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A computational comparison of formulations for a multi-period facility location problem with modular capacity adjustments and flexible demand fulfillment

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Abstract

We consider a multi-period facility location problem that takes into account changing trends in customer demands and costs. To this end, new facilities can be established at pre-specified potential locations and initially existing facilities can be closed over a planning horizon. Furthermore, facilities operate with modular capacities that can be expanded or contracted over multiple periods. A distinctive feature of our problem is that two customer segments are considered with different sensitivity to delivery lead times. Customers in the first segment require timely demand satisfaction, whereas customers in the second segment tolerate late deliveries. A tardiness penalty cost is incurred to each unit of demand that is satisfied with delay. We propose two alternative mixed-integer linear formulations to redesign the facility network over the time horizon at minimum cost. Additional inequalities are developed to enhance the original formulations. A computational study is performed with randomly generated instances and using a general-purpose solver. Useful insights are derived from analyzing the impact of several parameters on network redesign decisions and on the overall cost, such as different demand patterns and varying values for the maximum delivery delay tolerated by individual customers.

Keywords: Facility location, multi-period, capacity expansion and contraction, delivery lateness, mixed-integer linear models

1 Introduction

In today's globally competitive market, firms are faced with an increasing need to improve their flexibility, reliability, and responsiveness to satisfy the demands of their customers. In

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order to meet these challenges, it is crucial for firms to be able to adjust the configuration of their facility networks to changing market conditions. Important enablers include opening new facilities in markets with high demand growth and closing facilities in regions with demand decline. In addition, capacity scalability, i.e. adding or removing capacity to/from facilities, is also a meaningful strategy to adequately respond to fluctuations in the level of market demand.

In this study, we address a facility network that needs to be redesigned in order to effectively serve predicted variations in demand over time. To this end, gradual changes in the network structure and in the capacities of the facilities are considered over a planning horizon which is assumed to be finite and divided into several periods. The objective is to determine the minimal cost schedule for facility opening, facility closure, capacity expansion, and capacity contraction, and to allocate customer demands to operating facilities over time. A distinctive feature of the problem that we study is that customers are differentiated according to their sensitivity to delivery lead times. Customers having zero lead times require their demands to be satisfied in the time period they occur. Customers tolerating late deliveries specify a positive maximum delivery lead time. Delivery after the preferred due date and not beyond the latest acceptable time period is permitted, but incurs a tardiness penalty cost that depends on the length of delay. Customer segmentation on the basis of preferred delivery lead times can be encountered in various industries. Wang et al. [36] describe the case of a semiconductor equipment manufacturer that provides a two-class service policy for repairable parts. Customers with emergency demand pay a premium price to have their returned defective parts promptly repaired. Non-emergency service is provided to all other customers who accept a longer repair time in exchange for a lower price. This type of policy is also termed "demand postponement" by Wu and Wu [39] because the firm decides upon the actual delivery time for orders committed to customers who are less sensitive to lead times.

Integrated planning for facility location and capacity sizing under flexible conditions for demand fulfillment gives firms a framework to handle dynamic situations when significant changes in demand (and costs) over time are anticipated. Our work makes an important contribution toward the development of mathematical models to support the underlying decision-making process. From an economic viewpoint, the temporary adjustment of the capacity of an existing facility, either through expansion or contraction, may be more advantageous than installing a new facility in some other location or even closing the existing facility. However, trade-offs must be made between investments on facility location, capacity scalability, distribution costs, and tardiness costs for delayed demand satisfaction. We note that our problem arises in the context of sizing decisions being reversible in the medium term. This is the case, for example, when space and equipment can be rented or leased.

The contribution of our study is threefold. First, we address a new multi-period facility location problem which extends a particular case recently examined by Correia and Melo [8]. Second, we develop two mixed-integer linear programming (MILP) formulations: one is a natural way of formulating the problem and the other is inspired by a modeling framework recently introduced by Jena et al. [19]. We also describe various enhancements to the base models to improve the bound provided by their linear relaxation. In recent years, general-purpose MILP solvers have become an effective and reliable tool for solving many (real-world) problems. However, the capability of a solver to produce good, potentially optimal, solutions within acceptable computing time greatly depends on the selection of the right model. Therefore, the third contribution of our work is to perform a comparative analysis of the proposed formulations by using a state-of-the-art MILP solver. For this purpose, a large set of instances was randomly generated exhibiting different demand patterns. Important managerial insights will be provided on how delivery lead time restrictions affect the configuration of an existing facility system, the overall cost of redesign decisions, and the capacity usage of operating facilities.

The remainder of this paper is organized as follows. Section 2 gives a brief review of related literature. In Section 3, the problem that we study is formally described and two alternative MILP formulations are proposed. Various classes of additional inequalities are introduced in Section 4 for both formulations. The results of an extensive computational study are discussed in Section 5. Finally, Section 6 presents concluding remarks and outlines opportunities for further research.

2 Related research

In the multi-period (or "dynamic") facility location problem (MFLP), the objective is to determine the spatial distribution of facilities at each time period of a finite planning horizon so as to minimize the total fixed and variable costs for meeting customer demands over time. This natural extension of the single period or static version of the discrete location problem is particularly suited to handle situations with predicted changes in the parameters of the problem. If a network is already in place with a number of facilities being operated at fixed locations then location decisions also comprise the phase-out of initially existing facilities. Jacobsen [18] and, more recently, Nickel and Saldanha da Gama [29], discuss basic modeling aspects and address several variants of the MFLP.

Multi-period facility location has been a field of recurring interest as demonstrated by the surveys by Arabani and Farahani [3], Klose and Drexl [23], and Owen and Daskin [30]. Moreover, a number of models have been developed for a wide range of applications in many areas, both in the private and public sectors, including telecommunication network design (Chardaire et al. [6], Gourdin and Klopfenstein [15]), production-distribution system design (Canel et al. [5], Hinojosa et al. [16]), supply chain network design (Cortinhal et al. [10], Thanh et al. [33]), and public facility planning (Antunes and Peeters [2], Delmelle et al. [11]).

When market demand growth is anticipated and the capacities of existing facilities will not be sufficient to handle future customer requirements, firms face decisions about where and how to expand their capacities. This form of capacity scalability was addressed by Lee and Luss [24] and, more recently, by Julka et al. [22] and Martínez-Costa et al. [26]. Although in earlier works the location of facilities was not included in the decision set, the importance of integrating capacity acquisition decisions with facility location decisions has been widely recognized. In this case, the choice of the amount of capacity to be installed at a particular facility is often made by selecting a capacity level from a finite set of options. As argued by Correia and Captivo [7], this is an assumption with practical relevance since capacity is often purchased in the form of equipment which is only available in a few discrete sizes. Moreover, fixed and operating facility costs are frequently subject to economies of scale that depend on the capacity choices.

The MFLP with modular capacity expansion has been addressed by various authors. Syam [32] analyzes facility location and sizing decisions for an international firm and considers three levels of capacity expansion. At each location, capacity can be increased over successive time periods within the planning horizon. In the context of the forestry industry, Troncoso and Garrido [34] developed a mathematical model to determine the optimal location of a sawmill. The initially installed capacity for timber production can be gradually increased over a multi-period horizon provided that the total number of added modules does not exceed a given limit. Gourdin and Klopfenstein [15] also examined the problem of progressively expanding an existing telecommunications network through installing modular equipment over time. Delmelle et al. [11] proposed a model for redesigning a network of educational facilities through opening new schools and closing existing schools. The latter decision can be made on the condition that the school has reached a certain age. The student capacity of a school can also be raised by installing additional mobile units, each having the same size, for which leasing costs are incurred. Location and capacity acquisition decisions are constrained by an available total school budget over the planning horizon. Recently, Correia et al. [9] described a MILP model for the design of a two-echelon network. New facilities are established in the upper and intermediate echelons of the network and their capacities are gradually extended through the installation of storage areas dedicated to families of products. In particular, the same type of storage area can be selected more than once for a given family over the time horizon. Cortinhal et al. [10] also studied a multi-stage supply chain network redesign problem with location and capacity decisions. Modular capacity expansions can occur at a particular location as long as the overall capacity does not exceed a pre-specified global size. The problem examined by Shulman [31] differs from the works discussed before in that multiple facilities of different types can be established at a given location. At each time period, at most one facility of each type can be selected at a particular site but several facilities can be opened if they are of different types. This scheme is employed to gradually adjust the operating capacity of the facility network.

In a multi-period setting it may also be meaningful to dispose of capacity during periods of declining demand (Martínez-Costa et al. [26]). Therefore, capacity scalability does not only focus on expanding capacity but also involves removing capacity from operating facilities. A few authors have addressed both forms of capacity adjustments. The recent review by Martínez-Costa et al. [26] is devoted to mathematical programming models that deal with alternative capacity sizing strategies (including the two described above) in a manufacturing environment. To meet demographic trends and enrollment variations, Antunes and Peeters [2] suggested a MILP model that allows opening new educational facilities and closing existing locations, as well as expanding and contracting capacity. Over the planning horizon, the capacity of a particular school can be expanded or reduced, but not both. In the modeling framework proposed by Melo et al. [27] this feature is also adopted in the context of facility relocation. Capacity changes range from continuous increments to modular adjustments. Vila et al. [35] describe a case study from the lumber industry where strategic decisions on the configuration and size of the facility network are implemented at the beginning of the planning horizon and tactical decisions (e.g., for procurement, production, and demand allocation) are taken over the time horizon. To handle demand and price fluctuations, previously installed capacity modules can be removed. In the models developed by Wilhelm et al. [37] for a multi-echelon logistics network redesign problem, the configuration of a facility can be changed more than once over the planning horizon through adding or removing capacity. Dias et al. [12] designed a primal-dual heuristic for a MFLP with modular capacity expansion and reduction. Expansion (reduction) is achieved through locating (removing) multiple facilities at a particular location. In addition to facility opening and closing decisions, any facility can be reopened over the time horizon. This feature contrasts with most of the contributions in the dynamic facility location field, where typically one-time opening or closing of facilities is allowed. Recently, an extension of these settings was addressed by Jena et al. [20]. Motivated by a practical situation arising in the forestry industry, the authors studied a problem where different mechanisms for capacity sizing can be used for locating logging camps. The size of a camp is determined by the number of trailers it holds. Capacity adjustments take the form of adding or removing trailers and temporarily closed capacity may be reopened. Furthermore, an entire camp can also be relocated from one site to another. To solve this problem, Jena et al. [21] designed a hybrid heuristic based on the Lagrangian relaxation of a new MILP formulation. The latter relies on a modeling technique developed by the same authors that generalizes existing formulations for several variants of the MFLP (Jena et al. [19]). A framework for capturing capacity changes between all possible choices of capacity levels was proposed, which allows capacity scalability to be represented through expansion, contraction, temporary facility closing, and facility reopening. The authors show that the lower bound provided by the linear relaxation of their MILP model is stronger than the lower bounds of the linear relaxations of existing formulations for special problem variants.

The works discussed so far have in common the enforcement of demand satisfaction in every period of the planning horizon through appropriate constraints that are included in the associated MILP models. While the economic benefits of demand fulfillment flexibility have been widely recognized in the field of production planning and inventory control (see e.g., Hung et al. [17] and Merzifonluoğlu and Geunes [28]), this strategy has received far less attention in discrete facility location studies as it will be shown in the remainder of this section.

Demand fulfillment flexibility involves selecting the quantity and setting the delivery timing to meet individual customer demands (Geunes [14], Merzifonluoğlu and Geunes [28]). Hence, some or all of a customer's demand can be satisfied with delay. Gebennini et al. [13] examined a variant of the MFLP where demand satisfaction can be delayed by at most one period. This indicates that all customers have the same sensitivity to delays, which is an assumption seldom observed in practice. In contrast, in the problems addressed by Liang et al. [25] and Wilhelm et al. [37], no time limit is pre-specified on the maximum delay that a customer may experience. Therefore, deliveries may occur well beyond the maximum delivery date tolerated by the customer, even when very high tardiness or backorder costs are imposed. Recently, Correia and Melo [8] incorporated demand fulfillment flexibility into the MFLP through assuming that each individual customer specifies a desired maximum delivery lead time. Shipments that take place after the time period in which demand occurs but not later than the maximum lead time are subject to tardiness penalty costs. We adopt this setting in the present paper and extend the conditions under which facility location and capacity acquisition decisions can be made. In particular, we exploit capacity scalability by allowing the capacity of any facility to be expanded and/or reduced in multiple periods over the planning horizon. This extends the problem studied by Correia and Melo [8] which did not account for gradual capacity adjustments at each location.

As evidenced by the literature reviewed above, the integration of decisions related to dynamic modular capacity adjustments and flexible demand fulfillment into the MFLP has not been addressed so far. However, significant cost savings can often arise as a result of considering these decisions simultaneously in the planning phase. Our contribution aims at studying this important variant of the MFLP.

3 Problem statement and mathematical formulations

In this section, we state the problem settings that are considered and describe the assumptions made. Two alternative MILP formulations for our problem will be proposed.

We consider a company that operates a set of facilities at fixed locations to serve the demands of customers (or customer zones) for a single product (or product family). Due to projected variations in customer demands (regarding quantity and spatial distribution), it is anticipated that the company will not be able to provide adequate customer service in the future with the capacity currently available at its facilities. Therefore, the company will be compelled to modify the physical structure of its facility network. Over a strategic planning phase, multiple options for network redesign are available and the least overall cost reconfiguration is to be identified. Decisions include selecting locations from a given set of potential sites to establish new facilities, closing existing facilities, and gradually adjusting the operating capacity of the facility network over a given time horizon. Under these general settings, the following assumptions are made:

- A finite planning horizon is considered, being partitioned into a set of consecutive and discrete time periods. Strategic decisions related to facility location and capacity scalability can only be made at selected time periods, hereafter called *strategic periods*. In contrast, tactical decisions regarding the commodity flow from operating facilities to customers can be taken in any time period.

- Due to the sizeable investment associated with opening new facilities and closing existing facilities, locations cannot be temporarily closed and reopened. Accordingly, when a new facility is established at a potential site then it will be operational until the end of the planning horizon. Analogously, if an initially existing facility is closed then it cannot be reopened in a later time period. These assumptions are in line with existing literature on discrete facility location (see e.g., Hinojosa et al. [16] and Nickel and Saldanha da Gama [29]).

- The total capacity of a facility is the sum of the capacities of the modular units that the facility holds. For the sake of simplicity, only one type of module with a pre-specified size is considered. Examples of a modular unit include equipment and storage space. Each site cannot hold more than a pre-defined number of modules at any time. At the beginning of the planning horizon, the size of each existing facility is given by the number of modular units available.

- If a new facility is opened at a candidate site in a particular time period then at least one modular unit must be installed in the same period.

- Capacity scalability involves adding modular units to a location or removing modular units from a site. Capacity expansion and contraction can take place more than once, both at new and existing facilities over the time horizon.

- Facility location and capacity sizing costs are subject to economies of scale which favor larger capacities. In addition, economies of scale are also present in the variable cost for processing the product at each operating facility.

- Customers are distinguished on the basis of their sensitivity to delivery lead times. Customers receiving preferred service have zero delivery lead time, i.e. their demands must be satisfied in the time periods they occur. These customers typically contribute most to the company's sales revenue. Customers who are not averse to waiting for their demand requirements to be satisfied specify a maximum allowed delivery delay. These customers are compensated with a lower price which is translated into a tardiness penalty cost for delayed deliveries to reflect the negative impact on the company's profit margin.

- All relevant data (i.e. costs, demand requirements, and other parameters) are prepared by using appropriate forecasting methods and company-specific analyzes.

The possibility of adjusting the capacity of operating facilities over the time horizon is relevant when sizing decisions are reversible in the medium term. This is the case, for example, when operations can be subcontracted, equipment and space can be rented or leased, or changes in manpower requirements can be met. In this context, it is important to understand the trade-off between the costs incurred by capacity expansion and contraction and the level of service provided to the customers. For example, it may be economically attractive to invest in increasing capacity temporarily to guarantee shorter delays in demand fulfillment.

We also note that different time scales for strategic and tactical decisions are considered. This characteristic is also sometimes present in other problems that combine decisions at different levels, e.g. the dynamic location-routing problem studied by Albareda-Sambola et al. [1] and the multi-period logistics network design problem addressed by Badri et al. [4]. All strategic and tactical decisions are assumed to be taken at the beginning of the corresponding time period.

3.1 Notation

In this section, we review and extend the notation introduced by Correia and Melo [8]. Table 1 presents all index sets, while Table 2 describes the parameters related to modular capacity and customer demand.

Set	Description
T	Time periods in the planning horizon
$T_L \subset T$	*Strategic* time periods, i.e. periods in which location and capacity scalability decisions can be made
I^e	Existing facilities at the beginning of the planning horizon
I^n	Candidate sites for locating new facilities
$I = I^e \cup I^n$	All facility locations
J^0	Customers that receive preferred service (i.e. their demands must be satisfied in the time periods they occur)
J^1	Customers that tolerate delays in demand satisfaction
$J = J^0 \cup J^1$	All customers ($J^0 \cap J^1 = \emptyset$)

Table 1: Index sets

The first (last) strategic time period for taking decisions related to opening/closing facilities and expanding/removing capacity is denoted by $\ell = 1$ (ℓ_{max}). The configuration of a facility in time period $\ell \in T_L$ is the outcome of the decisions made in that period. This configuration remains unchanged over all periods up to the next strategic period, that is, up to period $\phi(\ell)$ with $\phi(\ell) = \max\{t \in T : t < \ell + 1\}$ if $\ell < \ell_{max}$, and $\phi(\ell) = |T|$ otherwise (i.e. $\ell = \ell_{max}$). In other words, $\phi(\ell)$ represents the tactical time period just before the next strategic period.

Symbol	Description
Q	Capacity of a modular unit
n_i	Maximum number of modular units that are available at location i in any time period $(i \in I)$
q_i	Number of modular units available at the existing facility i, $1 \leq q_i \leq n_i$ $(i \in I^e)$ at the beginning of the time horizon
d_j^t	Demand of customer j in time period t $(j \in J;\ t \in T)$
ρ_j	Maximum allowed delay (in number of time periods) to satisfy the demand of customer j $(j \in J)$
$\phi(\ell)$	Tactical time period just before the next strategic period $\ell + 1$

Table 2: Capacity and demand parameters

The capacity of an operating facility i can range from Q to n_iQ, in Q increments, at any given period. The adoption of a base modular size (here: Q) is not uncommon in problems involving capacity scalability decisions (see e.g. Delmelle et al. [11] and Gourdin and Klopfenstein [15]).

The two customer segments introduced in Table 1 are distinguished by differing values for parameter ρ_j. Accordingly, customers provided with preferred service impose $\rho_j = 0$ $(j \in J^0)$, whereas customers tolerating late deliveries set $\rho_j > 0$ $(j \in J^1)$. The time lag for demand satisfaction for customer j is also defined by ρ_j, namely demand for period $t \in T$ must be filled over periods $t, t+1, \ldots, t+\rho_j$. In case $t+\rho_j > |T|$, then the last delivery must occur at period $|T|$. This condition ensures that demand cannot be carried over to future periods beyond the planning horizon.

Table 3 presents the cost parameters. Economies of scale are reflected in all fixed cost terms, namely for opening, closing, and maintaining facilities (resp. FO_{ik}^{ℓ}, FC_{ik}^{ℓ}, M_{ik}^{t}), and for expanding or contracting the capacity of facilities (resp. FE_{ik}^{ℓ}, FR_{ik}^{ℓ}). Furthermore, variable processing costs at the facilities (o_{ik}^{t}) are also subject to economies of scale. This feature will impact the development of a mathematical formulation for the problem as it will be shown in Section 3.2.

Symbol	Description
FO_{ik}^{ℓ}	Fixed cost of opening a new facility at candidate site i with k modular units in time period ℓ ($i \in I^n$; $k = 1, \ldots, n_i$; $\ell \in T_L$)
FC_{ik}^{ℓ}	Fixed cost of closing the initially existing facility i with k modular units in time period ℓ ($i \in I^e$; $k = 1, \ldots, n_i$; $\ell \in T_L \setminus \{1\}$)
FE_{ik}^{ℓ}	Fixed cost of expanding the capacity of facility i through installing k modular units ($k = 1, \ldots, n_i - 1$) in time period ℓ; for an *existing* facility $i \in I^e$, we consider $\ell \in T_L$; for a *new* facility $i \in I^n$, we take $\ell \in T_L \setminus \{1\}$
FR_{ik}^{ℓ}	Fixed cost of removing k modular units ($k = 1, \ldots, n_i - 1$) from facility i in time period ℓ; for an *existing* facility $i \in I^e$, we consider $\ell \in T_L$; for a *new* facility $i \in I^n$, we take $\ell \in T_L \setminus \{1\}$
M_{ik}^{t}	Fixed maintenance cost incurred by operating facility i with k modular units in time period t ($i \in I$; $k = 1, \ldots, n_i$; $t \in T$)
o_{ik}^{t}	Cost of processing one unit of product at facility i with k modular units in time period t ($i \in I$; $k = 1, \ldots, n_i$; $t \in T$)
c_{ij}^{t}	Cost of distributing one unit of product from facility i to customer j in time period t ($i \in I$; $j \in J$; $t \in T$)
$p_j^{tt'}$	Tardiness penalty cost for satisfying one unit of demand of customer j in period t' that was originally demanded in period t ($j \in J^1$; $t \in T$; $t' = t, t+1, \ldots, \min\{t + \rho_j, \lvert T \rvert\}$); in particular, for $t' = t$, the tardiness penalty cost is equal to zero

Table 3: Fixed and variable cost parameters

3.2 Mixed-integer linear programming formulations

In this section, we propose two MILP formulations for our problem.

3.2.1 Model 1

For the problem variant without capacity scalability decisions, Correia and Melo [8] developed two MILP models and concluded that one of the formulations was computationally more attractive because of smaller optimality gaps and shorter computing times to identify optimal solutions. Therefore, the first MILP formulation that we propose is based on these findings.

The following two sets of binary variables are defined to represent facility location decisions.

$$z_{ik}^{\ell} = 1 \text{ if a new facility is established with } k \text{ modular units at candidate location } i \text{ in time period } \ell, \text{ 0 otherwise } (i \in I^n;\ k = 1, \ldots, n_i;\ \ell \in T_L) \tag{1}$$

$$z_{ik}^{\ell} = 1 \text{ if the initially existing facility } i \text{ is closed in time period } \ell, \text{ holding } k \text{ modular units at the time of its closure, 0 otherwise } (i \in I^e;\ k = 1, \ldots, n_i;\ \ell \in T_L \setminus \{1\}) \tag{2}$$

If a new facility is opened in time period ℓ then it will remain in operation in time periods $\ell, \ldots, |T|$. Analogously, if an initially existing facility is removed at the beginning of period ℓ then it was operational in periods $1, \ldots, \ell-1$. We consider that in this case the facility cannot be closed in the first time period (i.e. $\ell > 1$). However, an existing facility can have its size modified in the first strategic period either through capacity expansion or contraction.

Capacity scalability decisions are described by the following binary variables.

$$s_{ik}^{\ell} = 1 \text{ if a facility is operated at location } i \text{ with } k \text{ modular units at time period } \ell, \text{ 0 otherwise } (i \in I;\ k = 1, \ldots, n_i;\ \ell \in T_L) \tag{3}$$

$$e_{ik}^{\ell} = 1 \text{ if the capacity of a facility in site } i \text{ is expanded with } k \text{ modular units at time period } \ell, \text{ 0 otherwise; for an \textit{existing} facility: } i \in I^e; k = 1, \ldots, n_i - 1; t \in T_L; \text{ for a \textit{new} facility: } i \in I^n;\ k = 1, \ldots, n_i - 1;\ \ell \in T_L \setminus \{1\} \tag{4}$$

$r_{ik}^{\ell} = 1$ if the capacity of a facility in site i is reduced by removing k modular units at time period ℓ, 0 otherwise; for an *existing* facility: $i \in I^e$; $k = 1, \ldots, n_i - 1$; $\ell \in T_L$; for a *new* facility: $i \in I^n$; $k = 1, \ldots, n_i - 1$;

$$\ell \in T_L \setminus \{1\} \tag{5}$$

Tactical decisions on product flows are defined by continuous variables as follows.

x_{ij}^{t} : Amount of product distributed from facility i to customer j in time period t $(i \in I;\ j \in J^0;\ t \in T)$ (6)

$y_{ij}^{tt'}$: Amount of product distributed from facility i to customer j in time period t' to (partially) satisfy demand of period t $(i \in I;\ j \in J^1;\ t \in T;$ $t' = t, t+1, \ldots, \min\{t+\rho_j,\ |T|\})$ (7)

w_{ik}^{t} : Total quantity of product handled by facility i operating with k modular units in time period t $(i \in I;\ k = 1, \ldots, n_i;\ t \in T)$ (8)

Let $(\mathbf{P}_1)$ denote the following MILP formulation.

$$\begin{aligned}
\text{Min} \ & \sum_{\ell \in T_L} \sum_{i \in I^n} \sum_{k=1}^{n_i} FO_{ik}^{\ell}\, z_{ik}^{\ell} + \sum_{\ell \in T_L \setminus \{1\}} \sum_{i \in I^e} \sum_{k=1}^{n_i} FC_{ik}^{\ell}\, z_{ik}^{\ell} + \sum_{\ell \in T_L \setminus \{1\}} \sum_{i \in I^n} \sum_{k=1}^{n_i-1} FE_{ik}^{\ell}\, e_{ik}^{\ell} + \\
& \sum_{\ell \in T_L} \sum_{i \in I^e} \sum_{k=1}^{n_i-1} FE_{ik}^{\ell}\, e_{ik}^{\ell} + \sum_{\ell \in T_L \setminus \{1\}} \sum_{i \in I^n} \sum_{k=1}^{n_i-1} FR_{ik}^{\ell}\, r_{ik}^{\ell} + \sum_{\ell \in T_L} \sum_{i \in I^e} \sum_{k=1}^{n_i-1} FR_{ik}^{\ell}\, r_{ik}^{\ell} + \\
& \sum_{\ell \in T_L} \sum_{i \in I} \sum_{k=1}^{n_i} s_{ik}^{\ell} \sum_{t=\ell}^{\phi(\ell)} M_{ik}^{t} + \sum_{t \in T} \sum_{i \in I} \sum_{k=1}^{n_i} o_{ik}^{t}\, w_{ik}^{t} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J^0} c_{ij}^{t}\, x_{ij}^{t} + \\
& \sum_{t \in T} \sum_{i \in I} \sum_{j \in J^1} \sum_{t'=t}^{\min\{t+\rho_j,\, |T|\}} \left(c_{ij}^{t'} + p_j^{tt'} \right) y_{ij}^{tt'}
\end{aligned} \tag{9}$$

s.t.

$$\sum_{i \in I} x_{ij}^{t} = d_j^{t} \qquad j \in J^0,\ t \in T \tag{10}$$

$$\sum_{i \in I} \sum_{t'=t}^{\min\{t+\rho_j,\, |T|\}} y_{ij}^{tt'} = d_j^{t} \qquad j \in J^1,\ t \in T \tag{11}$$

$$\sum_{\ell \in T_L} \sum_{k=1}^{n_i} z_{ik}^{\ell} \leq 1 \qquad i \in I^n \tag{12}$$

$$\sum_{\ell \in T_L \setminus \{1\}} \sum_{k=1}^{n_i} z_{ik}^{\ell} \leq 1 \qquad i \in I^e \tag{13}$$

$$\sum_{k=1}^{n_i} s_{ik}^{\ell} = \sum_{\ell' \in T_L :\, \ell' \leq \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^n,\ \ell \in T_L \tag{14}$$

$$\sum_{k=1}^{n_i} s_{ik}^{\ell} = 1 - \sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^e,\ \ell \in T_L \tag{15}$$

$$\sum_{k=1}^{n_i-1} e_{ik}^{\ell} + \sum_{k=1}^{n_i-1} r_{ik}^{\ell} \leq \sum_{\ell' \in T_L:\, \ell' < \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^n,\ \ell \in T_L \setminus \{1\} \tag{16}$$

$$\sum_{k=1}^{n_i-1} e_{ik}^{\ell} + \sum_{k=1}^{n_i-1} r_{ik}^{\ell} \leq 1 - \sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^e,\ \ell \in T_L \tag{17}$$

$$\begin{aligned}\sum_{k=1}^{n_i} k\, s_{ik}^{\ell} = &\sum_{\ell' \in T_L:\, \ell' \leq \ell} \sum_{k=1}^{n_i} k\, z_{ik}^{\ell'} + \\ &\sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, e_{ik}^{\ell'} - \sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, r_{ik}^{\ell'} \qquad i \in I^n,\ \ell \in T_L\end{aligned} \tag{18}$$

$$\begin{aligned}\sum_{k=1}^{n_i} k\, s_{ik}^{\ell} \geq q_i + &\sum_{\ell' \in T_L:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, e_{ik}^{\ell'} - \\ &\sum_{\ell' \in T_L:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, r_{ik}^{\ell'} - n_i \sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^e,\ \ell \in T_L\end{aligned} \tag{19}$$

$$\begin{aligned}\sum_{k=1}^{n_i} k\, s_{ik}^{\ell} \leq q_i + &\sum_{\ell' \in T_L:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, e_{ik}^{\ell'} - \\ &\sum_{\ell' \in T_L:\, \ell' \leq \ell} \sum_{k=1}^{n_i-1} k\, r_{ik}^{\ell'} + n_i \sum_{\ell' \in T_L \setminus \{1\}:\, \ell' \leq \ell} \sum_{k=1}^{n_i} z_{ik}^{\ell'} \qquad i \in I^e,\ \ell \in T_L\end{aligned} \tag{20}$$

$$\sum_{k=1}^{n_i} k\, z_{ik}^{\ell} \geq \sum_{k=1}^{n_i} k\, s_{ik}^{\ell-1} - n_i \sum_{k=1}^{n_i} s_{ik}^{\ell} \qquad i \in I^e,\ \ell \in T_L \setminus \{1\} \tag{21}$$

$$\sum_{k=1}^{n_i} k\, z_{ik}^{\ell} \leq \sum_{k=1}^{n_i} k\, s_{ik}^{\ell-1} + n_i \sum_{k=1}^{n_i} s_{ik}^{\ell} \qquad i \in I^e,\ \ell \in T_L \setminus \{1\} \tag{22}$$

$$w_{ik}^{t} \leq k\, Q\, s_{ik}^{\ell} \qquad i \in I,\ k = 1, \ldots, n_i,\ t \in T,\ \ell = \max\{\ell' \in T_L :\ \ell' \leq t\} \tag{23}$$

$$\sum_{k=1}^{n_i} w_{ik}^{t} = \sum_{j \in J^0} x_{ij}^{t} + \sum_{j \in J^1} \sum_{t'=\max\{1,\, t-\rho_j\}}^{t} y_{ij}^{t't} \qquad i \in I,\ t \in T \tag{24}$$

$$x_{ij}^{t} \geq 0 \qquad i \in I,\ j \in J^0,\ t \in T \tag{25}$$

$$y_{ij}^{tt'} \geq 0 \qquad i \in I,\ j \in J^1,\ t \in T,\ t' = t, \ldots, \min\{t + \rho_j, |T|\} \tag{26}$$

$$w_{ik}^{t} \geq 0 \qquad i \in I,\ k = 1, \ldots, n_i,\ t \in T \tag{27}$$

$$z_{ik}^{\ell} \in \{0,1\} \qquad i \in I^n, k = 1, \ldots, n_i,\ \ell \in T_L \tag{28}$$

$$z_{ik}^{\ell} \in \{0,1\} \qquad i \in I^e,\ k = 1, \ldots,\ n_i,\ \ell \in T_L \setminus \{1\} \tag{29}$$

$$s_{ik}^{\ell} \in \{0,1\} \qquad i \in I,\ k = 1, \ldots,\ n_i,\ \ell \in T_L \tag{30}$$

$$e_{ik}^{\ell} \in \{0,1\} \qquad i \in I^n,\ k = 1, \ldots, n_i - 1,\ \ell \in T_L \setminus \{1\} \tag{31}$$

$$e_{ik}^{\ell} \in \{0,1\} \qquad i \in I^e,\ k = 1,\dots,n_i - 1,\ \ell \in T_L \tag{32}$$

$$r_{ik}^{\ell} \in \{0,1\} \qquad i \in I^n,\ k = 1,\dots,n_i - 1,\ \ell \in T_L \setminus \{1\} \tag{33}$$

$$r_{ik}^{\ell} \in \{0,1\} \qquad i \in I^e,\ k = 1,\dots,n_i - 1,\ \ell \in T_L \tag{34}$$

The objective function (9) describes the aim of the decision-making process, namely to identify the network reconfiguration with the least total cost. The first six terms account for all fixed costs that are incurred for decisions taken in strategic periods, i.e. for opening new facilities, closing existing facilities, and adjusting capacity through adding or removing modular units. The seventh term also represents fixed costs for maintaining facilities in all periods they are operated. The remaining terms in (9) are associated with variable costs for processing the product at operating facilities and distributing it to customers. In addition, tardiness penalty costs are incurred for delayed deliveries.

Constraints (10), resp. (11), enforce the satisfaction of demands for customer segment J^0, resp. J^1. According to constraints (12), resp. (13), opening a new facility, resp. closing an initially existing facility, is limited to take place at most once over the time horizon. Equalities (14)–(15) ensure that if a facility is operated then it must hold a positive number of modular units, otherwise capacity cannot be available. Constraints (16)–(17) guarantee that at most one type of capacity adjustment (either expansion or contraction) can occur at a location and in a strategic period provided that a facility is operating in that site. The number of available modular units at new facilities is described by equalities (18) for every time period. This number is the outcome of the sizing choice made when the facility was opened and the number of modular units that were added or removed afterward. Constraints (19)–(20) take a similar role but for existing facilities. Observe that these inequalities are redundant in all periods following a facility closure. Moreover, if an existing facility is closed at the beginning of a strategic period then the number of modular units it held at that time point is determined by constraints (21)–(22). This information is required to allocate the fixed closing costs correctly, since they depend on the size of the facility upon its closure (see second term in (9)). Capacity constraints are described by inequalities (23). Constraints (24) state that the total outflow from a facility at a given time period is split into deliveries to customers that receive preferred service and customers that accept delays in demand satisfaction. Finally, non-negativity and binary conditions are set by (25)–(34).

3.2.2 Model 2

Recently, Jena et al. [19] proposed a new modeling framework that captures several variants of the MFLP into a single formulation, including capacity expansion and reduction as well as temporarily opening and closing of facilities. Interestingly, the authors showed that their formulation provides stronger linear relaxation bounds than models that have been developed specifically for certain variants of the problem. This has been accomplished by using binary variables to represent capacity changes from one time period to the next at a particular location. Inspired by the work of Jena et al. [19], we now describe an alternative formulation for our problem. Instead of defining specific sets of variables to represent facility location and capacity scalability decisions, as done in the previous section (recall (1)–(5)), we introduce the following set of four-index binary variables:

$u^{\ell}_{ik_1k_2} = 1$ if a facility at site i has k_2 modular units in time period ℓ knowing that it held k_1 modular units before, 0 otherwise $(i \in I;\ k_1, k_2 = 0, \ldots, n_i;\ \ell \in T_L)$ (35)

At the start of the time horizon ($\ell = 1$), each existing facility $i \in I^e$ has q_i modular units and therefore, a capacity transition from $k_1 = q_i$ modules to k_2 modules, with $k_2 > 0$, can take place. Furthermore, at each candidate site $i \in I^n$ the initially available capacity is zero $(k_1 = 0)$ and a transition to k_2 modules, with $k_2 > 0$, occurs when a new facility is established.

All decisions about opening new facilities, closing initially existing facilities, and expanding or contracting capacities can be represented by the new binary variables for appropriate combinations of the indices k_1 and k_2:

- If an existing facility i is closed at the beginning of a strategic period ℓ then $k_1 > 0$ and $k_2 = 0$ and, therefore, $u^{\ell}_{ik_10} = 1$. In this case, at the time point of closure, the facility had a capacity of $k_1\, Q$ units.
- If a facility i is not available over two consecutive time periods then $k_1 = k_2 = 0$, meaning that $u^{\ell}_{i00} = 1$.
- If the positive capacity of a facility does not change over two consecutive strategic time periods then no capacity transition has occurred. Hence, $k_1 > 0$ and $k_2 = k_1$.

- Capacity expansion corresponds to having $k_1 > 0$ and $k_2 > k_1$. In this case, the capacity is increased by $(k_2 - k_1)\,Q$ units.

- Capacity contraction is represented by $k_2 > 0$ and $k_2 < k_1$. The capacity reduction amounts to $(k_1 - k_2)\,Q$ units.

Let $F^{\ell}_{ik_1k_2}$ denote the total fixed cost incurred at the beginning of time period $\ell \in T_L$ due to a capacity transition from k_1 to k_2 modular units for a facility at site $i \in I$. This parameter results from aggregating particular fixed location, maintenance, and capacity transition costs as follows:

$$F^{\ell}_{ik_1k_2} = \begin{cases} 0 & \text{if } i \in I;\ k_1 = k_2 = 0;\ \ell \in T_L \\ \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_1} & \text{if } i \in I;\ k_1 = k_2;\ k_1, k_2 = 1, \ldots, n_i; \\ & \ell \in T_L \\ FO^{\ell}_{ik_2} + \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_2} & \text{if } i \in I^n;\ k_1 = 0;\ k_2 = 1, \ldots, n_i;\ \ell \in T_L \\ FC^{\ell}_{ik_1} & \text{if } i \in I^e;\ k_1 = 1, \ldots, n_i;\ k_2 = 0; \\ & \ell \in T_L \setminus \{1\} \\ FE^{\ell}_{i(k_2-k_1)} + \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_2} & \text{if } i \in I^n;\ k_1 = 1, \ldots, n_i - 1; \\ & k_2 = k_1 + 1, \ldots, n_i;\ \ell \in T_L \setminus \{1\} \\ FE^{\ell}_{i(k_2-k_1)} + \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_2} & \text{if } i \in I^e;\ k_1 = 1, \ldots, n_i - 1; \\ & k_2 = k_1 + 1, \ldots, n_i;\ \ell \in T_L \\ FR^{\ell}_{i(k_1-k_2)} + \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_2} & \text{if } i \in I^n;\ k_1 = 1, \ldots, n_i; \\ & k_2 = 1, \ldots, k_1 - 1;\ \ell \in T_L \setminus \{1\} \\ FR^{\ell}_{i(k_1-k_2)} + \sum\limits_{t=\ell}^{\phi(\ell)} M^t_{ik_2} & \text{if } i \in I^e;\ k_1 = 1, \ldots, n_i; \\ & k_2 = 1, \ldots, k_1 - 1;\ \ell \in T_L \end{cases} \tag{36}$$

We now present an alternative formulation, denoted $(\mathbf{P}_2)$, that uses the new binary variables $u^{\ell}_{ik_1k_2}$ along with the flow variables x^t_{ij}, $y^{tt'}_{ij}$, and w^t_{ik} introduced in (6)–(8).

$$\text{Min} \sum_{\ell \in T_L} \sum_{i \in I} \sum_{k_1=0}^{n_i} \sum_{k_2=0}^{n_i} F^{\ell}_{ik_1k_2}\, u^{\ell}_{ik_1k_2} + \sum_{t \in T} \sum_{i \in I} \sum_{k=1}^{n_i} o^t_{ik}\, w^t_{ik} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J^0} c^t_{ij}\, x^t_{ij} +$$

$$+\sum_{t\in T}\sum_{i\in I}\sum_{j\in J^1}\sum_{t'=t}^{\min\{t+\rho_j,\,|T|\}}\left(c_{ij}^{t'}+p_j^{tt'}\right)y_{ij}^{tt'} \tag{37}$$

s.t.

$$(10),\ (11),\ (25)-(27)$$

$$\sum_{k=1}^{n_i} u_{iq_ik}^1 = 1 \qquad i\in I^e \tag{38}$$

$$\sum_{k_1=0,k_1\neq q_i}^{n_i}\ \sum_{k_2=0}^{n_i} u_{ik_1k_2}^1 = 0 \qquad i\in I^e \tag{39}$$

$$\sum_{k=0}^{n_i} u_{ik0}^{\ell} \leq u_{i00}^{\ell+1} \qquad i\in I^e,\ \ell\in T_L\setminus\{\ell_{max}\} \tag{40}$$

$$\sum_{k=0}^{n_i} u_{i0k}^1 = 1 \qquad i\in I^n \tag{41}$$

$$\sum_{k_1=1}^{n_i}\sum_{k_2=0}^{n_i} u_{ik_1k_2}^1 = 0 \qquad i\in I^n \tag{42}$$

$$\sum_{k=0}^{n_i} u_{ik0}^{\ell'} \leq 1-\sum_{k=1}^{n_i} u_{i0k}^{\ell} \qquad i\in I^n,\ \ell,\ell'\in T_L,\ \ell'>\ell \tag{43}$$

$$\sum_{k_1=0}^{n_i} u_{ik_1k}^{\ell-1} = \sum_{k_2=0}^{n_i} u_{ikk_2}^{\ell} \qquad i\in I,\ k=0,\dots,n_i,\ \ell\in T_L\setminus\{1\} \tag{44}$$

$$w_{ik}^t \leq Q\sum_{k_1=0}^{n_i} k\,u_{ik_1k}^{\ell} \qquad i\in I,\ k=1,\dots,n_i,\ t\in T,\ \ell=\max\{\ell'\in T_L:\ \ell'\leq t\} \tag{45}$$

$$u_{ik_1k_2}^{\ell}\in\{0,1\} \qquad i\in I,\ k_1,k_2=0,\dots,n_i,\ \ell\in T_L \tag{46}$$

The objective function (37) minimizes the fixed network operating costs and the variable customer distribution costs over all periods. Note that facility location and capacity scalability costs are combined in the first cost term. Constraints (38)–(40) involve exclusively initially existing facilities. Equalities (38) ensure that these facilities cannot be closed at the beginning of the planning horizon, while equalities (39) state that any transition from an initial capacity level other than q_i modular units is not permitted. Constraints (40) prevent an existing facility that was previously closed from being reopened. Constraints (41)–(43) are relevant to locating new facilities. Equalities (41) and (42) specify that only a given sub-set of capacity transitions are allowed in the first time period, namely those from level $k_1=0$ to level $k_2\geq 0$. Constraints (43) ensure that if a new facility is established at a candidate site then it must be operational throughout the time horizon. Finally, feasible capacity transitions are imposed by constraints (44) in any location, capacity constraints are described by inequalities (45), and binary conditions for the transition variables are defined by constraints (46).

The problem that we study is clearly NP-hard, since it includes the uncapacitated MFLP as a particular case (Jacobsen [18]). Additionally, the proposed formulations can also be used when the company does not employ customer segmentation. This is the case when every customer must have his demand requirements satisfied in the time periods they occur (i.e. $J^1 = \emptyset$, $J = J^0$) or when all customers tolerate late deliveries (i.e. $J^0 = \emptyset$, $J = J^1$). The particular "greenfield" situation, in which no facilities are open at the beginning of the time horizon, is also easily modeled by the proposed formulations. We decided to focus on the more general case by taking an existing infrastructure as input, as it is often the case in practice.

3.3 Comparison of formulations

Formulation $(\mathbf{P}_2)$ is more compact than formulation $(\mathbf{P}_1)$ owing to the variable redefinition (35). However, a closer look at the total number of variables and constraints in the two models reveals significant differences. Table 4 displays the size of each formulation.

Formulation	Number of variables		Number of constraints
	Binary	Continuous	
$(\mathbf{P}_1)$	$\mathcal{O}\Big(4\,\overline{n}\cdot\lvert I\rvert\cdot\lvert T_L\rvert - (\overline{n}-2)\cdot\lvert I^n\rvert - (\overline{n}+2\,\lvert T_L\rvert)\cdot\lvert I\rvert\Big)$	$\mathcal{O}\Big((\overline{n}+\lvert J^0\rvert)\cdot\lvert I\rvert\cdot\lvert T\rvert + (\overline{\rho}+1)\cdot\Big(\lvert T\rvert-\dfrac{\overline{\rho}}{2}\Big)\cdot\lvert I\rvert\cdot\lvert J^1\rvert\Big)$	$\mathcal{O}\Big(\lvert J\rvert\cdot\lvert T\rvert + 3\,\lvert T_L\rvert\cdot(\lvert I\rvert+\lvert I^e\rvert) + (1+\overline{n})\cdot\lvert I\rvert\cdot\lvert T\rvert - \lvert I^e\rvert\Big)$
$(\mathbf{P}_2)$	$\mathcal{O}\Big((\overline{n}+1)^2\cdot\lvert I\rvert\cdot\lvert T_L\rvert\Big)$	$\mathcal{O}\Big((\overline{n}+\lvert J^0\rvert)\cdot\lvert I\rvert\cdot\lvert T\rvert + (\overline{\rho}+1)\cdot\Big(\lvert T\rvert-\dfrac{\overline{\rho}}{2}\Big)\cdot\lvert I\rvert\cdot\lvert J^1\rvert\Big)$	$\mathcal{O}\Big(\lvert J\rvert\cdot\lvert T\rvert + 2\,\lvert I\rvert + \lvert I^e\rvert\cdot\lvert T_L\rvert + \lvert I^n\rvert\cdot\lvert T_L\rvert\cdot\dfrac{\lvert T_L\rvert-1}{2} + (\overline{n}+1)\cdot\lvert I\rvert\cdot(\lvert T_L\rvert-1) + \overline{n}\cdot\lvert I\rvert\cdot\lvert T\rvert - \lvert I^e\rvert\Big)$

Table 4: Summary of total number of variables and constraints in the proposed formulations

It can be seen that the formulations strongly differ in the number of binary variables. For a fixed number of facilities ($|I|$) and a fixed number of strategic periods ($|T_L|$), model $(\mathbf{P}_1)$ has $\mathcal{O}(\overline{n})$ binary variables, whereas formulation $(\mathbf{P}_2)$ has $\mathcal{O}(\overline{n}^2)$ binary variables, with $\overline{n} = max_{i\in I}\{n_i\}$. Let us consider a problem instance of moderate size to illustrate this characteristic. Suppose that the planning horizon is divided into 36 time periods and facility location and capacity scalability decisions can be taken in 6 of these periods (i.e. $|T| = 36$ and $|T_L| = 6$). Moreover, there are 12 candidate sites for locating new facilities and 3 facilities are operating at the start of the time horizon (i.e. $|I^n| = 12$ and $|I^e| = 3$). If any facility can hold at most $n_i = 5$ $(i \in I)$ modular units then model $(\mathbf{P}_1)$, resp. $(\mathbf{P}_2)$, has 1509, resp. 3240, binary variables. Increasing the maximum number of modular units to $n_i = 10$ yields 3174, resp. 10890, binary variables. Despite some of these binary variables being fixed at zero by constraints (39) and (42) in formulation $(\mathbf{P}_2)$, the overall number is still noteworthy.

The same number of continuous variables is present in both formulations. Parameter $\overline{\rho}$ that appears in the third column of Table 4 denotes the largest delivery lead time among all customers in segment J^1.

Regarding the number of constraints, there is not a striking difference between the two models. Using the above example with $n_i = 5$ $(i \in I)$ and considering 150 customers, there are 8,961 and 8,775 constraints with formulation $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$, respectively. When parameter n_i is increased to 10, $(\mathbf{P}_1)$ has fewer constraints compared to $(\mathbf{P}_2)$, namely 11,661 and 11,850, respectively. Variations in other parameters, such as $|T_L|$ and $|I^e|$, produce a similar ratio between the number of constraints of the two formulations.

Due to the significantly smaller number of binary variables, one may conjecture that it would be favorable to use formulation $(\mathbf{P}_1)$ to solve (large) problem instances using a state-of-the-art MILP solver. The validity of this conjecture will be investigated through a computational study that will be presented in Section 5. In addition, a comparison of the bounds provided by the linear relaxations of the two models, $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$, and the relative gaps between the optimal LP solutions and the optimal integer solutions will also be examined. Recall that Jena et al. [19] proved dominance relations between their formulation and specialized models for particular cases of the MFLP, showing that binary variables of the type of (35) play a predominant part in achieving stronger LP relaxation bounds.

4 Enhancing the mathematical formulations

In this section, we propose various ways of enhancing the mathematical formulations, $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$, with the purpose of improving the lower bounds provided by their linear relaxations. Extensive computational experience on MILP models suggests that when the LP lower bounds are tight, a state-of-the-art solver will most likely be computationally effective.

4.1 Additional inequalities for $(\mathbf{P}_1)$

Since each initially existing facility $i \in I^e$ has q_i modules at the beginning of the time horizon, it is clear that an expansion (contraction) of its capacity leading to more than n_i (less than 1) modular unit(s) is not permitted. This is stated by the following conditions.

$$\sum_{k=n_i-q_i+1}^{n_i-1} e_{ik}^1 = 0 \qquad i \in I^e \tag{47}$$

$$\sum_{k=q_i}^{n_i-1} r_{ik}^1 = 0 \qquad i \in I^e \tag{48}$$

In addition, if a new facility is opened in the first strategic period with k modular units then the following relationship between the binary variables z_{ik}^1 and s_{ik}^1 certainly holds.

$$s_{ik}^1 = z_{ik}^1 \qquad i \in I^n;\ k = 1, \ldots, n_i \tag{49}$$

If a facility has its capacity expanded through the installation of k additional modular units in a particular strategic period then it will operate with at least $k+1$ modular units in that period. This condition is ensured by the following inequalities.

$$e_{ik}^\ell \leq \sum_{k'=k+1}^{n_i} s_{ik'}^\ell \qquad i \in I^e;\ k = 1, \ldots, n_i - 1;\ \ell \in T_L \tag{50}$$

$$e_{ik}^\ell \leq \sum_{k'=k+1}^{n_i} s_{ik'}^\ell \qquad i \in I^n;\ k = 1, \ldots, n_i - 1;\ \ell \in T_L \setminus \{1\} \tag{51}$$

Capacity reduction through removing k modular units from a facility at a particular strategic period will result in a facility having at most $n_i - k$ modular units. This is expressed by inequalities (52) and (53).

$$r_{ik}^\ell \leq \sum_{k'=1}^{n_i-k} s_{ik'}^\ell \qquad i \in I^e;\ k = 1, \ldots, n_i - 1;\ \ell \in T_L \tag{52}$$

$$r_{ik}^\ell \leq \sum_{k'=1}^{n_i-k} s_{ik'}^\ell \qquad i \in I^n;\ k = 1, \ldots, n_i - 1;\ \ell \in T_L \setminus \{1\} \tag{53}$$

Given the configuration of a facility at time period ℓ, not all options for capacity expansion or contraction can be implemented in the next strategic period. The following inequalities make sure that certain capacity scalability choices are excluded in period $\ell + 1$.

$$\sum_{k'=n_i-k+1}^{n_i-1} e_{ik'}^{\ell+1} + \sum_{k'=k}^{n_i-1} r_{ik'}^{\ell+1} \leq 1 - s_{ik}^{\ell} \qquad i \in I;\ k = 1, \ldots, n_i;\ \ell \in T_L \setminus \{\ell_{max}\} \quad (54)$$

We illustrate these inequalities by means of a small example. We assume that the time horizon spans 3 years with each year being divided into 12 months. Facility location and capacity scalability decisions can be taken every six months. Hence, $|T| = 36$, $T_L = \{1, 7, 13, 19, 25, 31\}$ and $\ell_{max} = 31$. Furthermore, a facility can hold up to 5 modular units ($n_i = 5,\ i \in I$) at any given period. Let us suppose that at time period 7 a particular facility i is operating with 2 modular units, i.e. $s_{i2}^{7} = 1$, and that it will remain open in the next strategic period 13. If the facility is expanded at $\ell = 13$ then 1, 2 or 3 new modules can be installed. In the case that capacity contraction takes place then only one module can be removed. All other options for capacity expansion or reduction are not viable, hence $e_{i4}^{13} = r_{i2}^{13} = r_{i3}^{13} = r_{i4}^{13} = 0$. This is ensured by inequalities (54).

Let us assume, without loss of generality, that $|T|$ is a multiple of $|T_L|$. The total number of time periods between two consecutive strategic periods is thus given by $\tau = |T|/|T_L|$. For the above example we obtain $\tau = 6$. The minimum quantity of demand that must be served over the time interval covering one strategic period and all subsequent periods (before the next strategic period is reached) is given by

$$D^{\ell} = \begin{cases} \sum_{j \in J^0} \sum_{t=\ell}^{\ell+\tau-1} d_j^t + \sum_{j \in J^1} \sum_{t=\ell}^{\ell+\tau-\rho_j-1} d_j^t & \text{if } \ell \in T_L \text{ and } \ell < \ell_{max} \\ \sum_{j \in J} \sum_{t=\ell}^{|T|} d_j^t & \text{if } \ell = \ell_{max} \end{cases}$$

The above expression includes all demand requirements of the preferred customer segment J^0 as well as the minimum quantity that must be distributed to customers accepting delayed deliveries. Since customer demand cannot be lost, in the last time interval (comprising all periods from ℓ_{max} through $|T|$), the demands of both customer segments must be fully satisfied.

Knowing that each capacity module has size Q, the minimum demand requirements D^{ℓ} can only be filled provided that at least R^{ℓ} modular units are available in the facility network from period ℓ through the next strategic period with

$$R^{\ell} = \left\lceil \frac{D^{\ell}}{Q} \right\rceil \qquad (55)$$

and $\lceil x \rceil$ denoting the smallest integer greater than or equal to x. Therefore, the following inequalities are valid.

$$\tau \sum_{i \in I} \sum_{k=1}^{n_i} k\, s_{ik}^{\ell} \;\geq\; R^{\ell} \qquad \ell \in T_L$$

We now apply the Chvátal-Gomory rounding procedure a finite number of times (Wolsey [38]) and obtain the following set of valid inequalities.

$$\sum_{i \in I} \sum_{k=1}^{n_i} \left\lceil \frac{\tau\, k}{p} \right\rceil s_{ik}^{\ell} \;\geq\; \left\lceil \frac{R^{\ell}}{p} \right\rceil \qquad \ell \in T_L;\; p = 1, \ldots, \tau \max_{i \in I}\{n_i\} \tag{56}$$

Observe that these inequalities are derived from

$$\sum_{i \in I} \sum_{k=1}^{n_i} \frac{\tau k}{p} s_{ik}^{\ell} \;\geq\; \frac{R^{\ell}}{p} \qquad \ell \in T_L;\; p = 1, \ldots, \tau \max_{i \in I}\{n_i\}$$

Rounding up all coefficients on the left-hand side does not affect the validity of these inequalities. Since the left-hand side must be integer we can also round up the right-hand side, thus obtaining (56). When for a given ℓ and different values of p several inequalities (56) are obtained with the same right-hand side, it is only needed to consider the inequality with the strongest left-hand side.

4.2 Additional inequalities for $(\mathbf{P}_2)$

Due to the introduction of the transition variables $u_{ik_1k_2}^{\ell}$ in (35), the conditions described by (49)–(54) are implicitly present in formulation $(\mathbf{P}_2)$. Thus, further inequalities do not need to be imposed. Regarding the Chvátal-Gomory cuts, the counterpart of (56) is given by

$$\sum_{i \in I^n} \sum_{k=1}^{n_i} \left\lceil \frac{\tau\, k}{p} \right\rceil u_{i0k}^{1} + \sum_{i \in I^e} \sum_{k=1}^{n_i} \left\lceil \frac{\tau\, k}{p} \right\rceil u_{iq_ik}^{1} \geq \left\lceil \frac{R^{1}}{p} \right\rceil \tag{57}$$

$$\sum_{i \in I^n} \sum_{k_1=0}^{n_i} \sum_{k_2=1}^{n_i} \left\lceil \frac{\tau\, k_2}{p} \right\rceil u_{ik_1k_2}^{\ell} + \sum_{i \in I^e} \sum_{k_1=1}^{n_i} \sum_{k_2=1}^{n_i} \left\lceil \frac{\tau\, k_2}{p} \right\rceil u_{ik_1k_2}^{\ell} \geq \left\lceil \frac{R^{\ell}}{p} \right\rceil \qquad \ell \in T_L \setminus \{1\} \tag{58}$$

for $p = 1, \ldots, \tau \max_{i \in I}\{n_i\}$ and R^{ℓ} defined by (55). Again, we only consider the strongest inequalities (57)–(58) for each $\ell \in T_L$.

Compared to model $(\mathbf{P}_1)$, a smaller number of inequalities can be added to $(\mathbf{P}_2)$. Hereafter, the formulations strengthened with additional inequalities will be denoted by $(\mathbf{P}_1^+)$ and $(\mathbf{P}_2^+)$.

5 Computational study

In this section, we report on the computational experiments performed on randomly generated instances to evaluate the performance of the proposed formulations, both with and without enforcing additional inequalities, using a commercial MILP solver. In addition, further insights into the characteristics of the best solutions identified by the solver will be discussed in Section 5.3.

5.1 Characteristics of test instances

As benchmark instances are not available for the problem at hand, we randomly generated a set of test instances by combining the values indicated in Table 5.

Parameter	Value
$\lvert J\rvert$	100, 150
$\lvert J^0\rvert$	$\lceil \beta^J \lvert J\rvert \rceil$ with $\beta^J \in \{0.25,\ 0.5,\ 0.75,\ 1\}$
$\lvert I\rvert$	$0.1\,\lvert J\rvert$
$\lvert I^n\rvert$	$0.8\,\lvert I\rvert$
$\lvert T\rvert$	36
$\lvert T_L\rvert$	3, 6

Table 5: Cardinality of index sets

Observe that different interpretations can be given to a time period and thus to the length of the planning horizon. For example, a time period could represent one month, two months or one quarter. Taking $|T| = 36$, these choices correspond to a 3 year, 6 year or 9 year time frame, respectively. Accordingly, $T_L = \{1, 13, 25\}$ ($|T_L| = 3$) could be associated with annual, biennial or triennial strategic periods. By taking $|T_L| = 6$ with $T_L = \{1, 7, 13, 19, 25, 31\}$, the number of opportunities for making location and capacity scalability decisions would double.

Three different demand scenarios were considered. In scenario 1, customer demand is irregular, meaning that demand variations are allowed up to $\pm 20\%$ between two consecutive time periods. Scenarios 2 and 3 are associated with trapezoidal shapes. Scenario 2 represents a typical product life cycle with a growth stage followed by a maturity phase and ending with gradual decline. In scenario 3, customer demand rates follow an inverted trapezoid, the latter representing an economic downturn followed by market recovery. In this case, demand variations go through three phases, namely contraction, recession and growth. Each phase has a duration of 12 periods. Table 6 describes these three scenarios. In scenarios 2 and 3, demand can

Scenario	Demand	Periods	Coefficient
1,2,3	$d_j^1 = U[20,\, 100]$		
1	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 2, \ldots, \lvert T\rvert$	$\beta_d^t \in U[0.8,\, 1.2]$
2	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 2, \ldots, 12$	$\beta_d^t \in U[1.0,\, 1.2]$
	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 13, \ldots, 24$	$\beta_d^t \in U[0.99,\, 1.01]$
	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 25, \ldots, 36$	$\beta_d^t \in U[0.8,\, 1.0]$
3	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 2, \ldots, 12$	$\beta_d^t \in U[0.8,\, 1.0]$
	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 13, \ldots, 24$	$\beta_d^t \in U[0.99,\, 1.01]$
	$d_j^t = \beta_d^t\, d_j^{t-1}$	$t = 25, \ldots, 36$	$\beta_d^t \in U[1.0,\, 1.2]$

Table 6: Demand parameters

increase or decrease by at most 20% between two consecutive time periods during the growth and contraction phases. In the maturity (scenario 2) and recession (scenario 3) stages, small changes in customer demand are assumed ($\pm 1\%$). Although these rates are not typical of all situations, certain goods and services are known to have very high growth (or contraction) rates in demand, particularly in industrializing countries. The consumer technology industry is such an example.

Further details on the generation of the test instances are provided in the appendix. In particular, a scheme for obtaining the cost parameters, some of them reflecting economies of scale, is described.

For each demand scenario and each choice of $|J|$, six instances were generated, thus yielding a total of 108 instances. Each one of these instances was then considered with three different values for the maximum delay in demand fulfillment, namely $\rho = \rho_j = 0, 1, 2$ for every $j \in J^1$. Setting $\rho = 0$ corresponds to taking $\beta^J = 1$ (recall Table 5), which results in $J^0 = J$ and $J^1 = \emptyset$. Hence, in this case, we have the classical situation in facility location with all customers having their demands satisfied in the time periods they occur.

5.2 Numerical results

Formulations $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$, including their enhancements $(\mathbf{P}_1^+)$ and $(\mathbf{P}_2^+)$, were coded in C++ using IBM ILOG Concert Technology and solved with IBM ILOG CPLEX 12.6. The experiments were conducted on a workstation with a multi-core Intel Xeon E5-2650V3 processor (2.3 GHz, 10 cores), 32 GB RAM and running Ubuntu operating system (64-bit). Since the problem that we consider in this article has a strategic nature, fast solution times are not of paramount importance. Therefore, we set a limit of 10 hours of CPU time for each solver run. Furthermore, CPLEX was used with default settings under a deterministic parallel mode.

Table 7 provides a summary of the results obtained for the three demand scenarios (column 1) and different choices for the maximum allowed delay in demand fulfillment (column 2). Columns 3-6 report the number of instances solved to optimality (# opt sol.) and the number of instances not solved to proven optimality within the time limit (# non-opt sol.) for each of the four models. For the latter instances, the corresponding minimum, average and maximum optimality gaps reported by CPLEX are given for each type of formulation in columns 8-11 (MIP gap $= (z^{UB} - z^{LB})/z^{UB} \times 100\%$, where z^{UB} denotes the value of the best feasible solution and z^{LB} represents the best lower bound). Columns 13-16 display the minimum, average and maximum computing times, in seconds (MIP CPU). The last row of each demand scenario ('All') contains information about all test instances in that scenario. Finally, the best average values with respect to the evaluation criteria are shown in boldface.

The capability of CPLEX to identify an optimal solution within the time limit depends on the demand scenario. While optimality is achieved in 75.9% (82.4%) of the test instances in scenario 1 (scenario 2), optimality could only be proven in 52.8% of the instances with inverted trapezoidal demand rates (scenario 3). Moreover, it seems that the enhanced formulation $(\mathbf{P}_2^+)$ has the best average performance with respect to finding an optimal solution for each specific demand class. For those instances that were not solved to proven optimality, small MIP gaps were obtained with all demand patterns (4.3% was the largest MIP gap), despite the size of the test instances. Observe that with formulation $(\mathbf{P}_1)$, an instance has up to 1,509 binary variables (average: 920) and 8,961 constraints (average: 7,400). In contrast, instances with model $(\mathbf{P}_2)$ contain at most 3,240 binary variables (average: 2,025) and 8,775 constraints (average: 7,136). Both formulations have the same number of continuous variables, namely not more than 199,620 variables (average: 88,830).

Although the enhancements proposed in Section 4 do not have a major impact in decreasing the MIP gap, they are very useful for identifying optimal solutions (see columns 4 and 6 in Table 7). Furthermore, formulation $(\mathbf{P}_1^+)$ achieves on average lower MIP gaps for instances with trapezoidal demand rates (scenario 2), while formulation $(\mathbf{P}_2^+)$ provides better MIP gaps for the other two classes of instances. The enhancements do not necessarily incur an additional computational burden, as shown in Table 7. On average, shorter computing times are obtained

Demand scenario	ρ	# opt sol./# non-opt sol.					MIP gap (%)*					MIP CPU (sec.)			
		$(\mathbf{P}_1)$	$(\mathbf{P}_1^+)$	$(\mathbf{P}_2)$	$(\mathbf{P}_2^+)$		$(\mathbf{P}_1)$	$(\mathbf{P}_1^+)$	$(\mathbf{P}_2)$	$(\mathbf{P}_2^+)$		$(\mathbf{P}_1)$	$(\mathbf{P}_1^+)$	$(\mathbf{P}_2)$	$(\mathbf{P}_2^+)$
1	0	22/14	25/11	**27/9**	**27/9**	min	0.22	0.29	0.24	0.49	min	1361.7	848.6	235.5	431.6
						avg	0.90	0.84	**0.68**	0.81	avg	19709.4	16875.0	15749.5	**15654.7**
						max	2.18	2.17	1.04	1.34	max	36000.0	36000.0	36000.0	36000.0
	1	24/12	**26/10**	**26/10**	**26/10**	min	0.51	0.30	0.30	0.29	min	105.0	119.4	271.9	128.6
						avg	1.04	0.75	1.00	**0.71**	avg	18906.6	15405.3	18347.0	**13906.9**
						max	2.01	1.86	1.62	1.68	max	36000.0	36000.0	36000.0	36000.0
	2	28/8	**29/7**	27/9	**29/7**	min	0.35	0.30	0.34	0.25	min	85.8	141.2	99.2	81.8
						avg	0.71	0.69	0.70	**0.58**	avg	12726.3	10082.5	11964.4	**9588.3**
						max	1.33	1.49	1.63	1.18	max	36000.0	36000.0	36000.0	36000.0
	All	74/34	80/28	80/28	**82/26**	avg	0.91	0.77	0.80	**0.71**	avg	17114.1	14121.0	15353.62	**13050.0**
2	0	20/16	23/13	25/11	**30/6**	min	0.49	0.13	0.18	0.32	min	107.9	44.9	106.4	58.9
						avg	0.81	**0.36**	0.69	0.95	avg	22728.3	18381.2	20349.7	**10935.9**
						max	1.91	1.00	1.50	1.49	max	36000.0	36000.0	36000.0	36000.0
	1	29/7	**30/6**	29/7	**30/6**	min	0.31	0.11	0.38	0.33	min	88.5	108.3	221.2	148.3
						avg	0.98	**0.71**	0.96	0.89	avg	**9730.2**	9812.5	11938.1	10557.2
						max	1.41	1.34	1.25	1.26	max	36000.0	36000.0	36000.0	36000.0
	2	**31/5**	**31/5**	29/7	29/7	min	0.61	0.46	0.34	0.31	min	79.5	113.0	176.2	145.2
						avg	0.90	**0.75**	0.97	0.93	avg	**7850.1**	8660.3	10832.4	10520.8
						max	1.15	0.94	1.33	1.26	max	36000.0	36000.0	36000.0	36000.0
	All	80/28	84/24	83/25	**89/19**	avg	0.87	**0.53**	0.85	0.92	avg	13436.2	12284.7	14373.4	**10671.3**
3	0	10/26	11/25	**15/21**	14/22	min	0.11	0.14	0.07	0.16	min	179.6	531.2	342.5	441.4
						avg	1.06	1.12	0.94	**0.63**	avg	29115.7	28968.7	**25655.0**	26644.3
						max	4.30	4.11	2.44	1.77	max	36000.0	36000.0	36000.0	36000.0
	1	9/27	9/27	13/23	**14/22**	min	0.12	0.13	0.26	0.09	min	4358.8	3407.1	2111.5	3602.8
						avg	1.39	1.28	1.45	**1.26**	avg	32367.4	30781.2	**28719.5**	28807.6
						max	3.13	3.01	3.00	2.66	max	36000.0	36000.0	36000.0	36000.0
	2	25/11	28/8	27/9	**29/7**	min	0.17	0.11	0.15	0.30	min	494.1	372.3	167.8	508.3
						avg	1.21	1.28	**1.01**	1.30	avg	18231.2	17135.2	18481.0	**16100.8**
						max	2.53	3.72	2.03	2.10	max	36000.0	36000.0	36000.0	36000.0
	All	44/64	48/60	55/53	**57/51**	avg	1.23	1.21	1.17	**1.00**	avg	26571.4	25628.4	24285.2	**23850.9**

Table 7: Summary of the results obtained under different demand scenarios and varying values for the maximum delivery delay; *instances not solved to optimality within 10 hours

with model $(\mathbf{P}_2^+)$, mainly due to the fact that fewer instances reach the time limit when this formulation is chosen.

The maximum allowed delay in demand fulfillment has a significant impact on the performance of the four formulations. In particular, more challenging instances are obtained when all customer demands must be satisfied without incurring lateness. This is reflected in the larger number of instances not solved to proven optimality when $\rho = 0$ is imposed, compared to instances with a positive delivery delay. Increasing the number of periods for satisfying the demand of a customer results in more flexibility for redesigning the facility network, independently of the demand scenario considered. This aspect will be analyzed in more detail in Section 5.3.

The results shown in Table 7 clearly indicate that instances with inverted trapezoidal demand rates (scenario 3) are the most difficult to solve within the pre-specified time limit. This is possibly explained by the need to make more adjustments in the system configuration when this demand pattern occurs compared to other demand classes. To illustrate this characteristic, we selected two instances, one from scenario 2 and one from scenario 3, for which the corresponding optimal solutions are available. Table 8 presents the optimal schedules of facility openings, facility closures, and capacity adjustments over six strategic periods.

No. of facilities	Strategic time periods											
	Trapezoidal demand rate (scenario 2)						Inverted trapezoidal demand rate (scenario 3)					
	1	7	13	19	25	31	1	7	13	19	25	31
open	3	4		1			5				2	3
closed		1			1	1		2	1			
expanded (new)												1
expanded (existing)	2						2					
contracted (new)						3			1			
contracted (existing)								1				

Table 8: Optimal location and capacity scalability decisions in two instances with $|J^0| = 150$, $|J^1| = \emptyset$, $|I^e| = 3$, $|I^n| = 12$, and $|T_L| = 6$

In both instances, at the beginning of the time horizon, the capacity available at the three existing facilities is insufficient to meet the initial demand requirements. As a result, additional capacity is installed through opening new facilities and expanding the capacity at two existing locations. While in scenario 2 demand continues to grow until the end of time period 12, and thus four new facilities are established, in scenario 3 the opposite occurs. This leads to closing two existing facilities and reducing the capacity of an existing location in time period 7. From periods 13 to 24, demand rates are relatively regular in both instances. In the instance belonging to scenario 3 this leads to a further reduction of the overall capacity in period 13. In scenario 2, the facility network is also subjected to this type of capacity adjustment but later in periods 25 and 31, to respond to the decrease in demand in the last phase of the time horizon. To this end, two of the initially existing facilities are closed and three of the new facilities have their capacities reduced. In contrast, the last twelve periods coincide with a demand recovery phase in scenario 3. In this case, five new facilities are opened and the capacity of a previously established facility is expanded. In total, 16 (18) facility adjustments take place in scenario 2 (scenario 3). It seems that the more configuration changes are required, the more challenging the problem becomes for CPLEX.

The development of formulation $(\mathbf{P}_2)$ was motivated by the modeling framework proposed by Jena et al. [19]. For special variants of the MFLP, including those involving capacity expansion and reduction, Jena et al. [19] proved that their formulation provides stronger linear relaxation bounds than other existing models. We are now interested in analyzing if the same dominance relationship also exists between our formulations $(\mathbf{P}_1)$ and $(\mathbf{P}_2)$ (as well as between $(\mathbf{P}_1^+)$ and $(\mathbf{P}_2^+)$). Table 9 reports the LP gaps obtained for each demand scenario and different values of ρ in columns 4-7. The LP gap is defined as the relative percentage deviation between the LP-relaxation bound (z^{LP}) and the objective value of the best feasible solution available (z^{UB}), and is determined by $(z^{UB} - z^{LP})/z^{UB} \times 100\%$. Columns 9-12 give the minimum, average and maximum computing times (in seconds) required by the LP-relaxations of all formulations (LP CPU). Once again, the best average results with respect to the evaluation criteria are highlighted in boldface.

As can be seen in Table 9, no dominance relationship can be established for our MFLP with delayed demand satisfaction. Formulation $(\mathbf{P}_1^+)$ provides the best LP-relaxation bound for instances with irregular and trapezoidal demand patterns (scenarios 1 and 2), but it is outperformed by model $(\mathbf{P}_2^+)$ in the third class of instances. Table 9 further reveals that the LP bounds are always improved with the model enhancements. Moreover, their magnitude is insensitive to the choice of demand scenario and the length of the delivery delay. An additional important characteristic of all formulations is that their linear relaxations provide tight lower bounds. Hence, the LP-bound could be used to assess the quality of a feasible solution obtained, for example, by a tailored heuristic algorithm. This would be particularly important for large scale instances, when a near-optimal solution is not achieved within a reasonable time using a state-of-the-art optimization solver. Regarding the computational effort needed to solve the linear relaxation, it seems that instances with irregular demand patterns have slightly higher

Demand scenario	ρ		LP gap (%)					LP CPU (sec.)			
			$(\mathbf{P}_1)$	$(\mathbf{P}_1^+)$	$(\mathbf{P}_2)$	$(\mathbf{P}_2^+)$		$(\mathbf{P}_1)$	$(\mathbf{P}_1^+)$	$(\mathbf{P}_2)$	$(\mathbf{P}_2^+)$
1	0	min	1.84	1.83	2.04	1.89	min	0.5	0.9	1.1	2.3
		avg	2.96	**2.77**	3.21	3.05	avg	**6.9**	18.8	16.5	26.3
		max	4.75	4.24	4.94	4.63	max	26.2	79.4	54.7	56.9
	1	min	1.11	0.97	1.05	0.80	min	4.8	8.3	5.1	10.4
		avg	2.65	**2.14**	2.68	2.16	avg	33.2	45.7	**30.5**	46.2
		max	4.31	3.84	5.46	4.40	max	108.1	168.3	70.5	174.2
	2	min	1.27	0.98	0.96	0.69	min	9.3	12.0	6.3	9.7
		avg	2.21	1.81	2.20	**1.80**	avg	48.9	70.2	**45.3**	60.6
		max	3.74	2.77	3.70	3.30	max	126.4	277.4	133.6	207.2
	All	avg	2.61	**2.24**	2.70	2.34	avg	**29.7**	44.9	30.8	44.4
2	0	min	1.66	1.32	1.23	1.10	min	0.7	0.8	1.7	0.9
		avg	2.71	**2.07**	2.86	2.44	avg	**6.3**	9.7	15.3	21.3
		max	4.12	2.87	4.58	3.73	max	24.1	37.4	36.1	63.4
	1	min	0.76	0.71	0.91	0.91	min	6.4	7.4	5.4	8.9
		avg	1.97	**1.73**	2.64	2.46	avg	**27.8**	41.3	28.9	40.1
		max	3.83	3.79	4.64	4.35	max	110.7	111.7	68.7	103.5
	2	min	0.75	0.74	1.00	1.00	min	7.5	3.2	6.3	11.0
		avg	1.90	**1.79**	2.60	2.54	avg	48.8	64.6	**47.6**	69.0
		max	3.93	3.84	4.57	4.40	max	129.5	188.8	123.1	172.2
	All	avg	2.19	**1.86**	2.70	2.48	avg	**27.6**	38.5	30.6	43.5
3	0	min	1.26	1.15	0.81	0.81	min	0.6	0.8	0.8	6.8
		avg	3.21	3.09	2.91	**2.90**	avg	**7.3**	24.4	15.2	26.5
		max	4.79	4.66	4.37	4.59	max	22.2	119.6	41.7	63.0
	1	min	1.86	1.36	1.42	1.20	min	0.8	3.7	3.1	5.8
		avg	3.56	3.14	3.20	**2.87**	avg	**26.6**	37.3	27.1	33.0
		max	5.62	5.49	5.41	5.46	max	89.2	100.4	79.9	98.4
	2	min	0.88	0.77	0.87	0.77	min	5.7	3.8	3.4	3.5
		avg	2.69	2.35	2.29	**2.07**	avg	**41.6**	47.8	42.7	41.9
		max	5.18	5.07	4.79	4.68	max	124.0	119.9	116.6	123.4
	All	avg	3.15	2.86	2.80	**2.61**	avg	**25.2**	36.5	28.3	33.8

Table 9: Performance of the linear relaxation under different demand scenarios and varying values for the maximum delivery delay

time requirements than instances with trapezoidal demand scenarios. Nevertheless, the range of CPU times obtained is not significant given the strategic nature of the problem.

Even though formulation $(\mathbf{P}_2^+)$ did not always provide the best MIP gap across all demand classes, it seems to outperform formulation $(\mathbf{P}_1^+)$ since a larger number of optimal solutions were obtained and less computing time was required on average (recall Table 7). However, the differences between these two enhanced models are not so significant as it may appear at first sight. Figure 1 illustrates the percentage of best solutions that were identified by CPLEX within the specified time limit. In scenarios 1 and 2, the same high quality solutions are achieved with both models in most instances. The superiority of formulation $(\mathbf{P}_2^+)$ is only evident for some instances in scenario 3, namely those with $\rho \in \{0, 1\}$.

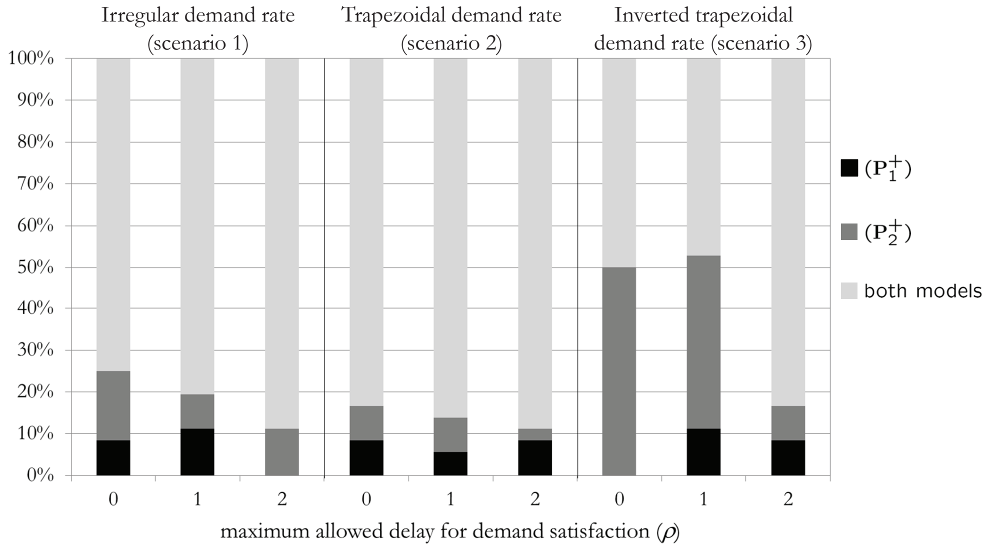

Figure 1: Percentage of instances with identified best solutions

5.3 Managerial implications

The aim of this section is to gain a broader insight into the characteristics of the best solutions identified by CPLEX. The analysis will help decision-makers to better understand the impact of permitting delays in demand fulfillment and the trade-offs derived from location and capacity scalability decisions.

Figure 2 depicts the relative average cost savings that are achieved in each demand scenario by taking $\rho > 0$ as opposed to $\rho = 0$. In each demand class, the average total cost was determined by considering the objective value of the best feasible solution available for each instance in that class. As expected, the total cost of redesigning the facility system decreases as the number of time periods for satisfying the demand of customer segment J^1 increases. This is explained by the trade-off between the high tardiness cost incurred by delayed deliveries and the lower requirement for location and capacity investments. Since CPLEX identified the least number of optimal solutions in instances belonging to scenario 3, the actual relative cost savings may be lower in this class than the values displayed in Figure 2. Interestingly, setting $\rho = 2$ does not have a significant impact on the total cost benefit compared to $\rho = 1$ in scenarios 1 and 2. Further insights on this characteristic are provided in Table 10, which reports the relative contribution of different cost categories to the average total cost in each demand class and for different values of parameter ρ. Relative fixed costs are given separately

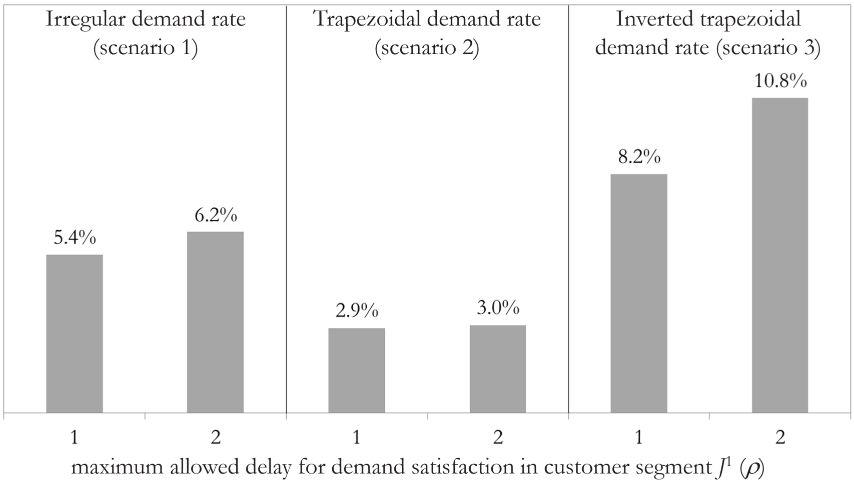

Figure 2: Average cost savings compared to base case (no delivery delay)

for new facilities (columns 4-6) and initially existing locations (columns 7-10). Moreover, the relative contribution of the variable costs is also presented in columns 11-13. The last column indicates the relative average amount of demand that is delivered with delay to customers in segment J^1.

When late deliveries are not allowed, the overall capacity available in a time period must be sufficient to cover all customer orders occurring in that particular period. This results in a larger relative spending on establishing new facilities and consequently, the relative cost of operating the new facilities also increases. At the same time, the higher facility investment is partly abated by a lower distribution cost since service is provided from more locations. This is a typical trade-off that is made when a facility network is redesigned.

In the context of delivery delays (i.e. $\rho > 0$), more emphasis is given to adjusting the capacity of operating facilities, especially at existing locations, rather than opening new facilities. This strategy results in higher maintenance costs at existing facilities due to the expansion of their capacities. Furthermore, the relative importance of the distribution cost also increases as expected, since fewer locations are operational. Interestingly, expenditures incurred by facility closures and capacity contractions are not significant. The total tardiness penalty cost also has a small relative contribution to the overall cost (0.4-1.2%). Naturally, when the delivery lead time increases for customers in segment J^1, also more demand is satisfied with some delay.

From a managerial perspective, information on the capacity utilization levels of operational facilities is important since it gives insight on the overall slack capacity available. For each de-

Demand scenario	ρ	% of total cost											% delayed demand*
		New facilities				Initially existing facilities							
		Opening cost	Maint. cost	Expansion cost	Contraction cost	Closing cost	Maint. cost	Expansion cost	Contraction cost	Processing cost	Distribution cost	Tardiness penalty cost	
1	0	4.1	25.6	-	0.1	0.3	8.7	0.5	-	24.9	35.7	-	-
	1	3.5	21.3	-	0.1	0.3	9.3	0.6	-	26.2	38.2	0.6	21.0
	2	3.2	19.9	-	-	0.3	9.9	0.6	-	26.4	38.6	1.0	30.8
2	0	3.1	21.0	-	0.1	0.2	8.2	0.4	-	21.7	45.2	-	-
	1	3.0	18.4	-	0.1	0.3	7.7	0.5	-	22.3	47.1	0.5	15.0
	2	3.0	18.1	-	0.1	0.3	7.8	0.6	-	22.3	47.2	0.6	18.4
3	0	7.0	39.2	2.0	0.5	0.3	5.6	0.6	-	21.7	23.0	-	-
	1	7.0	34.4	1.4	0.3	0.3	6.4	0.8	0.1	23.7	25.3	0.4	19.8
	2	7.1	31.8	1.0	0.2	0.3	6.9	0.9	0.1	24.3	26.1	1.2	32.4

Table 10: Average cost rates under different demand scenarios and varying values for the maximum delivery delay; *average over demand of customer segment J^1

mand scenario, columns 3-8 in Table 11 indicate the minimum, average and maximum capacity utilization rates at new facilities and existing locations that are associated with a specific value for the maximum allowed delivery delay. In addition, Table 11 also presents the mean number of new facilities (column 9), the mean number of closed facilities (column 10), and the mean number of facilities that undergo capacity adjustments over the time horizon (columns 11-14). The impact of a positive delivery lead time is reflected in a higher mean capacity usage. In all demand classes, this situation follows from fewer facilities being opened and more operational facilities having their capacity adjusted, either through expansion or contraction. However, when predicted variations in demand are more pronounced, as is the case in scenarios 2 and 3, it becomes increasingly more difficult to maintain a very high capacity utilization rate.

The highest level of slack capacity is achieved for $\rho = 0$. This is the price that must be paid for not permitting lateness in demand fulfillment, especially when large demand fluctuations occur over time (scenarios 2 and 3). Finally, Table 11 shows that redesigning a facility network under inverted trapezoidal demand rates (scenario 3) calls for more changes in the choice of facility locations and in their capacities in the course of the planning horizon. The examples previously presented in Table 8 already indicated this feature, which also yields more challenging instances.

6 Conclusions

In this article, we studied a problem that integrates three important network design decisions over a finite planning horizon: the location of facilities, the adjustment of capacity at existing sites and new locations, and the distribution of a product from operating facilities to customers taking into account the sensitivity of each individual customer to delivery lead times. Moreover, different time scales for strategic decisions (i.e. location and capacity scalability) and tactical decisions (i.e. demand allocation) were also considered. We proposed two MILP formulations for this new and important variant of the MFLP and developed additional inequalities to strengthen their linear relaxations. Computational testing on randomly generated instances using CPLEX indicated that the performance of all formulations is impacted by the shape of the demand distribution and the extent to which predicted demand variations occur over time. The performance of one of the enhanced models seems to be slightly superior to the other models since optimal solutions could be identified for more test instances and less computing time was required. It is noteworthy, however, that when optimality could not be proven, a small MIP gap

Demand scenario	ρ	Capacity usage (%)						No. of facilities		No. of facilities with capacity adjustments			
		new facilities			existing facilities			new	closed	new		existing	
		min	avg	max	min	avg	max			expansion	contraction	expansion	contraction
1	0	64.2	76.7	84.0	61.7	77.2	88.9	5.6	2.1	-	2.5	1.6	1.0
	1	78.2	89.4	97.4	61.9	86.2	99.6	4.5	1.9	-	1.4	1.5	1.1
	2	82.2	91.7	98.2	64.6	91.1	100.0	4.1	1.8	-	1.1	1.5	1.0
2	0	63.8	75.1	83.9	49.7	64.2	79.3	6.0	1.8	-	2.4	1.7	1.1
	1	76.2	85.3	93.2	75.8	79.5	87.0	5.5	2.4	-	2.3	1.8	1.0
	2	76.5	86.1	93.3	75.7	75.7	75.7	5.4	2.5	-	2.2	1.9	1.0
3	0	41.0	55.4	80.1	25.1	52.1	91.9	9.2	2.2	3.7	4.9	1.5	1.6
	1	44.4	66.4	92.9	34.3	64.1	93.8	8.6	2.3	2.8	3.0	1.7	2.0
	2	51.2	71.8	96.0	44.1	67.3	94.8	8.4	2.2	2.2	2.4	1.8	1.8

Table 11: Average capacity utilization rates and average number of facilities undergoing capacity adjustments

was obtained with *all* formulations. Our empirical study also indicated that cost savings and higher capacity utilization levels are achieved when delays in demand satisfaction are permitted. In this case, gradual changes in the network configuration are mainly determined by dynamically adjusting the capacities of operating facilities. The flexibility provided by positive delivery lead times to redesign a facility network is also reflected in lower computational effort.

A future research venue would be to examine our problem within a stochastic environment in order to explicitly account for the inherently uncertain nature of some parameters such as future customer demands and costs. In this context, the expected value of perfect information (EVPI) is an important measure of the gain obtained if complete and accurate information on the future were available at the moment decisions are made. To obtain the EVPI it is required to solve the deterministic counterpart for a finite set of scenarios. In that respect, our study makes an important contribution towards developing a deeper understanding of the deterministic problem.

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Appendix: Data generation

Test instances were generated using a random procedure based on that proposed by Correia and Melo [8] for a particular case of the problem that we study. In what follows, we denote by $U[a,\, b]$ the generation of random numbers over the range $[a,\, b]$ according to a continuous uniform distribution. The generation of random integer values in the same interval is denoted by $I[a,\, b]$. Table 12 describes how the capacity and demand parameters were obtained.

The generation of the variable costs relies on two random real numbers, β_1 and β_2, in the range 1.01 to 1.03 (details are given next).

Parameter	Value	
ρ_j	1, 2	$j \in J^1$
n_i	5	$i \in I$
q_i	$I[1,\, n_i]$	$i \in I^e$
Q	$\dfrac{1}{n_i\,\lvert I\rvert}\, U[2,\,3]\, \dfrac{\sum_{j\in J}\sum_{t\in T} d_j^t}{\lvert T\rvert}$	$i \in I^n$

Table 12: Selected parameters

- For $i \in I$ and $j \in J$, the variable distribution costs are set according to

$$
\begin{aligned}
c_{ij}^1 &= U[5,\,10] & & \\
c_{ij}^t &= c_{ij}^1 & & t = 2,\ldots,12 \\
c_{ij}^t &= \beta_1\, c_{ij}^{12} & & t = 13,\ldots,24 \\
c_{ij}^t &= \beta_2\, c_{ij}^{24} & & t = 25,\ldots,36.
\end{aligned}
$$

  It is assumed that the distribution costs are constant over one year (12 time periods) but they increase between 1% and 3% from one year to the next.

- The variable processing costs at the facilities are generated in order to reflect economies of scale by considering the number of modular units that are available at a particular location. Hence, the larger the capacity size, the lower the corresponding processing cost per unit of product. For $i \in I$, we set

$$
\begin{aligned}
o_{i1}^1 &= 100\,/\,\sqrt{Q} & & t = 1,\ldots,12 \\
o_{ik}^t &= 0.9\, o_{ik-1}^t & & t = 1,\ldots,12;\ k = 2,\ldots,n_i \\
o_{ik}^t &= \beta_1\, o_{ik}^{12} & & t = 13,\ldots,24;\ k = 1,\ldots,n_i \\
o_{ik}^t &= \beta_2\, o_{ik}^{24} & & t = 25,\ldots,36;\ k = 1,\ldots,n_i.
\end{aligned}
$$

  Cost fluctuations follow a pattern similar to that of the variable distribution costs.

We now describe how the fixed cost parameters were obtained.

- The fixed costs of opening new facilities at candidate sites $i \in I^n$ reflect economies of scale by taking into account the number of modular units with which the facilities are opened. In the first time period, these costs are set according to

$$
FO_{ik}^1 = \alpha_i + \gamma_i\,\sqrt{k\,Q} \qquad \alpha_i \in [500,\ 1000];\ \gamma_i \in [4000,\ 6000];\ k = 1,\ldots,n_i.
$$

In the remaining strategic periods $\ell \in T_L \setminus \{1\}$, we take $FO_{ik}^{\ell} = \beta_{|T_L|}\, FO_{ik}^{\ell-1}$ for every $k = 1, \ldots, n_i$. For $|T_L| = 3$, we set $\beta_{|T_L|} = U[1.01,\ 1.03]$ and for $|T_L| = 6$, we consider $\beta_{|T_L|} = U[\sqrt{1.01},\ \sqrt{1.03}\,]$.

- By taking $\gamma_i \in [4000,\ 6000]$, the fixed capacity expansion and contraction costs are determined as follows.

$$\begin{aligned} FE_{ik}^{1} &= \gamma_i \sqrt{k\,Q} && i \in I;\ k = 1, \ldots, n_i - 1 \\ FR_{ik}^{1} &= 0.2\, FE_{ik}^{1} && i \in I;\ k = 1, \ldots, n_i - 1. \end{aligned}$$

  Although capacity expansion or contraction is not permitted at a candidate site in the first strategic period, we generate the corresponding fixed costs for this period since they are required to obtain the costs in the following period. For all periods $\ell \in T_L \setminus \{1\}$, we use the same procedure as described above for the generation of the fixed opening costs.

- Fixed closing costs at existing facilities correspond to 20% of the opening costs:

$$FC_{ik}^{\ell} = 0.2\, FO_{ik}^{\ell} \qquad i \in I^e;\ k = 1, \ldots, n_i;\ \ell \in T_L \setminus \{1\}.$$

- Fixed facility maintenance costs correspond to 20% of the associated opening costs.

$$\begin{aligned} MC_{ik}^{t} &= 0.2\, FO_{ik}^{\ell} && i \in I;\ k = 1, \ldots, n_i;\ \ell \in T_L; \\ & && t = \ell, \ldots, t';\ t' \in T : t' < \ell + 1. \end{aligned}$$

Finally, we adapt the procedure developed by Correia and Melo [8] to select the tardiness penalty costs for orders delivered with delay to customers $j \in J^1$. These variable cost parameters result from combining the average maintenance, distribution, and processing costs in the following way:

$$p_j^{tt'} = 0.1\, \theta_j^t\, (t' - t)^2 \qquad t \in T;\ t' = t, \ldots, \min\{t + \rho_j,\ |T|\} \tag{59}$$

with

$$\theta_j^t = \frac{\sum_{i \in I} \sum_{k=1}^{n_i} M_{ik}^t}{TD_t\, |I| \sum_{i \in I} n_i} + \frac{\sum_{i \in I} c_{ij}^t}{|I|} + \frac{\sum_{i \in I} \sum_{k=1}^{n_i} o_{ik}^t}{|I| \sum_{i \in I} n_i}$$

and TD_t denoting the total quantity demanded in period t, that is, $TD_t = \sum_{j \in J} d_j^t$.

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