see also Table 11). This is particularly striking in the LND class. Table 10 also shows that the valid inequalities introduced in Sections 5.2 and 5.3 reduce the computational effort required to solve the problems. The largest improvement is achieved in instances of the LNRD class with 4 time periods.

Table 11 summarizes the quality of the best feasible solutions identified by CPLEX within the specified time limit. This is assessed by the value of the integrality gap as follows: MIP gap = $(z^{UB} - z^{LB})/z^{UB} \times 100\%$ with $z^{UB}$ denoting the objective value of the best feasible solution and $z^{LB}$ representing the best lower bound. This calculation was performed both for the base and the enhanced models. Given the complexity of the problems and the size of the instances considered, it is interesting to observe that the average gaps are rather small: less than 1% in the LNRD class and less than 2% in the LND class. Furthermore, the MIP gap is less than 9% over all experiments. Nevertheless, the gap further decreases when the inexpensive valid inequalities described in Sections 5.2 and 5.3 are added. The only exception is the LND class with $|T| = 4$, where approximately the same results are obtained with the two models. In contrast, the most difficult instances (LND, $|T| = 6$) clearly benefit from the proposed enhancements.

| | | $\|T\| = 4$ | | | | $\|T\| = 6$ | | | |
|---|---|---|---|---|---|---|---|---|---|
| Problem | | MIP gap (%) | | | # opt. | MIP gap (%) | | | # opt. |
| class | Model type | Avg | Min | Max | sol. | Avg | Min | Max | sol. |
| LNRD | Base | 0.62 | 0.00 | 4.35 | 67 | 0.98 | 0.00 | 7.58 | 61 |
| | Enhanced | 0.18 | 0.00 | 3.47 | 83 | 0.83 | 0.00 | 6.90 | 64 |
| LND | Base | 1.67 | 0.00 | 8.59 | 58 | 1.98 | 0.00 | 8.95 | 42 |
| | Enhanced | 1.69 | 0.00 | 8.38 | 53 | 1.51 | 0.00 | 5.81 | 44 |

Table 11: MIP gap and number of optimal solutions identified within the time limit

To further gain insight into the quality of the solutions obtained, Table 11 also presents the total number of optimal solutions identified in each problem class within the time limit of 8 h. The valid inequalities proved to be very useful in the LNRD class as 81.7% of the instances (147 out of 180) could be solved to optimality compared to only 71% (128 out of 180) with the base model. In the LND class, the number of optimal solutions identified by CPLEX with the enhanced model is slightly smaller than with the base model (97 against 100). This slight performance reduction can be explained by the unforeseeable behavior of CPLEX as a result of using several heuristics in an attempt to find good quality solutions early in the branch-and-cut tree. Hence, a proper comparison of different formulations is sometimes hindered by this phenomenon. Nonetheless, in 75.6% of the LND instances, a feasible solution was identified within 3% of optimality (in the LNRD class such high-quality solutions were achieved in 92.8% of the instances).

For the base model, we also determined the relative percentage deviation between the objective value of the best feasible solution ($z^{UB}$) and the linear programming relaxation bound ($z^{LR}$) as follows: $(z^{UB} - z^{LR})/z^{UB} \times 100\%$, since this serves as a measure of the tightness of the formulation. Table 12 reports the gaps obtained. Furthermore, to assess the extent to which the LP relaxation of the formulation is strengthened by adding valid inequalities, the table also presents the improvement of the LP relaxation bound of the enhanced model ($z^{LRE}$) over the base model. This is determined by $(z^{LRE} - z^{LR})/z^{LR} \times 100\%$. It can be seen that the impact is stronger on the instances belonging to the LNRD class.

| Problem class | Model type | $\|T\| = 4$ Avg | Min | Max | $\|T\| = 6$ Avg | Min | Max |
|---|---|---|---|---|---|---|---|
| LNRD | Base | 12.01 | 5.93 | 24.04 | 11.09 | 5.40 | 18.61 |
| LND | Base | 9.78 | 5.66 | 16.22 | 9.81 | 3.39 | 19.26 |
| Improvement (%) | | | | | | | |
| LNRD | Enhanced | 5.78 | 0.00 | 24.24 | 3.14 | 0.00 | 17.60 |
| LND | Enhanced | 2.64 | 0.00 | 6.51 | 2.06 | 0.00 | 5.91 |

Table 12: LP relaxation gap of base model (%) and improvement in enhanced model (%)

In summary, the introduction of a small-cardinality set of valid inequalities has a positive effect not only on the LP relaxation bound but also on the computational time and on the quality of the feasible solutions identified by CPLEX. Moreover, given the complexity of the LNRD and LND problems and the large size of the test instances, the results obtained indicate that it is worthwhile to invest computational time to solve these strategic planning problems with a mixed-integer programming solver such as CPLEX.

## 7.2 Managerial insights

In order to achieve deeper managerial insights and thus better help decision-makers to understand the economic benefits obtained from a logistics network (re-)design project, we now focus on "what-if" analyses. To this end, we considered the formulation with all enhancements and valid inequalities, and studied the impact of varying several parameters.

### 7.2.1 Cost analysis

In the so-called *base case*, direct product shipments from plants to customer zones are not possible ($\lambda_{t,p} = 0$, $\forall t \in T$, $p \in P$) and all products must be in-house manufactured ($\beta_{t,p} = 0$, $\forall t \in T$, $p \in P$). Problems with these characteristics are associated to logistics networks with limited flexibility. Table 13 shows the average contribution of various cost categories to the overall cost in both problem classes. The category "Location & cap. acquisition" corresponds to the total investment made on opening new facilities, closing existing facilities, acquiring/extending capacity, and operating facilities over the time horizon. This investment is determined by the sum of the first four components, (30)–(33), of the objective function. The category "Transportation" comprises all expenditures on moving raw materials and end products throughout the network. They depend on the selected distribution channels and on the choice of transportation modes. The categories "Procurement" and "Production" include the cost of purchasing raw materials and transforming them into end products at plants.

| Problem class | Location & cap. acquisition | | Logistics functions | | |
|---|---|---|---|---|---|
| | Plants | Wareh. | Transportation | Procurement | Production |
| LNRD | 9.74 | 21.65 | 45.37 | 6.78 | 16.46 |
| LND | 17.36 | 32.44 | 33.12 | 5.10 | 11.98 |

Table 13: Average contribution of different cost categories to total cost (%) - base case

Table 13 highlights the trade-offs inherent to each problem class, especially with respect to strategic location and capacity acquisition decisions. In a "greenfield" approach, the fixed costs for establishing and operating new facilities are significantly higher than those incurred in the context of network re-design. However, logistics costs, and in particular transportation expenditures, are considerably lower in an LND setting.

We examine next the effect of relaxing the assumptions made in the base case. To this end, a series of scenarios were considered and compared to the base case according to the following cost categories: location and capacity acquisition costs for plants and warehouses, transportation cost, procurement and production cost at plants, and total cost. Observe that the total cost also includes the cost of purchasing products from external sources. For all the instances of a particular scenario, the evaluation of each cost category is performed as follows: (avg. cost of scenario - avg. cost of base case)/(avg. cost of base case) $\times$ 100%. Figures 2–5 summarize the results obtained.

The strategic choice between in-house manufacturing (base case) versus a mixed approach with a limited outsourcing level for end products can be examined in Figure 2 for the LNRD class. In this scenario, all customer zones are supplied by warehouses ($\lambda_{t,p} = 0$). Increasing the maximum outsourcing level from zero to 20% does not have a significant impact on the overall costs. The largest effect is observed on the fixed costs for $\beta_{t,p} = 0.2$. In this case, less production capacity is required which results in lower investment spending on establishing new plants, installing capacity in these locations, and expanding the capacity of existing plants. As a consequence, procurement, production, and transportation costs also decrease. Despite these

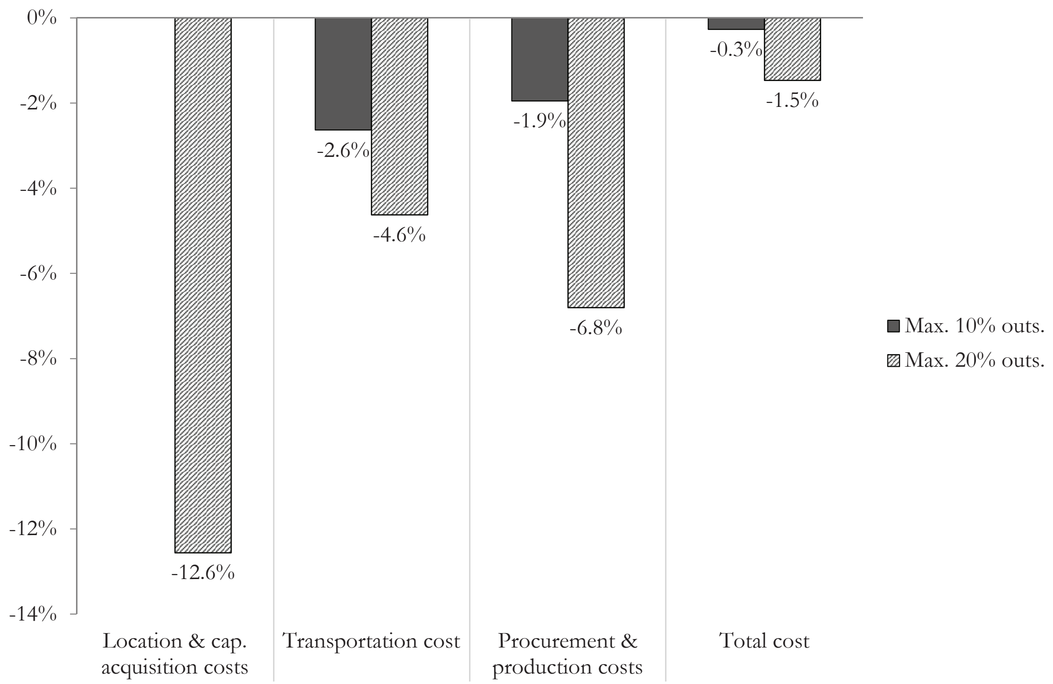


Figure 2: Cost comparison to base case: instances without direct shipments from plants to customer zones (LNRD class)

reductions, the total cost decrease is less than 1.5% since additional costs are incurred due to product outsourcing.

The impact of setting a 10% limit on the total demand that can be shipped from plants to customer zones is shown in Figure 3 for three different outsourcing levels in the LNRD class. In the scenario without outsourcing, the strategic and logistics costs decrease by 7.2%. As expected, this is due to lower transportation and warehouse costs. Since warehouses handle less products, capital expenditures on installing and/or expanding warehouse capacity also decrease. Additional cost savings are achieved when end products can be purchased up to a given limit from an external source ($\beta_{t,p} \in \{0.1, 0.2\}$). This feature reduces the capacity requirements at plants and as a result, the fixed costs associated with plant investments decrease. Moreover, less raw materials are needed which leads to a reduction in procurement and production costs. Lower production levels are also associated with smaller flows of raw materials to plants and of end products from plants to warehouses, thus further decreasing the transportation costs. In particular, the scenario with $\beta_{t,p} = 0.2$ benefits the most from these cost reductions.

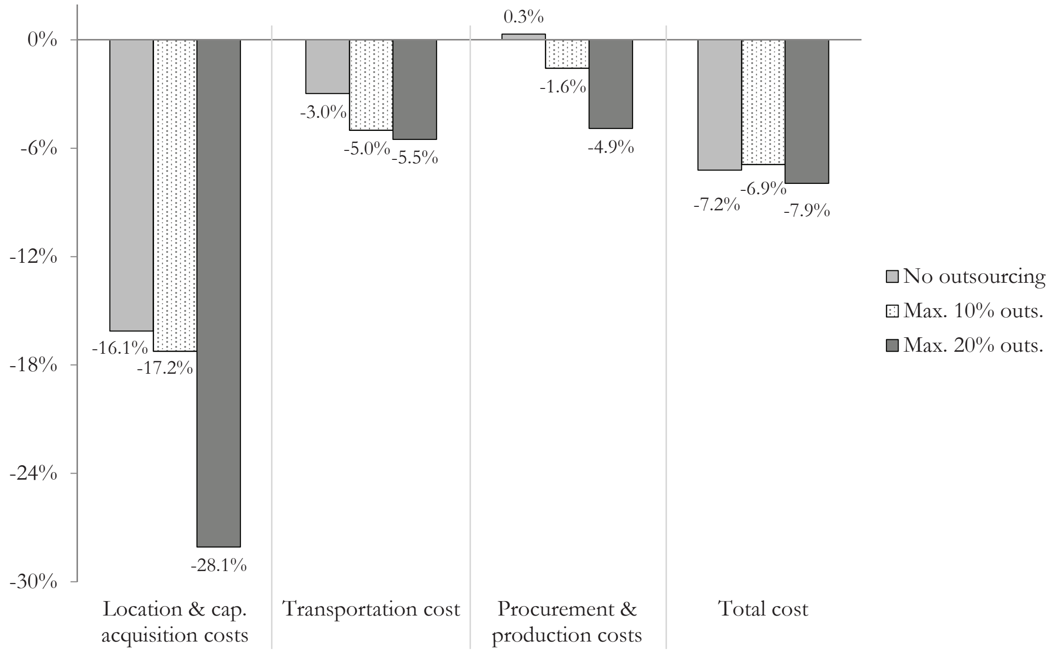


Figure 3: Cost comparison to base case: instances with at most 10% direct shipments from plants to customer zones (LNRD class)

In Figures 4 and 5, different scenarios involving instances of the LND class can be compared. In contrast to the LNRD test instances, here the transportation, procurement, and production costs seem to be mostly affected. This is due to a higher use of the available outsourcing opportunities (cf. Figure 4) which reduce the amount of raw materials and production levels required. Consequently, the transport volumes from suppliers to plants and from the latter to warehouses also decrease. Expenditures on opening new plants are not affected but lower requirements on production capacity lead to less investment in capacity acquisition.

The effect of combining in-house production with outsourcing and permitting direct shipments from plants to customer zones ($\lambda_{t,p} = 0.1$) induces further cost savings as shown in Figure 5. In addition to the cost reductions described above, also fixed costs for new warehouses decrease. For example, instead of establishing a new warehouse with a large capacity level in a given location, a smaller capacity size may be adequate, thus lowering the overall warehouse investment. We note that half of the instances in these scenarios could not be solved to optimality within the pre-specified time limit. This may explain why the procurement and production costs slightly increase in one of the scenarios ($\lambda_{t,p} = 0.1$, $\beta_{t,p} = 0$) compared to the base case.

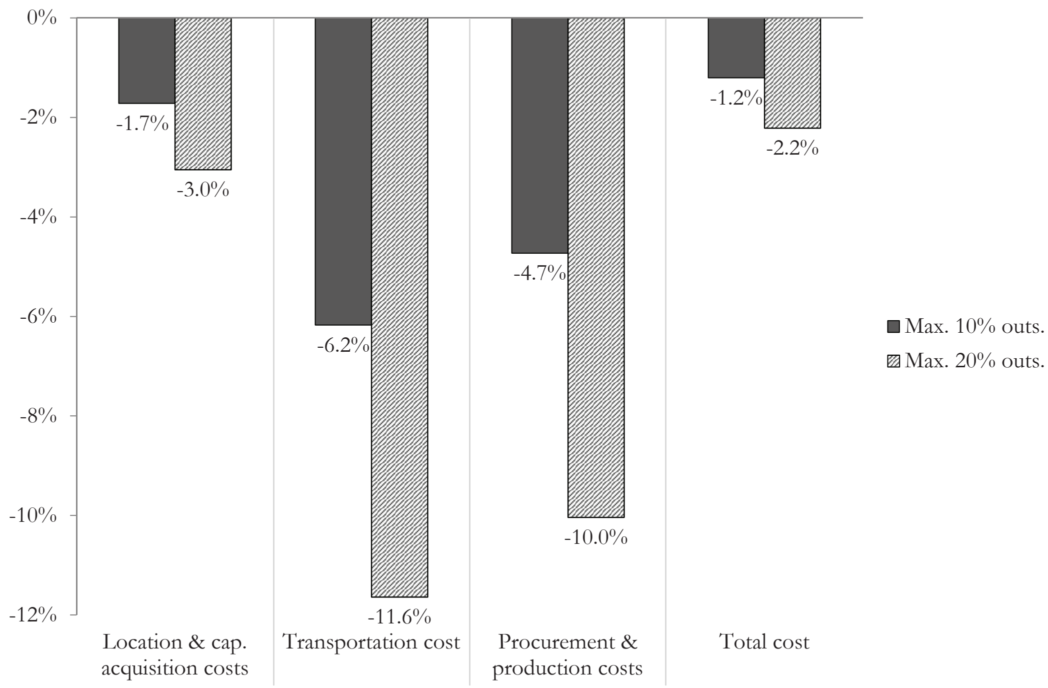


Figure 4: Cost comparison to base case: instances without direct shipments from plants to customer zones (LND class)

A further analysis of Figures 2–5 indicates that there is a striking difference between the two problem classes regarding the magnitude of the cost reduction related to strategic decisions compared to the base case. In particular, in those scenarios with the highest rate of product outsourcing, the decrease in expenditures on facility location and capacity expansion appears to be greater in the LNRD class than in the LND class. As mentioned before, this is mostly due to lower requirements on manufacturing capacity, which leads to reduced investments on opening new plants and/or extending the capacity of existing plants. In other words, there is less need to restructure the existing network and the available capacity is even better utilized (this aspect will be discussed in detail in the next section). In the LND class, investment spending on plant capacity also decreases with growing product outsourcing levels but the order of magnitude is lower than in the LNRD class. This is due to the fact that the new network configurations do not differ so much as in an LNRD context for different values of the parameter $\beta_{t,p}$.

The analysis of Figures 2–5 also indicates that the largest cost benefits are achieved when distribution channels are available from plants to customer zones and end products can be partially acquired from an external source. In contrast to the base case, these two features

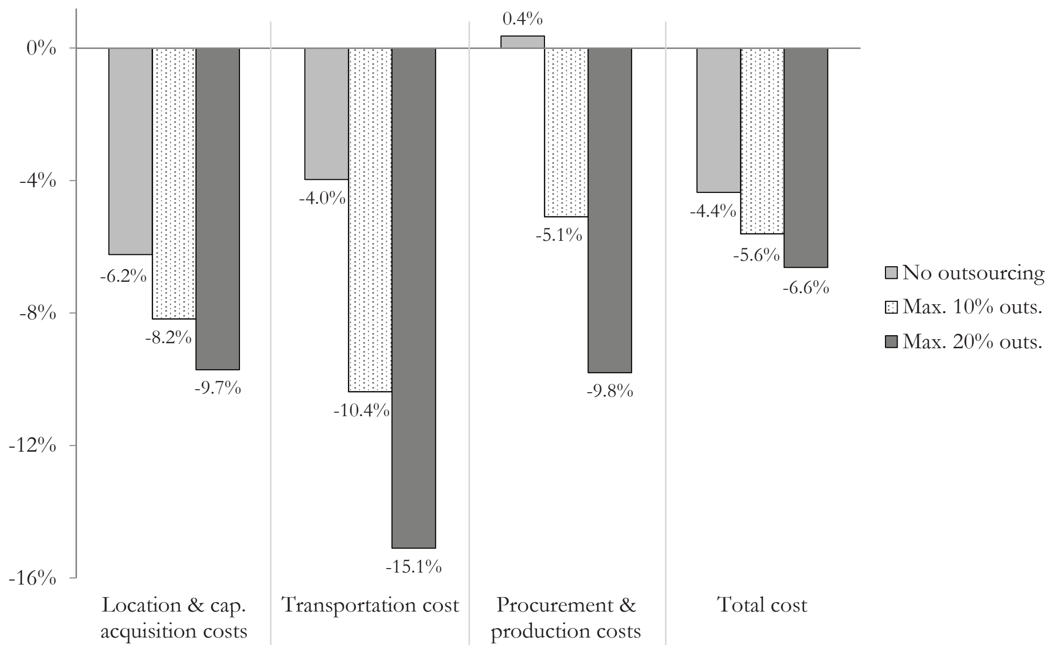


Figure 5: Cost comparison of instances with at most 10% direct shipments from plants to customer zones to base case (LND class)

reflect realistic settings and extend the scope of a network (re-)design project.

### 7.2.2 Capacity utilization and outsourcing levels

From a managerial perspective, an additional important aspect to be investigated is the capacity utilization level of the facilities in the logistics network. This metric gives insight into the overall slack capacity and therefore, it is an important indicator of whether a company has room to increase production or warehousing without incurring the expensive costs of establishing new facilities. Furthermore, deeper insight on the usage level of product outsourcing will also help a company to get a better understanding of the impact of this strategy on the structure of its supply chain and on the overall cost. Table 14 provides information on these two measures. For the LNRD problem class, we report the average capacity utilization rate per period at an existing and a new facility (plant/warehouse) associated with pre-specified limits on the amount of products that can be outsourced (i.e. $\beta_{t,p} \in \{0, 0.1, 0.2\}$, $\forall t \in T, p \in P$). In addition, the actual average outsourcing level per product and period is also given along with the corresponding standard deviation (columns "Avg" and "Std"). The last column of Table 14 gives the share of the outsourcing cost relative to the total cost. Similar information is also provided for the instances belonging to the LND problem class. All values reported in Table 14 refer to scenarios with at most 10% of all customer demand being directly delivered from plants, a total of 180 scenarios for the original 60 instances. Scenarios with $\lambda_{t,p} = 0, \forall t \in T, p \in P$, yielded comparable results.

| Problem | Max. outs. | Capacity usage (%) | | | | Outsourcing (%) | | |
|---|---|---|---|---|---|---|---|---|
| | | Plants | | Warehouses | | | | |
| class | level (%) | Exist | New | Exist | New | Avg | Std | Cost |
| | 0 | 72.3 | 99.5 | 88.5 | 100.0 | | | |
| LNRD | 10 | 71.8 | 99.4 | 88.7 | 100.0 | 1.6 | 3.3 | 1.2 |
| | 20 | 83.2 | 99.2 | 90.0 | 100.0 | 7.3 | 7.4 | 4.9 |
| | 0 | | 92.6 | | 94.6 | | | |
| LND | 10 | | 94.3 | | 96.0 | 5.5 | 4.3 | 2.9 |
| | 20 | | 94.5 | | 95.3 | 10.1 | 8.7 | 5.2 |

Table 14: Average capacity utilization rates and outsourcing statistics (scenarios with at most 10% direct shipments from plants to customer zones)

The type of network design project impacts the capacity utilization rate, especially at the plant echelon. "Greenfield" approaches lead on average to higher capacity usage than network re-design initiatives. This is explained by the fact that when a new logistics network is to be established there is more flexibility to choose locations to operate new facilities. For example, typical trade-offs with respect to proximity of plants to sources of raw materials and to warehouse locations as well as closeness of warehouses to customer zones are fully explored in an LND project. In contrast, if a network is already in place, many network re-design options are not attractive since additional fixed costs are incurred by closing existing facilities. Therefore, the trade-offs between the fixed facility costs and the variable logistics costs inherent in the selection of any given solution do not make it viable to replace the existing network configuration by a completely new structure. This feature is more noticeable when increasing outsourcing levels are considered because in this case the required production capacity decreases. Moreover, the capacity utilization rate at existing plants is significantly smaller than at new plants. In the lower echelon, the capacity requirements of warehouses are not so much affected by this feature since both in-house manufactured products and outsourced commodities are consolidated in these facilities. Existing warehouses have somewhat lower capacity utilization rates than new warehouses, while the latter are operated at full capacity. This indicates that cost benefits are achieved by only installing the required capacity, however capacity bottlenecks may occur with increasing product demand. Redesigned logistics networks are less prone to such situations as more slack production and warehouse capacity is available.

Table 14 also shows that there is a higher usage level of the outsourcing strategy when a limit of 20% is set compared to 10%. Interestingly, both network design and re-design problems make a selective use of this business strategy as indicated by the low average outsourcing rates. This aspect is also highlighted by the rather small share of outsourcing relative to the overall network (re-)design cost. Furthermore, the outsourcing rate varies significantly among products as indicated by the standard deviations.

# 8 Conclusions

Network design plays a crucial role for a company as the configuration of its logistics network defines the operating basis of the whole supply chain. In this paper, we presented a new mathematical formulation for a comprehensive multi-echelon logistics network design/re-design problem. The proposed model extends previous work by integrating key features needed to capture real-world situations. The aspects considered include, among others, location decisions involving plants and warehouses, capacity acquisition, expansion and contraction over a multi-period time horizon, transportation mode selection, and product outsourcing opportunities.

Our computational experiments showed that problems addressing the design of a new logistics network are more difficult to solve than problems focusing on the re-design of a network that is already in place. Despite the complexity of these problems and the large size of the test instances, the average integrality gaps produced by CPLEX are rather small (less than 2%). Further enhancements were achieved through the introduction of valid inequalities which helped strengthen the linear relaxation bound, identify better feasible solutions, and reduce the computational effort. In fact, feasible solutions within 3% of optimality were identified in 92.8%, resp. 75.6%, of the LNRD, resp. LND, instances.

In our empirical study, focus was also given to the impact of various parameters on different segments of the logistics network (location and capacity acquisition, procurement, production, distribution, and outsourcing). In particular, the trade-offs achieved by combining in-house manufacturing with product outsourcing were extensively analyzed. In addition, the advantages of using distribution channels to satisfy customer demands from upper echelon facilities were investigated, also in conjunction with an outsourcing strategy. The insights gained from the analyses performed illustrate the far-reaching implications of given strategies on the configuration of a logistics network and on the possibility of making adjustments in the network structure over the time horizon. In view of the substantial investment capital involved in LND/LNRD projects and the limited reversibility of strategic decisions, it is essential for a company to perceive the impact of certain decisions on the configuration and performance of its logistics network.

The proposed mathematical model is flexible and can easily be extended to handle further aspects of a strategic network design project. For example, single-sourcing could be imposed to meet customer demands. Some companies prefer this sourcing mode as this makes the management of the logistics network considerably simpler. Regarding the choice of suppliers, also alternatives to multiple sourcing could be considered, e.g. dual sourcing. However, single-sourcing requirements would significantly make the problem much harder to solve. In this case, further research would be required in order to develop a method that could produce near-optimal solutions within reasonable time limits. Future research could also focus on designing a specially tailored solution approach for the formulation introduced in this paper. As shown by our computational experiments, the length of the planning horizon has a significant effect on the computational effort required to solve an LND or LNRD instance. In such case, the computational requirements of a strategic planning process become even more important, particularly if several runs are needed to analyze the outcome of different scenarios.

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Das Institut für Supply Chain und Operations Management (ISCOM) der htw saar ist auf die Anwendung quantitativer Methoden in der Logistik und deren Implementierung in IT-Systemen spezialisiert. Neben öffentlich geförderten Forschungsprojekten zu innovativen Themen arbeitet ISCOM eng mit Projektpartnern aus der Wirtschaft zusammen, wodurch der Wissens- und Technologietransfer in die Praxis gewährleistet wird. Zu den Arbeitsgebieten zählen unter anderem Distributions- und Transportplanung, Supply Chain Design, Bestandsmanagement in Supply Chains, Materialflussanalyse und -gestaltung sowie Revenue Management.

Weitere Informationen finden Sie unter **http://iscom.htwsaar.de**

Qbing
Produktion | Logistik | Technologie

**Forschungsgruppe Qbing**

Qbing ist eine Forschungsgruppe an der Hochschule für Technik und Wirtschaft des Saarlandes, die spezialisiert ist auf interdisziplinäre Projekte in den Bereichen Produktion, Logistik und Technologie. Ein Team aus derzeit acht Ingenieuren und Logistikexperten arbeitet unter der wissenschaftlichen Leitung von Prof. Dr. Steffen Hütter sowohl in öffentlich geförderten Projekten als auch zusammen mit Industriepartnern an aktuellen Fragestellungen zur Optimierung von logistischen Prozessabläufen in Handel und Industrie unter Einbeziehung modernster Sensortechnologie und Telemetrie. Qbing hat auch und gerade auf dem Gebiet der angewandten Forschung Erfahrung in der Zusammenarbeit mit kleinen und mittelständischen Unternehmen.

Weitere Informationen finden Sie unter **http://www.qbing.de**

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