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Multi-period capacitated facility location under delayed demand satisfaction

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Abstract

We address an extension of the classical multi-period facility location problem in which customers are sensitive to delivery lead times. Accordingly, two customer segments are considered. The first segment comprises customers that require timely demand satisfaction, whereas customers accepting delayed deliveries make up the second segment. Each customer belonging to the latter segment specifies a maximum delivery time. A tardiness penalty is incurred to each unit of demand that is not satisfied on time. In the problem that we study, a network is already in place with a number of facilities being operated at fixed locations. The network can be expanded by establishing new facilities at a finite set of potential sites and selecting their capacity levels from a set of available discrete sizes. In addition, existing facilities may be closed over the time horizon. Two mixed-integer linear programming formulations are proposed to re-design the network at minimum cost and a theoretical comparison of their linear relaxations is provided. We also extend the mathematical models to the case in which each customer accepting delayed demand satisfaction requires late shipments to occur at most once over the delivery lead time. To gain insight into the complexity of the problems at hand, a computational study is performed with randomly generated instances that are solved with a general-purpose solver. Useful insights are derived from analyzing the impact of different delivery lead time restrictions on the network structure and cost.

Keywords: Location, multi-period, capacity choice, delivery lateness, MILP models

1 Introduction

Discrete facility location models are typically concerned with determining the number, location and capacities of facilities that should be established to serve the demands of a set of spatially

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distributed customers with least total cost. This field of location analysis has been an active and rich research area over the past decades. A wide variety of applications have emerged in many contexts such as strategic logistics planning (see e.g. Alumur et al. [5] and Melo et al. [16]) and telecommunications (see e.g. Fortz [11]), just to name a few.

Most discrete location models ensure the satisfaction of customer demands on time by imposing distance and/or time limits as service level requirements. However, in a number of practical settings not all customers have the same sensitivity to delivery lead time. For example, in e-commerce, companies often adopt different pricing policies, thereby offering price incentives to customers in exchange for longer order lead times (Agatz et al. [1]). On the one hand, lower unit prices may result in decreased total revenue for the company but on the other hand, in order to be able to satisfy the demands of those customers that are willing to pay a price premium for a shorter delivery time, the company will have to invest in increasing capacity. Following this line of reasoning, we propose a new modelling approach that captures the trade-off between the potential for increased timely demand fulfilment against the costs of re-designing and operating a network of facilities.

The starting point of our problem is a company that operates a set of facilities at fixed locations to fulfil the demand requirements for a given product. Customers are classified on the basis of their sensitivity to delivery lead time. To this end, two customer segments are considered. The first segment comprises customers that require timely demand satisfaction, whereas customers accepting delayed deliveries make up the second segment. Each customer belonging to the latter segment specifies a desired maximum delivery time. Changing market and business conditions, frequently in conjunction with increased cost pressure and service requirements, compel the company to restructure its network of facilities. This entails establishing new facilities at a finite set of potential sites over a multi-period planning horizon and selecting their capacity levels from a set of available discrete sizes. In addition, capacity contraction is also a viable option through closing one or several existing facilities over the time horizon. The objective is to determine the optimal network configuration over the planning horizon so as to minimize the total cost. The latter comprises fixed costs for facility siting and operation as well as fixed costs for capacity acquisition and contraction. In addition, variable processing and distribution costs along with penalty costs for delayed demand satisfaction are also considered.

Since both strategic and tactical decisions are combined in a single problem, we assume that location and capacity decisions can only be made over a subset of the time periods of the planning horizon. In contrast, decisions regarding the commodity flow from operating

facilities to customers may take place in any time period. This corresponds to considering two different time scales in a similar way as followed by Albareda-Sambola et al. [3] for a facility location-routing problem and by Badri et al. [6] in the context of logistics network design.

As mentioned before, the majority of mathematical models that have been proposed in the literature on discrete facility location impose the satisfaction of customer demand in a timely manner. In contrast, location problems with flexibility regarding demand fulfilment have received much less attention. The case of unfilled demand can be treated either with the lost sales assumption or with the backorder assumption. The former situation applies to contexts in which satisfying all customer demands may not be economically attractive due to high investment costs on establishing new facilities with appropriate capacities. In a static setting, Alumur et al. [5] describe a generic model for a facility location problem arising in logistics network design that includes this feature, whereas Correia et al. [10] address this issue in the design of a two-echelon production-distribution network over multiple time periods. The models developed by Badri et al. [6], Bashiri et al. [7], Canel and Khumawala [8], and Sousa et al. [18] also allow lost sales over a dynamic horizon. In the previous studies [6, 7, 8, 10, 18], strategic location and tactical logistics decisions are made under a profit maximization objective. In addition, Correia et al. [10] also investigate their problem from a cost minimization perspective with additional constraints enforcing a minimum rate of demand fulfilment. For a number of test instances with small and moderate sizes, the mixed-integer linear programming (MILP) formulations proposed in [5, 7, 8, 10, 18] could be solved to optimality by a commercial MILP solver within acceptable time. Badri et al. [6] developed a Lagrangian-based heuristic through dualizing a set of constraints that limit the expenditures for opening new facilities and expanding the capacity at existing locations.

The lost sales assumption is also present in the problem addressed by Altıparmak et al. [4] through the maximization of the overall fraction of demand that is delivered to customers. This objective is integrated with the minimization of the total cost of designing and operating a multi-stage network and the maximization of the capacity utilization of facilities. These three objectives are combined into a single-objective function by building a weighted sum and feasible solutions are determined with a genetic algorithm. The latter solution methodology was also adopted by Lieckens and Vandaele [14] for a facility location problem arising in reverse logistics with stochastic lead times for processing and moving used products. In this case, a fraction of the returned products may not be collected and demand for reused products may be only partially met. Cheong et al. [9] follow a different approach to deal with lost sales in an

uncapacitated two-echelon distribution network. To this end, each customer is offered a short and a long delivery lead time and his demand requirements for a single product decline as the lead time increases. Fixed costs for locating one central warehouse and several local warehouses as well as variable costs for inventory holding and product distribution are considered along with lost sales costs. The proposed MILP formulation is solved for randomly generated instances involving five customers. Contrary to the aforementioned works, which allow partial satisfaction of the demand of a particular customer, in the location-inventory problem addressed by Shen [17], the demand of each customer for a single product is either completely satisfied or not. This type of decision is impacted by the so-called customer's reserve price and the total price charged by the company to the customer. The latter comprises the selling price and the delivery cost, and both factors depend on the location, distribution and inventory decisions made. When the total price is higher than the customer's reserve price, the company does not supply the customer. The problem is solved with a branch-and-price algorithm.

Compared to the lost sales case, the backorder option has received much less attention in the literature dedicated to facility location. In the problem studied by Gebennini et al. [12], all customers tolerate a delay of at most one period for demand fulfilment. Moreover, a constant backorder cost representing a late-delivery penalty is considered in each period of the planning horizon for each customer. While this case may occur, it has a limited domain of applicability since different customers have different sensitivity to delays and the impact of the latter in terms of costs is also not the same for every customer. Wilhelm et al. [19] address the design of a multi-echelon production-distribution network over a multi-period horizon in an international context. Backorder costs are incurred to demand requirements that cannot be satisfied on time. Contrary to [12], no time limit is imposed on the delay for satisfying the demand of a particular customer. In other words, the unrealistic assumption is made that customers tolerate any time span between order placement and order delivery. Compared to the previous studies, Meisel et al. [15] adopt a broader view of customer sensitivity to delivery lead times. In their three-echelon production-distribution system, a maximum lead time is promised to each customer. Production and transportation times determine the actual lead time for satisfying the demand requirements of a customer. However, backorder costs for late shipments are not incurred, meaning that customers are insensitive to lateness in demand fulfilment, an assumption that has limited practical application.

To the best of the authors' knowledge, the mathematical models to be proposed in this paper are the first to embed customer segments having distinct sensitivity to delivery lead

times in a multi-period facility location problem. This new feature is combined with different time scales for strategic and tactical decisions. Furthermore, the decision space is extended with strategic facility sizing decisions, an aspect that is not often encountered in the literature. In fact, most facility location models consider capacity as an exogenous factor. However, from an application point of view, capacity is often purchased in the form of equipment which is only available at a few discrete sizes. Capacity choices incur specific fixed installation costs that are subject to economies of scale. Hence, we add three new dimensions (customer segments with distinct service requirements, different decision time scales and multiple capacity choices) to the classical multi-period facility location problem which is known to be an NP-hard problem. From a computational point of view, this results in a challenging problem for which the possibility of solving large-scale instances to optimality within acceptable time is rather limited. In such cases, one often resorts to heuristic methods to obtain feasible solutions. However, to be able to measure the quality of such solutions it is of paramount importance to have (good) lower bounds for the problem. Therefore, a further contribution of our work is to propose different formulations and to compare them in terms of the LP-relaxation bound they provide. Additional inequalities are also developed in an attempt to strengthen these bounds. Finally, we conduct an extensive computational study to obtain managerial insights that illustrate the far-reaching implications of delivery lead time restrictions on the network structure and its cost. Without the support of the models developed in this paper it would otherwise be difficult to obtain most of these insights. Given the typically high investment volumes and the limited reversibility of strategic decisions, it is essential for stakeholders to perceive the impact of such decisions on overall system performance.

The remainder of this article is organized as follows. In Section 2, we develop various MILP formulations for the problem under study and present a theoretical comparison of their linear relaxations. In particular, we also consider the special case in which customers accepting late deliveries wish to receive single shipments even if they arrive with some delay. In other words, partial, late deliveries are not allowed for such customers. In Section 3, additional inequalities are proposed to enhance the original formulations. Section 4 reports and discusses the results of an extensive computational study using general-purpose optimization software. Finally, in Section 5, conclusions are provided and directions for future research are identified.

2 Mathematical formulations

In this section, we propose and discuss two MILP models for the multi-period facility location problem with delayed demand satisfaction and multiple capacity levels (MFLPDDSM). Furthermore, we address a particular case of this problem in which the demand of a customer must be delivered as a single shipment when the customer tolerates late deliveries. Two MILP formulations will be developed for this case. Next, we introduce the notation that will be used hereafter.

Facilities, customers and planning horizon:

I^e	Set of existing facilities at the beginning of the planning horizon
I^n	Set of candidate sites for locating new facilities
I	Set of all facility locations, $I = I^e \cup I^n$
K_i	Set of discrete capacity levels that can be installed at candidate site i ($i \in I^n$)
K_i	Capacity type of existing facility i , $K_i = \{1\}$ ($i \in I^e$)
J^0	Set of customers whose demands must be satisfied on time
J^1	Set of customers that may experience delayed demand satisfaction
J	Set of all customers, $J = J^0 \cup J^1$
T	Set of discrete time periods
T_L	Set of time periods in which location and capacity acquisition decisions can be made, $T_L \subset T$

As mentioned in Section 1, time periods for strategic and tactical decisions are measured in different scales. In particular, tactical distribution decisions can be made in every time period, whereas strategic decisions related to opening/closing facilities and installing capacity levels at potential sites can only be made in a subset T_L of the time horizon T . To this end, $T_L \subset T$ represents the set of *strategic periods*. In all cases, we denote by $\ell = 1$, resp. ℓ_{max} , the first, resp. last, time period for making strategic decisions.

Capacity and demand parameters:

Q_{ik}	Capacity of level k that can be installed at candidate site i ($i \in I^n$; $k \in K_i$)
Q_{i1}	Capacity of existing facility i ($i \in I^e$) at the beginning of the time horizon
d_j^t	Demand of customer j in time period t ($j \in J$; $t \in T$)
ρ_j	Maximum number of time periods to satisfy the demand of customer j ($j \in J$)

Given the above definition of delivery lead time, the two categories of customers previously introduced correspond to $J^0 = \{j \in J : \rho_j = 0\}$ and $J^1 = \{j \in J : \rho_j > 0\}$. In particular, the demand of customer $j \in J^1$ in time period $t \in T$ must be filled within a maximum lead time ρ_j , but not later than period $t' = \min\{t + \rho_j, |T|\}$. Hence, demand satisfaction cannot be carried over to future periods beyond the planning horizon.

If a new facility is established at candidate location $i \in I^n$ then a capacity level has to be selected from the set of available discrete sizes K_i . We assume that the latter are sorted in non-decreasing order, that is, $Q_{i1} < Q_{i2} < \dots < Q_{i|K_i|}$.

Fixed and variable cost rates:

- FO_{ik}^ℓ Fixed cost of opening a new facility at candidate site i with capacity level k at the beginning of time period ℓ ($i \in I^n$; $k \in K_i$; $\ell \in T_L$)
- FC_{i1}^ℓ Fixed cost of closing the existing facility i at the end of time period ℓ ($i \in I^e$; $\ell \in T_L$)
- M_{ik}^t Fixed maintenance cost incurred by operating facility i with capacity level k in time period t ($i \in I$; $k \in K_i$; $t \in T$)
- c_{ij}^t Cost of distributing one unit of product from facility i to customer j in time period t ($i \in I$; $j \in J$; $t \in T$)
- o_{ik}^t Cost of processing one unit of product at facility i with capacity level k in time period t ($i \in I$; $k \in K_i$; $t \in T$)
- $p_j^{tt'}$ Penalty cost for satisfying one unit of demand of customer j in period t' that was originally demanded in period t ($j \in J^1$; $t \in T$; $t' = t, t+1, \dots, \min\{t + \rho_j, |T|\}$); in particular, for $t' = t$, the penalty cost is equal to zero

We assume that fixed facility opening and closing costs reflect economies of scale. Moreover, fixed maintenance costs account for business overhead costs such as staff and insurance expenditures. By combining the fixed facility and maintenance costs over an appropriate number of time periods, the following aggregated cost parameters are obtained:

$$F_{ik}^\ell = \begin{cases} FO_{ik}^\ell + \sum_{t=\ell}^{|T|} M_{ik}^t & \text{for } i \in I^n; k \in K_i; \ell \in T_L \\ FC_{i1}^\ell + \sum_{t=1}^{\ell} M_{i1}^t & \text{for } i \in I^e; \ell \in T_L \end{cases} \quad (1)$$

Observe that for $i \in I^n$, F_{ik}^ℓ represents the total cost of establishing a new facility at the beginning of period $\ell \in T_L$ and operating it until the end of the time horizon. In a similar way,

F_{ik}^ℓ gives the total cost of operating an existing facility $i \in I^e$ until the end of time period $\ell \in T_L$ when the facility is removed. We note that the earliest moment in time for closing an existing facility is at the end of the first time period.

2.1 Mixed-integer linear programming model

A natural formulation of the MFLPDDSM relies on binary variables to represent strategic facility location and capacity acquisition decisions as follows:

$$z_{ik}^\ell = 1 \text{ if a new facility is established at candidate location } i \text{ with capacity level } k \text{ at the beginning of time period } \ell, 0 \text{ otherwise } (i \in I^n; k \in K_i; \ell \in T_L) \quad (2)$$

$$z_{i1}^\ell = 1 \text{ if existing facility } i \text{ is closed at the end of time period } \ell, 0 \text{ otherwise } (i \in I^e; \ell \in T_L) \quad (3)$$

Observe that if a new facility is opened in period ℓ then it will operate in periods $\ell, \dots, |T|$. Analogously, if an existing facility is removed at the end of period ℓ then it operates in periods $1, \dots, \ell$. In addition, the formulation also includes two sets of continuous variables that rule tactical decisions:

$$x_{ijk}^t : \text{Amount of product distributed from facility } i \text{ with capacity level } k \text{ to customer } j \text{ in time period } t (i \in I; k \in K_i; j \in J^0; t \in T) \quad (4)$$

$$y_{ijk}^{tt'} : \text{Amount of product distributed from facility } i \text{ with capacity level } k \text{ to customer } j \text{ in time period } t' \text{ to (partially) satisfy demand of period } t (i \in I; k \in K_i; j \in J^1; t \in T; t' = t, t+1, \dots, \min\{t+\rho_j, |T|\}) \quad (5)$$

We denote by (P) the following MILP formulation for the MFLPDDSM:

$$\begin{aligned} \text{Min} \quad & \sum_{\ell \in T_L} \sum_{i \in I} \sum_{k \in K_i} F_{ik}^\ell z_{ik}^\ell + \sum_{t \in T} \sum_{i \in I^e} M_{i1}^t \left(1 - \sum_{\ell \in T_L} z_{i1}^\ell \right) + \\ & \sum_{t \in T} \sum_{i \in I} \sum_{k \in K_i} \sum_{j \in J^0} (c_{ij}^t + o_{ik}^t) x_{ijk}^t + \\ & \sum_{t \in T} \sum_{i \in I} \sum_{k \in K_i} \sum_{j \in J^1} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} (p_j^{tt'} + c_{ij}^{t'} + o_{ik}^{t'}) y_{ijk}^{tt'} \end{aligned} \quad (6)$$

s.t.

$$\sum_{i \in I} \sum_{k \in K_i} x_{ijk}^t = d_j^t \quad j \in J^0, t \in T \quad (7)$$

$$\sum_{i \in I} \sum_{k \in K_i} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} y_{ijk}^{tt'} = d_j^t \quad j \in J^1, t \in T \quad (8)$$

$$\sum_{\ell \in T_L} \sum_{k \in K_i} z_{ik}^\ell \leq 1 \quad i \in I \quad (9)$$

$$\sum_{j \in J^0} x_{ijk}^t + \sum_{j \in J^1} \sum_{t'=\max\{1, t-\rho_j\}}^t y_{ijk}^{t't} \leq Q_{ik} \sum_{\ell \in T_L: \ell \leq t} z_{ik}^\ell \quad i \in I^n, k \in K_i, t \in T \quad (10)$$

$$\sum_{j \in J^0} x_{ij1}^t + \sum_{j \in J^1} \sum_{t'=\max\{1, t-\rho_j\}}^t y_{ij1}^{t't} \leq Q_{i1} \left(1 - \sum_{\ell \in T_L: \ell < t} z_{i1}^\ell \right) \quad i \in I^e, t \in T \quad (11)$$

$$x_{ijk}^t \geq 0 \quad i \in I, j \in J^0, k \in K_i, t \in T \quad (12)$$

$$y_{ijk}^{tt'} \geq 0 \quad i \in I, j \in J^1, k \in K_i, t \in T, \quad t' = t, \dots, \min\{t + \rho_j, |T|\} \quad (13)$$

$$z_{ik}^\ell \in \{0, 1\} \quad i \in I, k \in K_i, \ell \in T_L \quad (14)$$

The objective function (6) minimizes the sum of the fixed and variable costs. The former include the costs incurred for establishing new facilities and installing capacity levels, removing existing facilities and operating facilities, whereas the latter account for the costs of processing and shipping the product to customers along with the tardiness costs resulting from delayed deliveries. Constraints (7), resp. (8), guarantee the satisfaction of the demand over the time horizon for customer segment J^0 , resp. J^1 . For each candidate site $i \in I^n$, constraints (9) impose that at most one new facility can be established with a given capacity level over the time horizon. Moreover, once open, new facilities cannot be closed. Constraints (9) also allow each existing facility $i \in I^e$ to be closed at most once over the planning horizon. Inequalities (10), resp. (11), are capacity constraints for new, resp. existing, facilities. Observe that since an existing facility can only be closed at the end of a given time period, say t , its capacity is not available in any subsequent period. This is described in (11) by considering all strategic periods $\ell \in T_L$ such that $\ell < t$ for every $t \in T$. In contrast, if a new facility is established in time period t then its capacity also becomes available in the same period. Therefore, in constraints (10) we consider all periods $\ell \in T_L$ such that $\ell \leq t$ for every $t \in T$. Finally, constraints (12)–(14) state non-negativity and binary conditions.

The formulation that we propose covers multiple situations. In particular, it generalizes the classical multi-period uncapacitated facility location problem (MUFLP). The latter corresponds to setting $T = T_L$, $J^1 = \emptyset$, $I^e = \emptyset$, $|K_i| = 1$, and $Q_{i1} = \infty$ ($i \in I$). Since the MUFLP is an NP-hard problem (see e.g. Jacobsen [13]), the MFLPDDSM is also NP-hard. If $J = J^0$ then model (P) reduces to a classical case in multi-period capacitated facility location in which all

customers must have their demands satisfied on time. If $J = J^1$, the opposite case is captured, namely all customers accept a delay in product delivery. In case $J^0 \subset J$ and $J^1 \subset J$, an intermediate situation is modelled by (P). In particular, this variant ensures that all important customers for the company (i.e. the members of set J^0) receive preferred service. Another distinctive feature of our model, that results from considering different time scales for strategic and tactical decisions, is the extended length of the time horizon compared to classical multi-period location problems where typically only instances with a reduced number of time periods can be solved exactly within acceptable computing times. As it will be shown in Section 2.4, this characteristic has a significant impact on the overall size of the model. This has prompted us to develop an alternative formulation in an attempt to reduce the size and complexity of the resulting problem.

2.2 Alternative formulation

Another way of modelling the MFLPDDSM is to keep the binary variables (2)–(3) and to replace the tactical decision variables x_{ijk}^t and $y_{ijk}^{tt'}$ by the following variables:

$$r_{ij}^t : \text{Total quantity of product shipped from facility } i \text{ to customer } j \text{ in time period } t \ (i \in I; j \in J^0; t \in T) \quad (15)$$

$$s_{ij}^{tt'} : \text{Amount of product distributed from facility } i \text{ to customer } j \text{ in time period } t' \text{ to (partially) satisfy demand of period } t \ (i \in I; j \in J^1; t \in T; t' = t, \dots, \min\{t + \rho_j, |T|\}) \quad (16)$$

$$w_{ik}^t : \text{Total quantity of product that is shipped from facility } i \text{ with capacity level } k \text{ in time period } t \ (i \in I; k \in K_i; t \in T) \quad (17)$$

The relationship between the new variables and variables (4)–(5) is straightforward:

$$r_{ij}^t = \sum_{k \in K_i} x_{ijk}^t \quad i \in I, j \in J^0, t \in T \quad (18)$$

$$s_{ij}^{tt'} = \sum_{k \in K_i} y_{ijk}^{tt'} \quad i \in I, j \in J^1, t \in T, t' = t, \dots, \min\{t + \rho_j, |T|\} \quad (19)$$

$$w_{ik}^t = \sum_{j \in J^0} x_{ijk}^t + \sum_{j \in J^1} \sum_{t' = \max\{1, t - \rho_j\}}^t y_{ijk}^{t't} \quad i \in I, k \in K_i, t \in T \quad (20)$$

Under the transformations (18)–(20), the following formulation, denoted $(\mathbf{P}_a)$, is obtained:

$$\begin{aligned}
\text{Min} \quad & \sum_{\ell \in T_L} \sum_{i \in I} \sum_{k \in K_i} F_{ik}^\ell z_{ik}^\ell + \sum_{t \in T} \sum_{i \in I^e} M_{i1}^t \left(1 - \sum_{\ell \in T_L} z_{i1}^\ell \right) \\
& + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J^0} c_{ij}^t r_{ij}^t + \sum_{t \in T} \sum_{i \in I} \sum_{k \in K_i} o_{ik}^t w_{ik}^t \\
& + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J^1} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} \left(p_j^{tt'} + c_{ij}^{t'} \right) s_{ij}^{tt'} \quad (21)
\end{aligned}$$

s.t.

$$(9), (14)$$

$$\sum_{i \in I} r_{ij}^t = d_j^t \quad j \in J^0, t \in T \quad (7')$$

$$\sum_{i \in I} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} s_{ij}^{tt'} = d_j^t \quad j \in J^1, t \in T \quad (8')$$

$$w_{ik}^t \leq Q_{ik} \sum_{\ell \in T_L: \ell \leq t} z_{ik}^\ell \quad i \in I^n, k \in K_i, t \in T \quad (10')$$

$$w_{i1}^t \leq Q_{i1} \left(1 - \sum_{\ell \in T_L: \ell < t} z_{i1}^\ell \right) \quad i \in I^e, t \in T \quad (11')$$

$$\sum_{k \in K_i} w_{ik}^t = \sum_{j \in J^0} r_{ij}^t + \sum_{j \in J^1} \sum_{t'=\max\{1, t-\rho_j\}}^t s_{ij}^{t't} \quad i \in I, t \in T \quad (22)$$

$$r_{ij}^t \geq 0 \quad i \in I, j \in J^0, t \in T \quad (23)$$

$$s_{ij}^{tt'} \geq 0 \quad i \in I, j \in J^1, t \in T, t' = t, \dots, \min\{|T|, t + \rho_j\} \quad (24)$$

$$w_{ik}^t \geq 0 \quad i \in I, k \in K_i, t \in T \quad (25)$$

The original demand satisfaction constraints are replaced by equalities (7') and (8'), while the capacity constraints are now imposed by conditions (10') and (11'). The new set of constraints (22) link the newly defined continuous variables. They state that the total product outflow from a facility in a given time period is split into deliveries to customers with high service requirements (the first term on the right-hand side) and deliveries to customers accepting delays in demand satisfaction (the last term on the right-hand side). Finally, non-negativity and binary conditions are given by (14) and (23)–(25).

2.3 The single shipment case

For the customer segment J^1 , formulations $(\mathbf{P})$ and $(\mathbf{P}_a)$ allow an order to be split over multiple periods of time for the same customer. However, in some cases, the customer may prefer to receive a single shipment even if it arrives with some delay. To model this requirement, we introduce the following binary variables for every $j \in J^1$, $t \in T$ and $t' = t, \dots, \min\{t + \rho_j, |T|\}$:

$$v_j^{tt'} = \begin{cases} 1 & \text{if all the demand of customer } j \text{ in period } t \text{ is delivered in period } t' \\ 0 & \text{otherwise} \end{cases} \quad (26)$$

We denote the single shipment case by MFLPDDSM-S and adapt formulations $(\mathbf{P})$ and $(\mathbf{P}_a)$ accordingly. In the first case, the demand satisfaction constraints (8) for customers that accept late deliveries are replaced by the following three sets:

$$\sum_{i \in I} \sum_{k \in K_i} y_{ijk}^{tt'} = d_j^t v_j^{tt'} \quad j \in J^1, t \in T, t' = t, \dots, \min\{t + \rho_j, |T|\} \quad (27)$$

$$\sum_{t'=t}^{\min\{t+\rho_j, |T|\}} v_j^{tt'} = 1 \quad j \in J^1, t \in T \quad (28)$$

$$v_j^{tt'} \in \{0, 1\} \quad j \in J^1, t \in T, t' = t, \dots, \min\{t + \rho_j, |T|\} \quad (29)$$

Let us denote by $(\mathbf{Q})$ the MILP model defined by (6)–(7), (9)–(14), (27)–(29).

The counterpart of formulation $(\mathbf{P}_a)$ is obtained by replacing constraints (8') by

$$\sum_{i \in I} s_{ij}^{tt'} = d_j^t v_j^{tt'} \quad j \in J^1, t \in T, t' = t, \dots, \min\{t + \rho_j, |T|\} \quad (30)$$

As a result, the new MILP formulation, $(\mathbf{Q}_a)$, has the objective function (21) and constraints (7'), (9), (10'), (11'), (14), (22)–(25), (28)–(30).

2.4 Comparison of formulations

Let $\bar{k}$ be the largest number of capacity levels that are available, that is, $\bar{k} = \max_{i \in I} \{|K_i|\}$. Moreover, we define $\bar{\rho} = 1 + \max_{j \in J^1} \{\rho_j\}$. This is the maximum time span between order placement and order delivery for a customer belonging to segment J^1 . Recall that the demand of such a customer that occurs in period t can be delivered over periods $t, \dots, \min\{t + \rho_j, |T|\}$. Table 1 displays the size of the proposed formulations with respect to the number of their variables, whereas Table 2 reports on the number of constraints. In the MFLPDDSM, there is a significant reduction in the total number of continuous variables and a marginal increase

Problem	Formulation	Number of decision variables	
		binary	continuous
MFLPDDSM	(P)	$O(T_L \cdot (\bar{k} \cdot I^n + I^e))$	$O(T \cdot (\bar{k} \cdot I^n + I^e) \cdot (J^0 + \bar{p} \cdot J^1))$
	(P _a)	$O(T_L \cdot (\bar{k} \cdot I^n + I^e))$	$O(T \cdot (I \cdot J^0 + \bar{p} \cdot I \cdot J^1 + \bar{k} \cdot I^n + I^e))$
MFLPDDSM-S	(Q)	$O(T_L \cdot (\bar{k} \cdot I^n + I^e) + \bar{p} \cdot J^1 \cdot T)$	$O(T \cdot (\bar{k} \cdot I^n + I^e) \cdot (J^0 + \bar{p} \cdot J^1))$
	(Q _a)	$O(T_L \cdot (\bar{k} \cdot I^n + I^e) + \bar{p} \cdot J^1 \cdot T)$	$O(T \cdot (I \cdot J^0 + \bar{p} \cdot I \cdot J^1 + \bar{k} \cdot I^n + I^e))$

Table 1: Number of decision variables in the proposed formulations

in the number of constraints (namely, $|I| \cdot |T|$) in model (P_a) compared to (P). In fact, if n denotes the number of continuous variables in the latter formulation then (P_a) has only $n/\bar{k} + \bar{k} \cdot |I| \cdot |T|$ such variables. The requirement of single shipments for customers accepting delayed demand satisfaction results in a considerable increase in the number of binary variables, namely in $\bar{p} \cdot |J^1| \cdot |T|$ additional variables. The number of constraints also increases by the same factor. The alternative formulation (Q_a) benefits from the same variable reduction as (P_a). As it will be shown in Section 4, these differences will have a significant impact on the computing times.

Problem	Formulation	Number of constraints
MFLPDDSM	(P)	$O(T \cdot (J + \bar{k} \cdot I^n + I^e) + I)$
	(P _a)	$O(T \cdot (I + J + \bar{k} \cdot I^n + I^e) + I)$
MFLPDDSM-S	(Q)	$O(T \cdot (J + \bar{k} \cdot I^n + I^e + \bar{p} \cdot J^1) + I)$
	(Q _a)	$O(T \cdot (I + J + \bar{k} \cdot I^n + I^e + \bar{p} \cdot J^1) + I)$

Table 2: Number of constraints in the proposed formulations

Let $(\bar{P})$, resp. $(\bar{P}_a)$, denote the linear relaxation of formulation (P), resp. (P_a). The following result states that formulations $(\bar{P})$ and $(\bar{P}_a)$ are equally strong in terms of the lower bounds they provide. We denote by $v(\cdot)$ the optimal objective function value of a model.

Theorem. $v(\bar{P}) = v(\bar{P}_a)$

Proof. See Appendix A. □

In spite of the above result, our computational results (see Section 4) indicate that formulation (P_a) outperforms formulation (P).

3 Additional inequalities

In this section, we develop two groups of valid inequalities in an attempt to strengthen the bounds of the linear relaxations of the mathematical models introduced in the previous section. Moreover, adding the new conditions may also increase the chances of obtaining good feasible solutions in the course of solving the problem with general-purpose optimization software.

Both groups of inequalities strengthen the setting of the strategic decisions (i.e. the binary variables) by imposing a lower bound for the total number of facilities that must operate at given time periods. The first group of inequalities is associated with the location periods $\ell \in T_L$, whereas the second group concerns those time periods immediately after $\ell \in T_L$.

3.1 Inequalities involving strategic time periods

Denoting by R^ℓ the minimum number of facilities that must operate in time period $\ell \in T_L$, the following inequalities can be added to the MILP formulations:

$$|I^e| + \sum_{i \in I^n} \sum_{k \in K_i} z_{ik}^1 \geq R^1 \quad (31)$$

$$\sum_{i \in I^e} \left(1 - \sum_{\ell' \in T_L: \ell' < \ell} z_{i1}^{\ell'} \right) + \sum_{i \in I^n} \sum_{k \in K_i} \sum_{\ell' \in T_L: \ell' \leq \ell} z_{ik}^{\ell'} \geq R^\ell \quad \ell \in T_L \setminus \{1\} \quad (32)$$

Condition (31) is established for the first period in the time horizon by assuming, without loss of generality, that the latter corresponds to the first opportunity to make location and capacity acquisition decisions. Inequalities (32) apply to the remaining strategic periods. In both cases, the left-hand side represents the total number of facilities operating in time period ℓ . Since the decision to remove any existing facility takes place at the end of a strategic period, all existing locations are available in period $\ell = 1$.

In order to determine an appropriate value for the lower bound R^ℓ , we need to consider the minimum quantity D^ℓ that must be delivered in time period ℓ . This corresponds to the total demand requirements of the preferred customer segment:

$$D^\ell = \sum_{j \in J^0} d_j^\ell \quad \ell \in T_L$$

At the beginning of the time horizon, the total capacity available in the network is equal to $\sum_{i \in I^e} Q_{i1}$. Hence, if $D^1 \leq \sum_{i \in I^e} Q_{i1}$ then $R^1 = |I^e|$, meaning that the minimum demand requirements can be met by the capacity provided by the existing facilities. However, in the

case that $D^1 > \sum_{i \in I^e} Q_{i1}$, it is necessary to expand the network through opening new facilities. For this purpose, we consider the largest capacity level that can be installed in each candidate location $i \in I^n$ and build a sequence with these capacity sizes $Q_{i|K_i|}$ sorted by non-increasing order. Let us denote this sequence by $\tilde{Q}_{[1]} \geq \tilde{Q}_{[2]} \geq \dots \geq Q_{[|I^n|]}$. After having identified the number m of required capacity levels such that the following inequalities hold

$$\sum_{i=1}^{m-1} \tilde{Q}_{[i]} < D^1 - \sum_{i \in I^e} Q_{i1} \leq \sum_{i=1}^m \tilde{Q}_{[i]}$$

we set $R^1 = |I^e| + m$. By restricting the selection of capacity levels to the largest sizes that could be installed, we ensure that R^1 is indeed a lower bound for the actual number of facilities that must operate in the first period.

In all strategic periods ℓ other than the first, a similar sequence of capacity levels needs to be created in order to determine an appropriate value for R^ℓ . The only difference lies in the fact that the capacities of all existing facilities must also be considered in addition to the largest capacity levels that can be installed at candidate sites. With the sorted sequence of capacity sizes thus obtained (i.e. $\tilde{Q}_{[1]} \geq \tilde{Q}_{[2]} \geq \dots \geq Q_{[|I^n|+|I^e|]}$), we identify the minimum number of facilities that must be available in time period ℓ by employing similar conditions as above. In other words, for every $\ell \in T_L \setminus \{1\}$ we take $R^\ell = m$ such that

$$\sum_{i=1}^{m-1} \tilde{Q}_{[i]} < D^\ell \leq \sum_{i=1}^m \tilde{Q}_{[i]}$$

3.2 Inequalities involving selected tactical time periods

We now derive a second group of valid inequalities that serve a similar purpose to (31)–(32) but apply to those time periods immediately after the strategic decisions are made. Observe that the status of each facility remains unchanged over all intermediate periods between two consecutive strategic periods ℓ and $\ell' \in T_L$. Therefore, at the end of period ℓ , all strategic decisions made until then define the subset of facilities that are open over this time interval along with their capacities. These facilities will have to serve at least part of the demand occurring over the intermediate periods, namely all orders placed by customers $j \in J^0$ as well as those orders placed by customers $j \in J^1$ that must be satisfied before the next strategic period, even if some of these customers may experience delivery delays.

To illustrate this case, suppose that the time horizon spans 3 years with each year being divided into 12 months. Moreover, location and capacity acquisition decisions can be taken

at the beginning of each year. Therefore, $|T| = 36$, $T_L = \{1, 13, 25\}$ and $\ell_{max} = 25$. Let us assume that the maximum delivery lead time ρ_j is the same for all customers $j \in J^1$ and it is equal to two periods. For example, over the time interval covering the tactical periods $t = 2, \dots, 12$, all demands of the preferred customer segment must be filled. In addition, the orders placed by customers $j \in J^1$ in periods $t = 2, \dots, 10$ must also be served. Demand requirements of these customers for periods $t = 11$ and $t = 12$ could be satisfied in periods beyond this time interval through using capacity that would only become available in the next strategic period $\ell = 13$. Therefore, these demands do not have to be considered. This line of reasoning also applies to the intermediate periods $t = 14, \dots, 24$ but not to the last tactical time interval. In the latter case, all demand requirements of the customers that accept delayed deliveries must also be met.

Let us assume without loss of generality that $|T|$ is a multiple of $|T_L|$ and let us denote the number of time periods between two consecutive strategic periods by $\tau = |T|/|T_L|$. In the above example, $\tau = 12$. The minimum quantity of demand that must be served between two strategic time periods is determined as follows:

$$\overline{D}^{\ell+1} = \begin{cases} \sum_{j \in J^0} \sum_{t=\ell+1}^{\ell+\tau-1} d_j^t + \sum_{j \in J^1} \sum_{t=\ell+1}^{\ell+\tau-\rho_j-1} d_j^t & \text{if } \ell \in T_L \text{ and } \ell < \ell_{max} \\ \sum_{j \in J} \sum_{t=\ell+1}^{|T|} d_j^t & \text{if } \ell = \ell_{max} \end{cases}$$

We also need to sort the capacity levels of all facilities $i \in I$ according to the same procedure presented in Section 3.1 and use the ordered sequence to identify the minimum number of facilities that must operate in time period $\ell + 1$ ($\ell \in T_L$) as follows:

$$\sum_{i=1}^{m-1} (\tau - 1) \tilde{Q}_{[i]} < \overline{D}^{\ell+1} \leq \sum_{i=1}^m (\tau - 1) \tilde{Q}_{[i]}$$

Clearly, $R^{\ell+1} = m$ for every $\ell \in T_L$. Finally, the following inequalities are valid for the mathematical formulations introduced in Section 2:

$$\sum_{i \in I^e} \left(1 - \sum_{\ell' \in T_L: \ell' \leq \ell} z_{i1}^{\ell'} \right) + \sum_{i \in I^n} \sum_{k \in K_i} \sum_{\ell' \in T_L: \ell' \leq \ell} z_{ik}^{\ell'} \geq R^{\ell+1} \quad \ell \in T_L \quad (33)$$

Inequalities (31)–(33) are computationally inexpensive since in total only $2|T_L|$ constraints need to be added to each formulation. The resulting MILP models are denoted by $(\mathbf{P}^+)$, $(\mathbf{P}_a^+)$, $(\mathbf{Q}^+)$, and $(\mathbf{Q}_a^+)$.

4 Computational study

In this section, we describe the data generation scheme developed for our computational experiments followed by a discussion of the numerical results.

4.1 Description of test instances

Since the problems studied in this paper are new, benchmark instances are not available and consequently we randomly generated a set of test instances. The size of each instance is mainly dictated by the length of the planning horizon and the total number of customers as shown in Table 3.

Parameter	Value
$ J $	100, 150
$ J^0 $	$\lceil \beta^J J \rceil$ with $\beta^J \in \{0.25, 0.5, 0.75\}$
$ I $	$0.1 J $
$ I^n $	$0.8 I $
$ K_i $	3 ($i \in I^n$)
$ T $	36
$ T_L $	3, 6

Table 3: Cardinality of index sets

Three capacity levels are considered for each candidate location representing small, medium and large sizes. The planning horizon spans 3 years, each comprising 12-month periods, which yields in total 36 time periods. Location decisions may be taken either once a year ($|T_L| = 3$) or once every six months ($|T_L| = 6$). In the first case, the location periods are $T_L = \{1, 13, 25\}$ and in the second case $T_L = \{1, 7, 13, 19, 25, 31\}$.

In what follows, we denote by $U[a, b]$ the generation of random numbers over the range $[a, b]$ according to a continuous uniform distribution. Table 4 describes how customer demands and the sizes of the capacity levels are obtained. We note that both downward and upward demand fluctuations are possible over the time horizon. In fact, changes are allowed up to $\pm 5\%$ between two consecutive time periods. In the customer segment J^1 , lateness in demand fulfilment ranges from zero (no delays) to three time periods. Regarding the capacity choices, the small ($k = 1$), resp. medium ($k = 2$), size corresponds to 49%, resp. 70%, of the largest size. Observe that the latter depends on the mean total demand per period. To determine the

Parameter	Value	
d_j^1	$U[20, 100]$	
d_j^t	$\beta_d^t d_j^{t-1}$	$t = 2, \dots, T ; \beta_d^t \in [0.95, 1.05]$
ρ_j	0, 1, 2, 3	$j \in J^1$
$Q_{i K_i }$	$\frac{1}{ I } U[2, 3] \frac{\sum_{j \in J} \sum_{t \in T} d_j^t}{ T }$	$i \in I^n$
Q_{ik}	$0.7 Q_{i,k+1}$	$i \in I^n; k \in K_i \setminus \{ K_i \}$

Table 4: Demand and capacity parameters

initial capacity of each existing facility, also three capacity levels are generated according to the procedure in Table 4. One of these capacities is then selected at random.

The generation of the variable costs relies on two random numbers, β_1 and β_2 , both belonging to $U[1.01, 1.03]$ (details are given next).

- For $i \in I$ and $j \in J$, the variable distribution costs are set according to

$$\begin{aligned}
c_{ij}^1 &= U[5, 10] \\
c_{ij}^t &= c_{ij}^1 & t = 2, \dots, 12 \\
c_{ij}^t &= \beta_1 c_{ij}^{12} & t = 13, \dots, 24 \\
c_{ij}^t &= \beta_2 c_{ij}^{24} & t = 25, \dots, 36
\end{aligned}$$

It is assumed that the distribution costs are constant over one year (12 periods) but they increase between 1% and 3% from one year to the next.

- The variable processing costs at the facilities are generated in order to reflect economies of scale by considering the available capacity levels. Hence, the larger the capacity size, the lower the corresponding processing cost per unit of product. For $i \in I$ we set

$$\begin{aligned}
o_{i1}^1 &= 100 / \sqrt{Q_{i1}} & t = 1, \dots, 12 \\
o_{ik}^t &= 0.9 o_{ik-1}^t & t = 1, \dots, 12; k \in K_i \setminus \{1\} \\
o_{ik}^t &= \beta_1 o_{ik}^{12} & t = 13, \dots, 24; k \in K_i \\
o_{ik}^t &= \beta_2 o_{ik}^{24} & t = 25, \dots, 36; k \in K_i
\end{aligned}$$

Cost fluctuations follow a pattern similar to that of the variable distribution costs.

To obtain the fixed facility costs given in (1), we describe next how the various components are generated.

- The fixed costs of opening new facilities at candidate sites $i \in I^n$ reflect economies of scale by taking into account the three capacity levels. In the first time period, these costs are set according to

$$\begin{aligned} FO_{i1}^1 &= \alpha_i + \gamma_i \sqrt{Q_{i1}} & \alpha_i &\in [0, 1000]; \\ & & \gamma_i &\in [6000, 6500] \\ FO_{ik}^1 &= 1.01 (FO_{ik-1}^1 + o_{ik-1}^1 Q_{ik-1}) - o_{ik}^1 Q_{ik-1} & k &\in K_i \setminus \{1\} \end{aligned}$$

In the remaining strategic periods $\ell \in T_L \setminus \{1\}$, we take $FO_{ik}^\ell = \beta_{|T_L|} FO_{ik}^{\ell-1}$ for every $k \in K_i$. For $|T_L| = 3$, we set $\beta_{|T_L|} = U[1.01, 1.03]$ and for $|T_L| = 6$, we consider $\beta_{|T_L|} = U[\sqrt{1.01}, \sqrt{1.03}]$.

- A similar scheme is used to generate the fixed closing costs. Recall that the capacity of an existing facility $i \in I^e$ is randomly chosen among three sizes (small, medium and large, see Table 4). For these capacity levels $k \in \{1, 2, 3\}$, auxiliary fixed costs FO_{ik}^ℓ are determined for the existing facility i through applying the above procedure. The fixed closing cost of that facility corresponds to 20% of the associated auxiliary cost, that is, $FC_{i1}^\ell = 0.2 FO_{is}^\ell$ ($\ell \in T_L$) with s denoting the randomly selected capacity size ($s \in \{1, 2, 3\}$).
- Facility maintenance costs correspond to 5% of the associated opening/auxiliary costs. On the one hand, if a new facility is established at candidate site $i \in I^n$ with size $k \in K_i$ in time period $\ell \in T_L$ then the maintenance cost $MC_{ik}^t = 0.05 FO_{ik}^\ell$ is incurred in every period $t = \ell, \dots, |T|$. On the other hand, closing an existing facility $i \in I^e$ in period $\ell \in T_L$ incurs maintenance costs $MC_{i1}^t = 0.05 FO_{is}^\ell$ in all periods $t = 1, \dots, \ell$ (recall that s denotes the capacity size that was randomly selected for facility i , $s \in \{1, 2, 3\}$).

Finally, tardiness penalty costs for orders delivered with delay to customers $j \in J^1$ result from combining the average maintenance, distribution and processing costs in the following way:

$$p_j^{tt'} = 0.1 \theta_j^t (t' - t)^2 \quad t \in T; t' = t, \dots, \min\{t + \rho_j, |T|\}$$

with

$$\theta_j^t = \frac{\sum_{i \in I} \sum_{k \in K_i} M_{ik}^t}{TD_t |I| \sum_{i \in I} K_i} + \frac{\sum_{i \in I} c_{ij}^t}{|I|} + \frac{\sum_{i \in I} \sum_{k \in K_i} o_{ik}^t}{|I| \sum_{i \in I} K_i}$$

and TD_t denoting the total quantity demanded in period t , that is, $TD_t = \sum_{j \in J} d_j^t$. The above scheme was motivated by a procedure used by Albareda-Sambola et al. [2] in the context of a facility location problem with lost sales.

Preliminary tests showed that the procedure described above leads to meaningful instances to the problems that we investigate.

4.2 Numerical results

For each choice of the total number of customers ($|J|$) according to Table 3, six test instances were randomly generated, thus yielding a total of 36 instances. Each one of these instances was considered with the four values given in Table 4 for the maximum delivery lead time. The formulations including their enhancements were implemented in C++ using IBM ILOG Concert Technology and solved with IBM ILOG CPLEX 12.3. All experiments were conducted on a PC with a 3.4 GHz Intel Core i7-2600K processor, 8 GB RAM and running Windows 7 (64-bit). A limit of 10 hs of CPU time was set for each instance. CPLEX was used with default settings, as it is typically the case in practice, making full use of its MIP heuristics in an attempt to find high quality solutions at an early stage of the branch-and-cut algorithm. Finally, a deterministic parallel mode was selected to ensure that multiple runs with the same instance reproduce the same solution path and results.

4.2.1 Formulations $(\mathbf{P})$, $(\mathbf{P}_a)$ and $(\mathbf{P}_a^+)$

Table 5 summarizes the sizes of the test instances with formulations $(\mathbf{P})$ and $(\mathbf{P}_a)$ for different choices of the maximum delivery lead time. The latter is denoted by ρ in the first column. Each value of ρ indicates that the demands of all customers in segment J^1 must be satisfied within the same number of time periods ($\rho_j = \rho$, $j \in J^1$).

The number of binary variables z is identical in both formulations. As discussed in Section 2.4, instances solved with formulation $(\mathbf{P}_a)$ contain significantly less continuous variables than formulation $(\mathbf{P})$. In fact, the number of continuous variables in $(\mathbf{P}_a)$ is on average only 39% of the corresponding number in model $(\mathbf{P})$. However, instances run with formulation $(\mathbf{P}_a)$ have on average 7.9% more constraints. The introduction of the additional inequalities described in Section 3 has little impact on the model sizes and therefore, the corresponding information is omitted from the table. Instances with $|T_L| = 3$, resp. $|T_L| = 6$, strategic periods have 6, resp. 12, more constraints.

ρ		# binary variables			# continuous variables			# constraints		
		min	avg	max	min	avg	max	min	avg	max
0	(P)	78	146.3	234	93600	152100	210600	4546	5682.5	6819
	(P _a)	78	146.3	234	36936	59670	82404	4906	6132.5	7359
1	(P)	78	146.3	234	116350	225810	363480	4546	5682.5	6819
	(P _a)	78	146.3	234	45686	88020	141204	4906	6132.5	7359
2	(P)	78	146.3	234	138450	297414	511992	4546	5682.5	6819
	(P _a)	78	146.3	234	54186	115560	198324	4906	6132.5	7359
3	(P)	78	146.3	234	159900	366912	656136	4546	5682.5	6819
	(P _a)	78	146.3	234	62436	142290	253764	4906	6132.5	7359

Table 5: Sizes of test instances with formulations (P) and (P_a) for the MFLPDDSM under different delivery lead time limits

To evaluate the effectiveness of the proposed formulations (P), (P_a) and (P_a⁺), we compare them by means of their LP-relaxation bounds, the time necessary to solve the LP-relaxations, the optimality gaps reported by CPLEX, and the computation times required to solve the instances. Table 6 summarizes the results obtained for different values of the maximum delivery lead time (ρ). For each one of the three formulations, the table reports the number of instances that were solved to optimality (# opt) as well as the number of instances where the optimal solution was not identified within the given time limit of 10 hs (# non-opt). For the latter instances, we also present the minimum, average and maximum optimality gaps as reported by CPLEX for each type of formulation (MIP gap = $(z^{UB} - z^{LB})/z^{UB} \times 100\%$ with z^{UB} denoting the optimal objective value or the value of the best feasible solution and z^{LB} representing the best lower bound). In addition, the relative percentage deviation between the objective value of the best feasible solution and the LP-relaxation bound (z^{LP}) is presented in columns 9 and 10 (LP gap = $(z^{UB} - z^{LP})/z^{UB} \times 100\%$). Since the LP-relaxations of models (P) and (P_a) provide the same lower bound, the latter is shown in column 9. The minimum, average and maximum computing times (in seconds) to solve the instances for each one of the formulations are displayed in columns 11-13 (MIP CPU). Finally, the minimum, average and maximum computing times (also in seconds) required by the LP-relaxations are given in the last three columns (LP CPU). The last row of Table 6 reports the average values over all instances (in total 144). For each choice of the parameter ρ , the best average result is highlighted by boldface. Detailed results are given in Tables 11–14 in Appendix B for each value of the parameter ρ .

A closer look at Table 6 indicates that the proposed alternative formulations (P_a) and (P_a⁺) perform consistently better than formulation (P). Not only more instances were solved

ρ	# opt sol./# non-opt sol.				MIP gap (%)*			LP gap (%)		MIP CPU (sec)			LP CPU (sec)		
	(P)	(P _a)	(P _a ⁺)		(P)	(P _a)	(P _a ⁺)	(P), (P _a)	(P _a ⁺)	(P)	(P _a)	(P _a ⁺)	(P)	(P _a)	(P _a ⁺)
0	33/3	34 /2	34 /2	min	0.38	0.43	0.59	1.98	1.32	55.43	21.40	9.28	0.67	0.33	0.53
				avg	0.84	0.44	1.05	3.82	2.94	7021.07	4957.04	3612.76	1.84	0.80	1.52
				max	1.13	0.44	1.50	10.99	6.12	36000.00	36000.00	36000.00	3.93	1.36	3.43
1	23/13	30 /6	30 /6	min	0.26	0.27	0.76	0.65	0.65	89.89	23.09	11.90	3.57	1.19	0.98
				avg	1.21	1.04	1.22	2.93	2.85	15743.01	11014.94	10851.84	12.64	3.18	3.44
				max	2.77	1.73	1.74	5.65	5.65	36000.00	36000.00	36000.00	28.59	6.01	6.19
2	24/12	29 /7	29 /7	min	1.06	0.39	0.58	0.65	0.65	56.75	32.20	19.91	5.23	1.58	1.62
				avg	1.61	1.43	1.81	2.85	2.78	15796.96	12309.21	12624.36	18.16	4.26	4.48
				max	3.09	2.69	4.82	5.65	5.65	36000.00	36000.00	36000.00	43.82	8.99	10.11
3	22/14	27 /9	27 /9	min	1.18	0.57	0.60	0.65	0.65	96.60	39.52	35.65	6.55	1.86	2.00
				avg	1.77	1.58	1.46	2.84	2.83	16762.89	13555.61	13569.86	23.06	5.17	5.40
				max	3.49	4.61	2.89	5.65	5.65	36000.00	36000.00	36000.00	67.06	12.09	12.42
All	102/42	120 /24	120 /24	avg	1.36	1.12	1.39	3.11	2.85	13830.98	10459.20	10614.52	13.92	3.35	3.71

Table 6: Summary of results for the MFLPDDSM under different delivery lead time limits; *instances not solved to optimality within 10 hs

to optimality but also substantially shorter computing times were obtained with models $(\mathbf{P}_a)$ and $(\mathbf{P}_a^+)$. Furthermore, the optimality gaps decreased for those instances where the pre-specified time limit was attained. Hence, although formulations $(\mathbf{P})$ and $(\mathbf{P}_a)$ have the same linear relaxation values, $(\mathbf{P}_a)$ clearly outperforms $(\mathbf{P})$. The lower bound provided by the LP-relaxation of formulation $(\mathbf{P}_a^+)$ is slightly better, as expected. This is achieved at the cost of marginally higher computing times compared to formulation $(\mathbf{P}_a)$.

The results further reveal that increasing the maximum delivery time for customers accepting delayed shipments yields more difficult instances. This feature is reflected in the growing number of instances that could not be solved to optimality within the given time limit and also in the increasing average integrality gaps. Interestingly, the LP-bounds seem to be insensitive to the parameter ρ . This aspect is very important, especially regarding formulation $(\mathbf{P}_a^+)$. The tight linear relaxation bounds provided by this model (on average, 2.85%) indicate that the LP-relaxation of $(\mathbf{P}_a^+)$ could be used to evaluate the quality of feasible solutions obtained, for example, by means of a tailored heuristic method. In addition, tight LP-bounds can also be very helpful to accelerate the solution process when a branch-and-bound or branch-and-cut method has been especially designed to solve the problem to optimality. The differences in the computing times of formulations $(\mathbf{P}_a)$ and $(\mathbf{P}_a^+)$ are not significant for $\rho \geq 1$. The largest variation occurs in those instances where all customers must have their demands satisfied on time ($\rho = 0$). In these cases, the overall computational effort is considerably smaller with model $(\mathbf{P}_a^+)$ than with model $(\mathbf{P}_a)$.

4.2.2 Numerical results for the single shipment case

Since the proposed alternative models $(\mathbf{P}_a)$ and $(\mathbf{P}_a^+)$ have shown to be clearly superior in terms of overall computational performance, we decided to study the MFLPDDSM-S by evaluating the effectiveness of formulation $(\mathbf{Q}_a^+)$ (recall that for the MFLPDDSM the best LP-relaxation bound is obtained with the additional constraints introduced in Section 3). As shown in Table 7, the set of binary variables increases by a factor ranging from 31.3 ($\rho = 1$) to 59.8 ($\rho = 3$) compared to the problem variant where late deliveries can be split over several time periods. Formulations $(\mathbf{P}_a)$, $(\mathbf{P}_a^+)$, $(\mathbf{Q}_a)$, and $(\mathbf{Q}_a^+)$ have the same number of continuous variables (recall Table 1). In contrast, formulation $(\mathbf{Q}_a^+)$ has roughly twice as many constraints as model $(\mathbf{P}_a^+)$. Hence, the problem instances result in large-scale model representations.

Table 8 summarizes the results obtained under the requirement of single shipments for customers accepting late deliveries and for maximum lead times ranging from one to three

ρ	# binary var.			# continuous var.			# constraints		
	min	avg	max	min	avg	max	min	avg	max
1	1853	4571.9	8186	45686	88020	141204	6687	10567.2	15323
2	2703	6691.3	11994	54186	115560	198324	7537	12686.5	19131
3	3528	8748.3	15690	62436	142290	253764	8362	14743.5	22827

Table 7: Sizes of instances with formulation ($\mathbf{Q}_a^+$) for the MFLPDDSM-S under different delivery lead time limits

periods. The structure of the table is similar to that of Table 6 but it only includes information about a single formulation, namely ($\mathbf{Q}_a^+$). Since CPLEX has run out of memory in some of the instances, we also display in column 2 the number of instances where this error has occurred. Given the large size of the MILPs, it is not surprising that CPLEX’s ability to prove optimality within the given time limit greatly declines. This occurs with 74.1% of the instances and thus, the computing times increase significantly compared to the MFLPDDSM. Nevertheless, despite the large number of variables and constraints, the integrality gaps are rather small, ranging from 0.07% to 4.99%. Furthermore, the delivery lead time limit does not seem to impact the MIP gap although the size of the instances increases considerably as the value of ρ grows. The linear relaxation also provides very good lower bounds, even if the LP gaps are slightly larger compared to those given in Table 6. This results from the fact that proven optimal solutions are only available for 15.7% of the instances. The major difference lies in the increasing computing

ρ	# opt sol./ # non-opt sol./ # out of memory		Gap (%)		CPU (sec)	
			MIP*	LP	MIP	LP
1	12/16/8	min	0.50	1.69	990.43	1.34
		avg	2.09	3.60	26112.41	4.02
		max	4.90	8.41	36000.00	7.77
2	3/32/1	min	0.07	0.99	5144.35	2.40
		avg	2.13	3.60	34268.23	7.11
		max	4.99	6.68	36000.00	16.86
3	2/32/2	min	0.08	0.99	3168.02	2.45
		avg	2.05	3.38	35005.87	8.68
		max	4.75	6.54	36000.00	19.05
All	17/80/11	avg	2.09	3.52	32172.53	6.77

Table 8: Summary of results with formulation ($\mathbf{Q}_a^+$) for the MFLPDDSM-S under different delivery lead time limits; *instances not solved to optimality within 10 hs

times of the LP-relaxation but for such large MILPs this does not pose a limitation in practice, since all LP-relaxations required less than 20 seconds to be solved to optimality. One important drawback is that no feasible solution could be identified in 10.2% of the instances (11 out of 108) due to insufficient memory of the computer used. In this case, the importance of obtaining good LP-bounds becomes even greater in order to be able to evaluate, for example, the quality of upper bounds produced by a heuristic method.

4.2.3 Managerial insights

To further gain insight into the characteristics of the optimal or near-optimal solutions identified by CPLEX, Table 9 reports the contribution of various cost components to the overall cost for each problem type and different maximum delivery time limits. The results shown in the table refer to averages determined with respect to the best feasible solution available for each instance. Fixed facility cost rates are given separately for new and existing facilities according to (1). In addition, the average processing (o_{ik}^t), distribution (c_{ij}^t) and tardiness penalty ($p_j^{tt'}$) cost rates are also displayed. The last column of Table 9 presents the average percentage of demand that is not satisfied on time.

Problem	ρ	% of total cost							% delayed demand
		New facilities		Existing facilities		Processing cost	Distribution cost	Penalty cost	
		Opening cost	Maint. cost	Closing cost	Maint. cost				
MFLPDDS	0	14.3	25.5	0.7	11.3	16.4	31.7	-	-
	1	13.7	23.8	0.6	11.7	16.9	33.0	0.2	6.1
	2	13.8	23.9	0.6	11.3	16.9	33.0	0.4	8.8
	3	13.8	23.9	0.6	11.4	16.9	33.0	0.4	8.6
MFLPDDS-S	1	13.7	23.8	0.6	11.9	16.8	32.9	0.1	3.5
	2	14.0	24.6	0.7	11.1	16.7	32.8	0.1	3.1
	3	14.0	24.4	0.7	11.0	16.7	33.0	0.2	3.3

Table 9: Average cost rates under different delivery lead time limits

For the test instances of the MFLPDDS, the effect of enforcing timely demand fulfilment can be compared to the opposite situation in which some customers accept late shipments. The former case ($\rho = 0$) calls for a larger investment on establishing new facilities, choosing appropriate capacity levels for the new locations and on closing existing facilities (see also columns 5 and 6 in Table 10). This investment arises from increasing capacity needs in order to be able to meet all customer demands on time. In contrast, when a subset of the customers may experience delayed demand satisfaction, less capacity is installed which results in lower

investment spending on facilities. Nevertheless, as shown in columns 4–6 of Table 9, this reduction is not substantial due to the negative impact of late deliveries. This can be seen by the low contribution of the tardiness penalty cost to the total cost (0.2%–0.4%) and by the small percentage of delayed demand. Naturally, when the delivery time limit increases, also more demand is satisfied with some delay. The typical trade-off between the fixed facility costs and the variable processing and distribution costs is also illustrated in Table 9. A higher investment volume is partly abated by lower expenditures through selecting facilities that are less expensive and/or closer to the demand markets.

Regarding the MFLPDDS-S, since late deliveries cannot be split over several time periods, delays in demand satisfaction occur at a lower degree. However, the required adjustments in the network configuration do not yield substantially different fixed and variable cost rates from those in the MFLPDDS. Observe that the single shipment case lies in between two extreme situations, namely one in which timely demand fulfilment is enforced and another where the satisfaction of customer demand may be split over at most four periods (in the case of $\rho = 3$).

From a managerial perspective, an additional important aspect to be investigated is the capacity utilization level at the facilities. This metric gives insight into the overall slack capacity and therefore, it is an important indicator of whether it is possible to process larger product amounts without incurring the expensive costs of establishing new facilities and installing additional capacity. Table 10 provides information on this metric for both types of problems. Furthermore, it also presents the mean number of new facilities that are opened and the mean number of existing facilities that are closed over each set of instances. For the MFLPDDS, the impact of increasing delivery lead time limits is reflected in a higher capacity usage. This results from operating less facilities as highlighted by the last two columns of Table 10. Hence, by allowing delays in demand satisfaction cost benefits can be achieved (cf. Table 9) through the acquisition of the required capacity. As expected, this feature is also present in the MFLPDDS-S but at a slightly lower level. In contrast, timely demand satisfaction ($\rho = 0$) yields more slack capacity in a larger network of facilities.

5 Conclusions

In this paper, we introduced an extension of the classical multi-period facility location problem by considering customer segments with distinct sensitivity to delivery lead times and by incorporating different time scales for strategic and tactical decisions into the time horizon. For

Problem	ρ	Capacity usage (%)		# facilities	
		new facilities	existing facilities	new	closed
MFLPDDS	0	90.6	86.4	4.0	1.4
	1	96.0	92.8	3.7	1.3
	2	95.9	94.8	3.7	1.3
	3	95.8	94.8	3.7	1.3
MFLPDDS-S	1	95.3	91.1	3.8	1.3
	2	95.2	90.8	3.8	1.4
	3	94.7	92.6	3.8	1.4

Table 10: Average capacity utilization rates and average number of new facilities and closed facilities

each candidate site for locating a new facility, it is assumed that a set of discrete capacity levels is available. A variant of the problem was also studied in which customers accepting delayed demand satisfaction require late shipments to occur at most once over the delivery lead time. We proposed two mathematical programming formulations for each problem and developed additional inequalities to strengthen their linear relaxations. A theoretical comparison between the models without additional inequalities showed that they are equally strong in terms of the lower bounds provided by their linear relaxations. However, using randomly generated test instances, our computational experiments with a state-of-the-art MILP solver demonstrated the superiority of one of the formulations over the other. Furthermore, for medium-sized test instances high quality solutions could be identified by the optimization solver in acceptable computing times. In our empirical study, additional insights were gained by analyzing several characteristics of the best solutions obtained. In particular, our analysis illustrated the far-reaching implications of the delivery lead time for customers accepting delayed shipments with respect to the overall cost and the capacity usage of the operating facilities.

Based on our numerical results, a future line of research would be to develop a heuristic procedure to find good quality solutions for large problem instances within reasonable computing times. This would be particularly important for the more difficult problem variant where single shipments are imposed. Another line of research could be directed towards the development of more comprehensive multi-period facility location models by including multiple commodities and by considering several facility layers. These aspects play an important role in the context of supply chain network design.

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Appendix A: Theoretical result

We show that the LP-relaxations of formulations $(\mathbf{P})$ and $(\mathbf{P}_a)$ provide the same lower bound.

Theorem. $v(\bar{\mathbf{P}}) = v(\bar{\mathbf{P}}_a)$

Proof. Let $(\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}})$ be a feasible solution to $(\bar{\mathbf{P}})$. Using relations (18)–(20), it is easy to construct a feasible solution to $(\bar{\mathbf{P}}_a)$. Moreover, both solutions have the same objective function value. Therefore, $v(\bar{\mathbf{P}}_a) \leq v(\bar{\mathbf{P}})$.

Let us now consider any feasible solution $(\bar{\mathbf{r}}, \bar{\mathbf{s}}, \bar{\mathbf{w}}, \bar{\mathbf{z}})$ to $(\bar{\mathbf{P}}_a)$. We will show that $v(\bar{\mathbf{P}}) \leq v(\bar{\mathbf{P}}_a)$. To this end, we describe next how this solution can be used to obtain a feasible solution to $(\bar{\mathbf{P}})$. In particular, the values of variables $\bar{x}_{ijk}^t$ ($j \in J^0$; $k \in K_i$, $t \in T$) and $\bar{y}_{ijkt'}^{tt'}$ ($j \in J^1$; $k \in K_i$; $t \in T$, $t' = t, \dots, \min\{t + \rho_j, |T|\}$) will be determined by solving a sequence of single commodity minimum cost flow problems, one for each facility $i \in I$ in a directed multi-layer network $N_i = (V_i, E_i)$ defined by the set of vertices V_i and the set of arcs E_i ($E_i \subset V_i \times V_i$). Figure 1 depicts the general structure of such a network for a given $i \in I$. In addition to a source α_i and a sink β_i , the vertex set also includes three types of transshipment vertices denoted by V_i^1, V_i^2 and V_i^3 , distributed across two layers. Hence, $V_i = \{\alpha_i, \beta_i\} \cup V_i^1 \cup V_i^2 \cup V_i^3$. For each arc $(u, v) \in E_i$, let $\underline{m}_{uv}$, resp. $\overline{m}_{uv}$, be the lower, resp. upper, capacity bound and let c'_{uv} be a non-negative cost per unit of flow.

The first layer in network N_i ($i \in I$) consists of arcs pointing from the source α_i to transshipment vertices in V_i^1 and V_i^2 . The latter are associated respectively with variables $\bar{\mathbf{r}}$ and $\bar{\mathbf{s}}$ as follows. For each variable $\bar{r}_{ij}^t$ taking a positive value, a vertex is considered which is denoted by the pair (j, t) in Figure 1 ($j \in J^0$; $t \in T$). If $\bar{r}_{ij}^t = 0$ for a given j and t then we set $\bar{x}_{ijk}^t = 0$ for every $k \in K_i$. Hence, there are at most $|J^0| \cdot |T|$ vertices in V_i^1 corresponding to variables $\bar{\mathbf{r}}$. Each arc $\alpha_i \rightarrow u$ ($u \in V_i^1$) has the following characteristics:

$$\underline{m}_{\alpha_i u} = \overline{m}_{\alpha_i u} = \bar{r}_{ij}^t \quad \text{and} \quad c'_{\alpha_i u} = c_{ij}^t.$$

Observe that the cost per unit of flow on the arc is the unit distribution cost from facility i to customer j in time period t .

Transshipment vertices associated with all positive variables $\bar{s}_{ij}^{tt'}$ make up the set V_i^2 . Such vertices are displayed in Figure 1 as triples (j, t, t') for $j \in J^1$, $t \in T$ and $t' =$

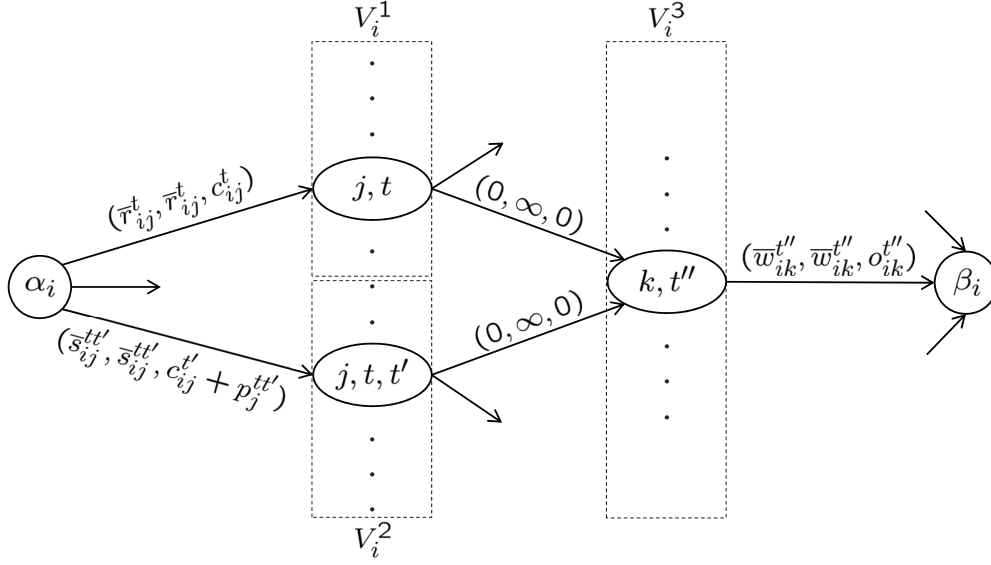


Figure 1: General structure of the network $N_i = (V_i, E_i)$ associated with $i \in I$

$t, \dots, \min\{t + \rho_j, |T|\}$. An arc from the source α_i to vertex $u \in V_i^2$ has the following capacity bounds and unit cost:

$$\underline{m}_{\alpha_i u} = \overline{m}_{\alpha_i u} = \overline{s}_{ij}^{tt'} \quad \text{and} \quad c'_{\alpha_i u} = c_{ij}^{t'} + p_j^{tt'}.$$

In this case, the cost of delivering one unit of product to customer j in time period t' from facility i accounts for the cost per unit of flow. This is determined by the distribution cost ($c_{ij}^{t'}$) and by the tardiness penalty cost ($p_j^{tt'}$). Again, if $\overline{s}_{ij}^{tt'} = 0$ for a given j, t and t' then we consider $\overline{g}_{ijk}^{tt'} = 0$ for every $k \in K_i$ and it is not necessary to include the corresponding vertex in V_i^2 .

Regarding the second layer of transshipment vertices V_i^3 , there are in total $|K_i| \cdot |T|$ such vertices that are depicted in Figure 1 by the pairs (k, t'') with $k \in K_i$ and $t'' \in T$. As we will show later, they are related to variables $\overline{w}$. From any vertex $(j, t) \in V_i^1$, we draw in total $|K_i|$ arcs, one to each vertex $(k, t'') \in V_i^3$ such that $k \in K_i$ and $t = t''$. These arcs are associated with timely deliveries to customers $j \in J^0$ in time period t from facility i with capacity size k . Furthermore, each vertex $(j, t, t') \in V_i^2$ is connected with $|K_i|$ vertices $(k, t'') \in V_i^3$ such that $k \in K_i$ and $t' = t''$. In this case, the arcs represent late-deliveries to customers $j \in J^1$ in time period t' from facility i with capacity level k . An arc $u \rightarrow v$ with $u \in V_i^1 \cup V_i^2$ and $v \in V_i^3$ has a lower bound capacity $\underline{m}_{uv} = 0$, an upper bound capacity $\overline{m}_{uv} = \infty$ and a cost per unit of flow $c'_{uv} = 0$ (see Figure 1).

Finally, all vertices $v \in V_i^3$ are linked to the sink β_i . As shown in Figure 1, each arc $v \rightarrow \beta_i$ with $v = (k, t'')$ has the following characteristics:

$$\underline{m}_{v\beta_i} = \overline{m}_{v\beta_i} = \overline{w}_{ik}^{t''} \quad \text{and} \quad c'_{v\beta_i} = o_{ik}^{t''}.$$

The cost per unit of flow on the arc $v \rightarrow \beta_i$ is the unit processing cost at facility i with capacity level k in time period t'' .

According to the structure of the network N_i , the total amount of flow generated by the source α_i for a given facility $i \in I$ is given by

$$b(\alpha_i) = \sum_{t \in T} \sum_{j \in J^0} \bar{r}_{ij}^t + \sum_{t \in T} \sum_{j \in J^1} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} \bar{s}_{ij}^{tt'}$$

whereas the sink β_i has the total inflow

$$b(\beta_i) = \sum_{t \in T} \sum_{k \in K_i} \bar{w}_{ik}^t$$

Since $(\bar{\mathbf{r}}, \bar{\mathbf{s}}, \bar{\mathbf{w}}, \bar{\mathbf{z}})$ is a feasible solution to $(\bar{\mathbf{P}}_a)$, constraints (22) hold. For each $i \in I$, summing up the left-hand side and the right-hand side of (22) over $t \in T$ yields $b(\alpha_i) = b(\beta_i)$. This means that the available supply equals the total demand and therefore, the single-commodity minimum cost flow problem is feasible for each $i \in I$. The optimal solutions to these $|I|$ problems contain the values of the variables $\bar{\mathbf{x}}$ and $\bar{\mathbf{y}}$ of model $(\bar{\mathbf{P}})$. For each network associated with $i \in I$, the flow passing through the arc $u \rightarrow v$ such that $u = (j, t) \in V_i^1$ and $v = (k, t) \in V_i^3$ sets the value of the variable $\bar{x}_{ijk}^t$. Moreover, the flow in the arc $u \rightarrow v$ with $u = (j, t, t') \in V_i^2$ and $v = (k, t') \in V_i^3$ sets the value of the variable $\bar{y}_{ijk}^{tt'}$.

Due to the structure of each network N_i ($i \in I$), it is now easy to establish a relationship between the variables $\bar{\mathbf{r}}$ and $\bar{\mathbf{x}}$. The flow into vertex $u = (j, t) \in V_i^1$ is equal to $\bar{r}_{ij}^t$, whereas the total outflow is determined by $\sum_{k \in K_i} \bar{x}_{ijk}^t$. In other words, relations (18) hold for every $i \in I$, $j \in J^0$ and $t \in T$. In a similar way, we can show that variables $\bar{\mathbf{s}}$ and $\bar{\mathbf{y}}$ are linked by considering the inflows and outflows of vertices V_i^2 . For $v = (j, t, t') \in V_i^2$, it follows that $\bar{s}_{ij}^{tt'} = \sum_{k \in K_i} \bar{y}_{ijk}^{tt'}$. Hence, relations (19) are valid for every $i \in I$, $j \in J^1$, $t \in T$, and $t' = t, \dots, \min\{t + \rho_j, |T|\}$. Finally, the relationship between variables $\bar{\mathbf{r}}$, $\bar{\mathbf{x}}$ and $\bar{\mathbf{w}}$ is determined by considering the inflows and outflows of vertices V_i^3 . Observe that each vertex $v = (k, t'') \in V_i^3$ receives flow from vertices $u = (j, t) \in V_i^1$ for $j \in J^0$ and $t = t''$, and from vertices $u = (j, t, t') \in V_i^2$ such that $j \in J^1$, $t = \max\{1, t'' - \rho_j\}$ and $t' = t''$. Hence, the total inflow to vertex $v \in V_i^3$ is determined by $\sum_{j \in J^0} \bar{x}_{ijk}^{t''} + \sum_{j \in J^1} \sum_{t=\max\{1, t''-\rho_j\}}^{t''} \bar{y}_{ijk}^{tt''}$. This equals the outflow, $\bar{w}_{ik}^{t''}$, and so relations (20) hold for every $i \in I$, $k \in K_i$ and $t'' \in T$.

Regarding the location variables $\bar{\mathbf{z}}$, it is clear that their values in the solution to $(\bar{\mathbf{P}}_a)$ coincide with the values of the corresponding variables in $(\bar{\mathbf{P}})$.

We have shown how to construct a feasible solution to $(\bar{\mathbf{P}})$ from any feasible solution to $(\bar{\mathbf{P}}_a)$. Moreover, for every network $N_i = (V_i, E_i)$, $i \in I$, the optimal flow has total cost

$$\sum_{t \in T} \sum_{j \in J^0} c_{ij}^t \bar{r}_{ij}^t + \sum_{t \in T} \sum_{j \in J^1} \sum_{t'=t}^{\min\{t+\rho_j, |T|\}} (c_{ij}^{t'} + p_j^{tt'}) \bar{s}_{ij}^{tt'} + \sum_{t \in T} \sum_{k \in K_i} o_{ik}^t \bar{w}_{ik}^t$$

Summing up the above cost over $i \in I$ and adding the fixed facility costs with respect to variables $\bar{z}$, we obtain the objective function value associated with the feasible solution $(\bar{\mathbf{r}}, \bar{\mathbf{s}}, \bar{\mathbf{w}}, \bar{\mathbf{z}})$ to $(\bar{\mathbf{P}}_a)$. Due to relations (18)–(20), it follows that this coincides with the objective function value of the corresponding feasible solution $(\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}})$ to $(\bar{\mathbf{P}})$. Therefore, $v(\bar{\mathbf{P}}) \leq v(\bar{\mathbf{P}}_a)$.

□

Appendix B: Detailed results

Tables 11–14 report additional results for the various choices of the maximum delivery lead time and for all the formulations that were tested. In particular, the total number of customers in each segment ($|J^0|, |J^1|$), the number of existing facilities ($|I^e|$) and the number of candidate locations for new facilities ($|I^n|$) are shown. For $\rho = 0$ (Table 11), each row represents 18 instances, whereas for $\rho > 0$ (Tables 12–14) the results of six instances are summarized in each row.

Formulation	$ J $	$ I^e $	$ I^n $	# opt sol./ # non-opt sol.	Gap (%)		CPU (sec)	
					MIP*	LP	MIP	LP
(P)	100	2	8	18/0		4.67	1061.59	1.10
	150	3	12	15/3	0.84	2.97	12980.54	2.58
All				33/3	0.84	3.82	7021.07	1.84
(P _a)	100	2	8	18/0		4.67	808.00	0.47
	150	3	12	16/2	0.44	2.97	9106.08	1.14
All				34/2	0.44	3.82	4957.04	0.80
(P _a ⁺)	100	2	8	18/0		3.16	939.19	0.98
	150	3	12	16/2	1.05	2.71	9884.91	2.06
All				34/2	1.05	2.94	5412.05	1.52

Table 11: Average results for timely demand satisfaction ($J^1 = \emptyset$, $J = J^0$); *instances not solved to optimality within 10 hs

References

- [1] N.A.H. Agatz, M. Fleischmann, and J.A.E.E. van Nunen. E-fulfillment and multi-channel distribution - A review. *European Journal of Operational Research*, 187:339–356, 2008.
- [2] M. Albareda-Sambola, A. Alonso-Ayuso, L.F. Escudero, E. Fernández, Y. Hinojosa, and

Formulation	$ J^0 $	$ J^1 $	$ I^e $	$ I^n $	# opt sol./ # non-opt sol./ # out of memory	Gap (%)		CPU (sec)	
						MIP*	LP	MIP	LP
(P)	25	75	2	8	5/1/0	0.26	3.89	11641.21	6.87
	50	50	2	8	5/1/0	1.42	3.76	8557.84	6.01
	75	25	2	8	6/0/0		3.24	1705.52	5.43
	38	112	3	12	1/5/0	1.60	2.21	30075.73	24.76
	75	75	3	12	3/3/0	0.49	2.22	20686.23	19.84
	113	37	3	12	3/3/0	1.05	2.27	21791.55	12.93
All					23/13/0	1.21	2.93	15743.01	12.64
(P _a)	25	75	2	8	6/0/0		3.89	4775.39	2.14
	50	50	2	8	6/0/0		3.76	5574.62	1.81
	75	25	2	8	6/0/0		3.24	927.03	1.42
	38	112	3	12	2/4/0	1.25	2.21	27258.47	5.65
	75	75	3	12	5/1/0	0.16	2.22	12374.46	4.53
	113	37	3	12	5/1/0	0.27	2.27	15179.65	3.55
All					30/6/0	1.04	2.93	11014.94	3.18
(P _a ⁺)	25	75	2	8	6/0/0		3.68	4879.97	1.91
	50	50	2	8	6/0/0		3.48	4316.44	1.89
	75	25	2	8	6/0/0		3.24	722.84	1.75
	38	112	3	12	2/4/0	1.36	2.21	27034.19	5.97
	75	75	3	12	5/1/0	0.19	2.22	12283.55	4.91
	113	37	3	12	5/1/0	0.78	2.25	15874.04	4.20
All					30/6/0	1.22	2.85	10851.84	3.44
(Q _a ⁺)	25	75	2	8	1/3/2	3.05	4.96	27247.61	2.81
	50	50	2	8	4/1/1	1.85	3.86	10775.90	2.17
	75	25	2	8	4/0/2		3.83	10943.94	1.74
	38	112	3	12	0/5/1	1.77	2.98	36000.00	7.28
	75	75	3	12	1/4/1	2.11	3.73	33201.34	5.86
	113	37	3	12	2/3/1	1.01	2.55	35699.02	3.57
All					12/16/8	2.09	3.60	26112.41	4.02

Table 12: Average results for $\rho_j = 1$ ($j \in J^1$); *instances not solved to optimality within 10 hs

C. Pizarro-Romero. A computational comparison of several formulations for the multi-period incremental service facility location problem. *TOP*, 18:62–80, 2010.

- [3] M. Albareda-Sambola, E. Fernandez, and Stefan Nickel. Multiperiod location-routing with decoupled time scales. *European Journal of Operational Research*, 217:248–258, 2012.
- [4] F. Altıparmak, M. Gen, L. Lin, and T. Paksoy. A genetic algorithm approach for multi-objective optimization of supply chain networks. *Computers & Industrial Engineering*, 51: 197–216, 2006.
- [5] S.A. Alumur, B.Y. Kara, and M.T. Melo. Location and logistics. In G. Laporte, S. Nickel, and F. Saldanha da Gama, editors, *Location Science*, chapter 16, pages 419–441. Springer, Heidelberg, 2015.

Formulation	$ J^0 $	$ J^1 $	$ I^e $	$ I^n $	# opt sol./ # non-opt sol./ # out of memory	Gap (%)		CPU (sec)	
						MIP*	LP	MIP	LP
(P)	25	75	2	8	5/1/0	1.42	3.89	13979.35	10.22
	50	50	2	8	5/1/0	1.67	3.62	9332.14	8.68
	75	25	2	8	6/0/0		3.05	1988.60	6.55
	38	112	3	12	2/4/0	2.15	2.35	27814.79	38.88
	75	75	3	12	3/3/0	0.59	2.14	22519.29	26.74
	113	37	3	12	3/3/0	1.36	2.03	19147.64	17.87
All					24/12/0	1.61	2.85	15796.96	18.16
(P _a)	25	75	2	8	6/0/0		3.89	7891.36	2.66
	50	50	2	8	6/0/0		3.62	4611.84	2.32
	75	25	2	8	6/0/0		3.05	789.88	1.73
	38	112	3	12	2/4/0	1.78	2.35	29229.80	8.50
	75	75	3	12	5/1/0	0.21	2.14	15588.77	6.10
	113	37	3	12	4/2/0	0.87	2.03	15743.65	4.27
All					29/7/0	1.43	2.85	12309.21	4.26
(P _a ⁺)	25	75	2	8	6/0/0		3.68	7667.03	2.61
	50	50	2	8	6/0/0		3.45	5805.24	2.32
	75	25	2	8	6/0/0		3.02	1165.21	2.00
	38	112	3	12	2/4/0	2.41	2.35	29621.76	8.81
	75	75	3	12	5/1/0	0.19	2.14	15018.36	6.46
	113	37	3	12	4/2/0	0.96	2.03	16468.54	4.70
All					29/7/0	1.81	2.78	12624.36	4.48
(Q _a ⁺)	25	75	2	8	1/5/0	2.55	4.46	34245.85	4.36
	50	50	2	8	2/4/0	2.47	3.84	27652.16	3.65
	75	25	2	8	0/5/1	1.90	4.54	36000.00	2.53
	38	112	3	12	0/6/0	2.76	3.58	36000.00	15.14
	75	75	3	12	0/6/0	1.95	2.98	36000.00	9.95
	113	37	3	12	0/6/0	1.29	2.34	36000.00	6.26
All					3/32/1	2.13	3.60	34268.23	7.11

Table 13: Average results for $\rho_j = 2$ ($j \in J^1$); *instances not solved to optimality within 10 hs

- [6] H. Badri, M. Bashiri, and T.H. Hejazi. Integrated strategic and tactical planning in a supply chain network design with a heuristic solution method. *Computers & Operations Research*, 40:1143–1154, 2013.
- [7] M. Bashiri, H. Badri, and J. Talebi. A new approach to tactical and strategic planning in production-distribution networks. *Applied Mathematical Modelling*, 36:1703–1717, 2012.
- [8] C. Canel and B.M. Khumawala. A mixed-integer programming approach for the international facilities location problem. *International Journal of Operations & Production Management*, 4:49–68, 1996.
- [9] M.L.F. Cheong, R. Bhatnagar, and S. Graves. Logistics network design with differenti-

Formulation	$ J^0 $	$ J^1 $	$ I^e $	$ I^n $	# opt sol./ # non-opt sol./ # out of memory	Gap (%)		CPU (sec)	
						MIP*	LP	MIP	LP
(P)	25	75	2	8	5/1/0	1.50	3.89	13750.74	13.66
	50	50	2	8	5/1/0	1.85	3.62	9721.33	10.71
	75	25	2	8	6/0/0		3.05	2035.32	7.94
	38	112	3	12	1/5/0	2.10	2.35	30117.90	52.31
	75	75	3	12	2/4/0	0.97	2.15	25282.64	33.46
	113	37	3	12	3/3/0	1.71	1.98	19669.38	20.28
All					22/14/0	1.77	2.84	16762.89	23.06
(P _a)	25	75	2	8	6/0/0		3.89	8805.16	3.33
	50	50	2	8	6/0/0		3.62	5781.67	2.75
	75	25	2	8	6/0/0		3.05	1299.46	1.97
	38	112	3	12	1/5/0	2.10	2.35	30027.29	10.45
	75	75	3	12	5/1/0	0.23	2.15	16940.32	7.52
	113	37	3	12	3/3/0	0.80	1.98	18479.75	5.01
All					27/9/0	1.58	2.84	13555.61	5.17
(P _a ⁺)	25	75	2	8	6/0/0		3.89	9078.29	3.21
	50	50	2	8	6/0/0		3.62	6432.30	2.79
	75	25	2	8	6/0/0		3.02	822.50	2.12
	38	112	3	12	1/5/0	2.10	2.35	30025.68	11.37
	75	75	3	12	5/1/0	0.23	2.15	16635.81	7.57
	113	37	3	12	3/3/0	0.80	1.98	18424.55	5.35
All					27/9/0	1.46	2.83	13569.86	5.40
(Q _a ⁺)	25	75	2	8	1/5/0	2.87	4.62	35838.60	5.42
	50	50	2	8	1/5/0	2.61	3.96	30528.00	4.53
	75	25	2	8	0/4/2	1.78	3.77	36000.00	2.72
	38	112	3	12	0/6/0	2.50	3.35	36000.00	17.34
	75	75	3	12	0/6/0	1.32	2.38	36000.00	12.73
	113	37	3	12	0/6/0	1.35	2.34	36000.00	7.36
All					2/32/2	2.05	3.38	35005.87	8.68

Table 14: Average results for $\rho_j = 3$ ($j \in J^1$); *instances not solved to optimality within 10 hs

ated delivery lead-time: Benefits and insights. In *Proceedings of 2005 SMA Conference, Singapore*, pages 1–20, 2005.

- [10] I. Correia, T. Melo, and F. Saldanha-da-Gama. Comparing classical performance measures for a multi-period, two-echelon supply chain network design problem with sizing decisions. *Computers & Industrial Engineering*, 64:366–380, 2013.
- [11] B. Fortz. Location problems in telecommunications. In G. Laporte, S. Nickel, and F. Saldanha da Gama, editors, *Location Science*, chapter 20, pages 537–554. Springer, Heidelberg, 2015.
- [12] E. Gebennini, R. Gamberini, and R. Manzini. An integrated production-distribution model

- for the dynamic location and allocation problem with safety stock optimization. *International Journal of Production Economics*, 122:286–304, 2009.
- [13] S.K. Jacobsen. Multiperiod capacitated location models. In P.B. Mirchandani and R.L. Francis, editors, *Discrete Location Theory*, chapter 4. Wiley, New York, 1990.
 - [14] K. Lieckens and N. Vandaele. Reverse logistics network design with stochastic lead times. *Computers & Operations Research*, 34:395–416, 2007.
 - [15] F. Meisel, W. Rei, M. Gendreau, and C. Bierwirth. The design of supply networks under maximum customer order lead times. Working paper CIRRELT-2011-57, 2011.
 - [16] M.T. Melo, S. Nickel, and F. Saldanha da Gama. Facility location and supply chain management – A review. *European Journal of Operational Research*, 196:401–412, 2009.
 - [17] Z.-J. M. Shen. A profit-maximizing supply chain network design model with demand choice flexibility. *Operations Research Letters*, 34:673–682, 2006.
 - [18] R. Sousa, N. Shah, and L.G. Papageorgiou. Supply chain design and multilevel planning – An industrial case. *Computers & Industrial Engineering*, 32:2643–2663, 2008.
 - [19] W. Wilhelm, D. Liang, B. Rao, D. Warriier, X. Zhu, and S. Bulusu. Design of international assembly systems and their supply chains under NAFTA. *Transportation Research Part E*, 41:467–493, 2005.

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