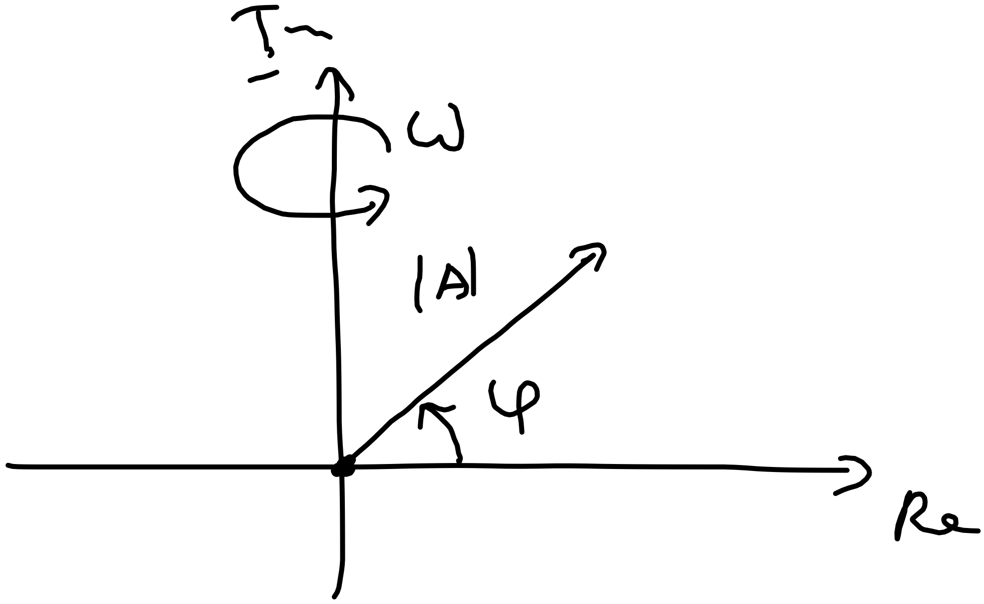
$$y(t) = A \sin(\omega t + \varphi)$$

$$z(t) = \underline{y}(t) = A e^{i(\omega t + \varphi)}$$

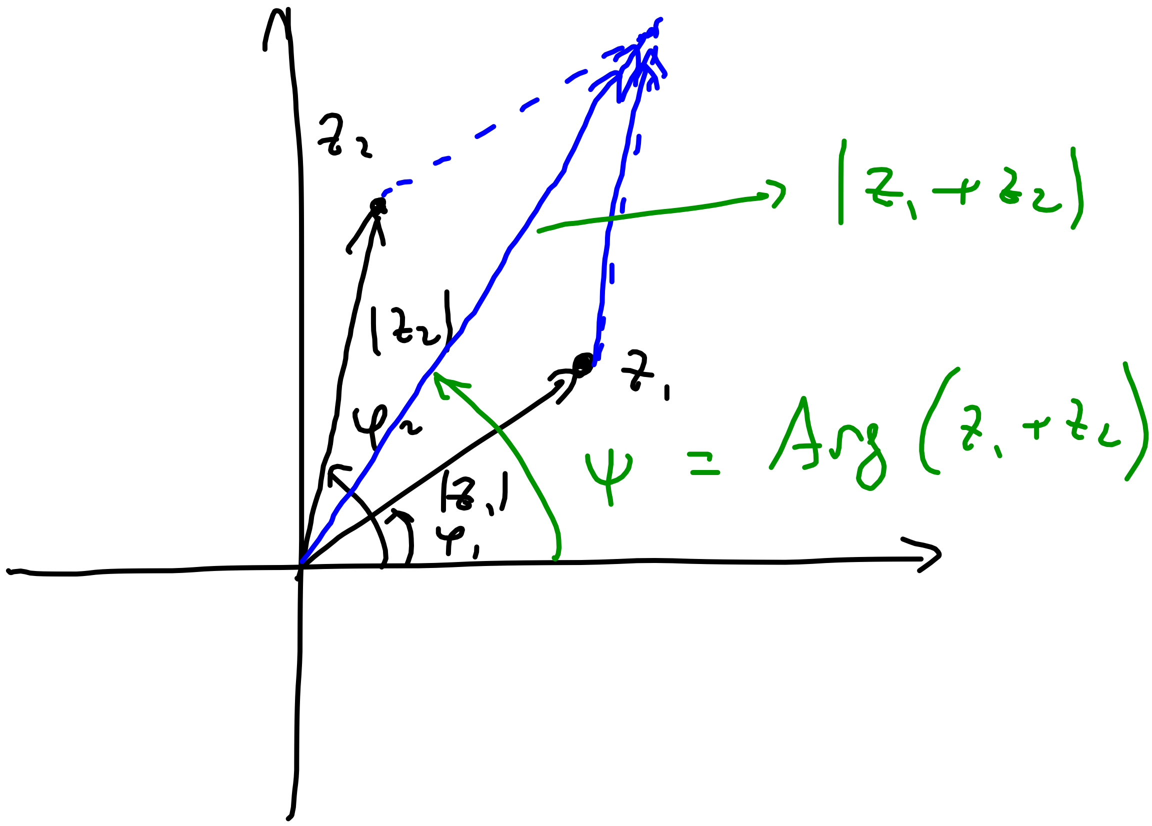
$$z(t) : \mathbb{R} \longrightarrow \mathbb{C}$$

$$A e^{i(\omega t + \varphi)} = \underbrace{A e^{i\varphi}}_{\text{Komplexe Ampl.}} \cdot e^{i\omega t}$$



$$\left| A e^{i\varphi} \right| = |A|$$

$$\operatorname{Arg}\left(A e^{i\varphi}\right) = \varphi$$


z_2
$|z_2|$
φ_2
z_1
$|z_1|$
φ_1
$|z_1 + z_2|$
$\psi = \mathrm{Arg}(z_1 + z_2)$

1.5. ~~Der~~ Addition / Überlagerung / Superposition gleichfrequenter Schwingungen

$$y_1(t) = A_1 \sin(\omega t + \varphi_1)$$

$$y_2(t) = A_2 \sin(\omega t + \varphi_2)$$

$$y(t) = y_1(t) + y_2(t) = ?$$

$$? = A \sin \ldots ?$$

<u>Satz:</u> Sei gegeben:

$$y_1(t) = A_1 \sin(\omega t + \varphi_1)$$

$$y_2(t) = A_2 \sin(\omega t + \varphi_2)$$

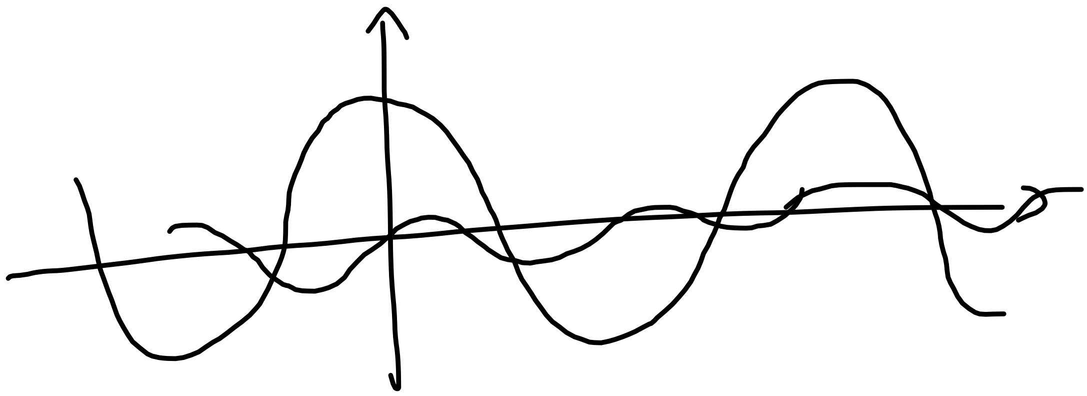
Dann gilt:

$$y(t) = y_1(t) + y_2(t) =$$

$$= A \sin(\omega t + \varphi)$$

<u>Gesucht</u>: A, φ

Im reellen Bereich:



sehr SCHWER!

$\downarrow$

$\mathbb{C}$

(1) $y_1(t) = A_1 \sin(\omega t + \varphi_1) =$

$= \operatorname{Im}(\underline{y}_1(t)) =$

$= \operatorname{Im}\left(A_1 e^{i(\omega t + \varphi_1)}\right)$

(2) $z_1 = a + ib$
$+ \; z_2 = c + id$

$$z = z_1 + z_2 = a + ib + c + id =$$

$$= (a+c) + i(b+d) =$$

$$= \underbrace{(\mathrm{Re}\, z_1 + \mathrm{Re}\, z_2)}_{\mathrm{Re}(z_1+z_2)} + i\underbrace{(\mathrm{Im}\, z_1 + \mathrm{Im}\, z_2)}_{\mathrm{Im}(z_1+z_2)}$$

$$\mathrm{Re}(z_1+z_2) = \mathrm{Re}\, z_1 + \mathrm{Re}\, z_2$$

$$\boxed{\mathrm{Im}(z_1+z_2) = \mathrm{Im}\, z_1 + \mathrm{Im}\, z_2}$$

Also:

$$y_1(t) = A_1 \sin(\omega t + \varphi_1) \xrightarrow{T} \underline{y_1}(t)$$

$$+$$

$$y_2(t) = A_2 \sin(\omega t + \varphi_2) \xrightarrow{T} \underline{y_2}(t)$$

$$\underline{y_1}(t) + \underline{y_2}(t)$$

$$y_1(t) + y_2(t) \xleftarrow{T^{-1}} \underline{y_1}(t) + \underline{y_2}(t)$$

$$\underline{y_1}(t) = z_1(t) =$$

$$= A_1 e^{i(\omega t + \varphi_1)} =$$

$$= A_1 e^{i\varphi_1} \cdot e^{i\omega t}$$

$$\underline{y_2}(t) = z_2(t) = A_2 e^{i\varphi_2} \cdot e^{i\omega t}$$

$$z(t) = \underline{y_1}(t) + \underline{y_2}(t) =$$

$$= A_1 e^{i\varphi_1} \cdot e^{i\omega t} + A_2 e^{i\varphi_2} \cdot e^{i\omega t} =$$

$$= e^{i\omega t} \underbrace{\left(A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2} \right)}_{\text{komplexe Amplitude}}$$

wieder eine kompl. Schwingung

Die Frequenz hat sich nicht geändert: ω

$\Longrightarrow$ Wir müssen die beiden kompl. Startwerte addieren!

$$A_1 e^{i\varphi_1} + A_2 e^{i\varphi_2} =$$

Transf. $EF \xrightarrow{TF} NF$

$$= \left(A_1 \cos\varphi_1 + i A_1 \sin\varphi_1\right) +$$

$$+ \left(A_2 \cos\varphi_2 + i A_2 \sin\varphi_2\right) =$$

$$= \overbrace{\left(A_1 \cos\varphi_1 + A_2 \cos\varphi_2\right)}^{Re =: a} +$$

$$+ i \underbrace{\left(A_1 \sin\varphi_1 + A_2 \sin\varphi_2\right)}_{Im =: b} =$$

$$= a + ib$$

NF

$\longrightarrow$ EF

$$= A\, e^{i\varphi} = \underline{y}(0) = z(0)$$

$$A = |z(0)| =$$

$$= \sqrt{a^2 + b^2} =$$

$$= \sqrt{(A_1 \cos\varphi_1 + A_2 \cos\varphi_2)^2 + (A_1 \sin\varphi_1 + A_2 \sin\varphi_2)^2}$$

$\varphi = \arc\ldots$

Quadrant!

Ergebnis:

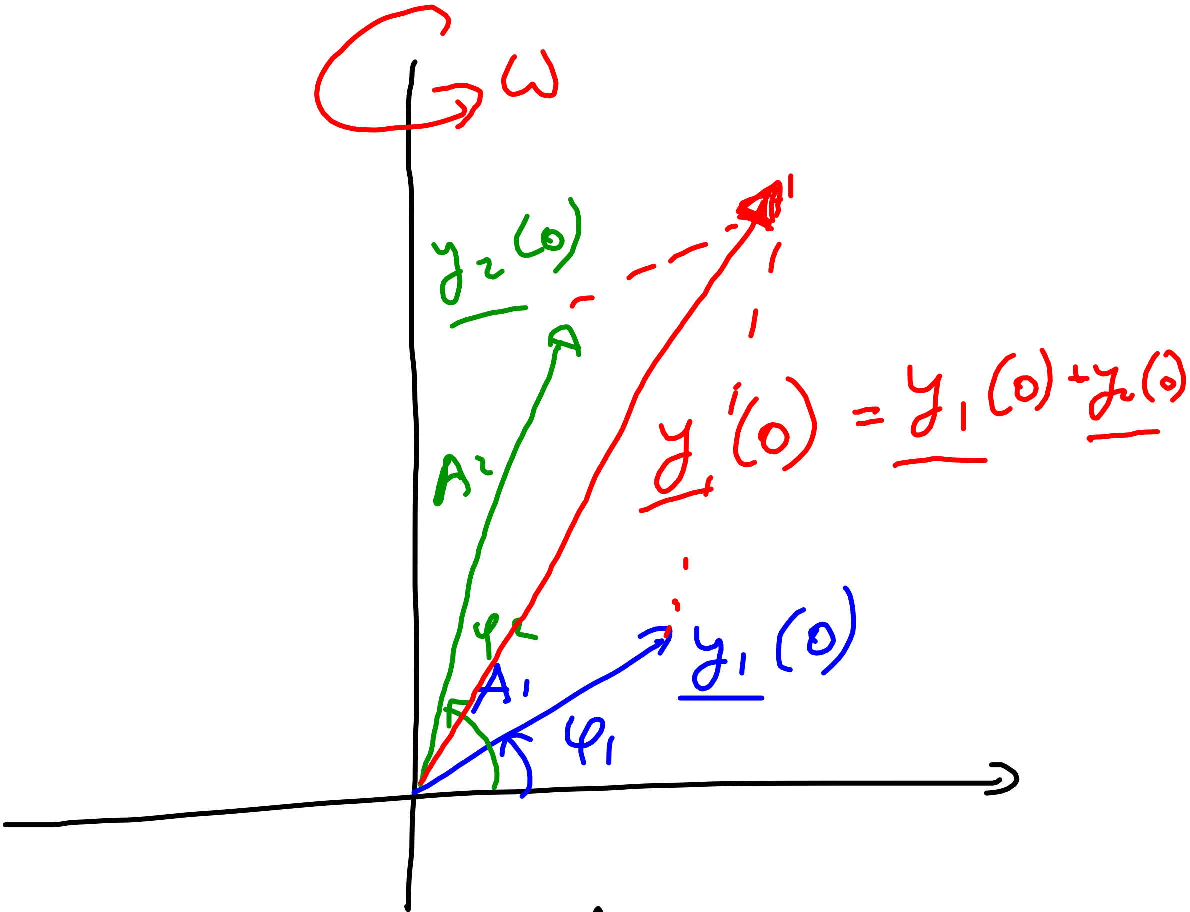
$$\underline{y_1}(t) + \underline{y_2}(t) = A e^{i\varphi} \cdot e^{i\omega t} =$$

$$= A e^{i(\omega t + \varphi)}$$

$$\Big\downarrow T^{-1}$$

$$y_1(t) + y_2(t) = A \sin(\omega t + \varphi)$$

Geometrische Bedeutung:



Vektoraddition der komplexen Zeiger

Bsp!:

$$y_1(t) = 2 \sin(50t + \pi/4)$$

$$y_2(t) = 5 \sin\left(50t + \frac{3\pi}{4}\right)$$

$$y = y_1 + y_2 ?$$

$$y_1(t) \longrightarrow \underline{y_1}(t) = z_1(t):$$

$$z_1(t) = 2\, e^{i(50t + \pi/4)}$$

Analog: $y_2 \longrightarrow z_2$

$$z_2(t) = 5\, e^{i\left(50t + \frac{3\pi}{4}\right)}$$

$$z_1(t) + z_2(t) =$$

$$= 2e^{i(50t + \pi/4)} + 5e^{i(50t + 3\pi/4)} =$$

$$= 2e^{i50t} \cdot e^{i\pi/4} + 5e^{i50t} \cdot e^{i3\pi/4} =$$

$$= e^{i50t} \underbrace{\left(2e^{i\pi/4} + 5e^{i3\pi/4}\right)}_{\in \mathbb{C}}$$

$$2e^{i\pi/4} + 5e^{i3\pi/4}$$

$$2e^{i\pi/4} = 2\left(\cos\pi/4 + i\sin\pi/4\right)$$

$$= 2\left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) =$$

$$= \sqrt{2} + i\sqrt{2}$$

$$5e^{i\frac{3\pi}{4}} = 5\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$$

$$= 5\left(\frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right) = \frac{-5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i$$

Also: $2e^{i\pi/4} + 5e^{i3\pi/4} =$

$$= \left(\sqrt{2} + i\sqrt{2}\right) + \left(\frac{-5}{\sqrt{2}} + \frac{5}{\sqrt{2}}i\right) =$$

$$= \left(\frac{2}{\sqrt{2}} - \frac{5}{\sqrt{2}}\right) + i\left(\frac{2}{\sqrt{2}} + \frac{5}{\sqrt{2}}\right) =$$

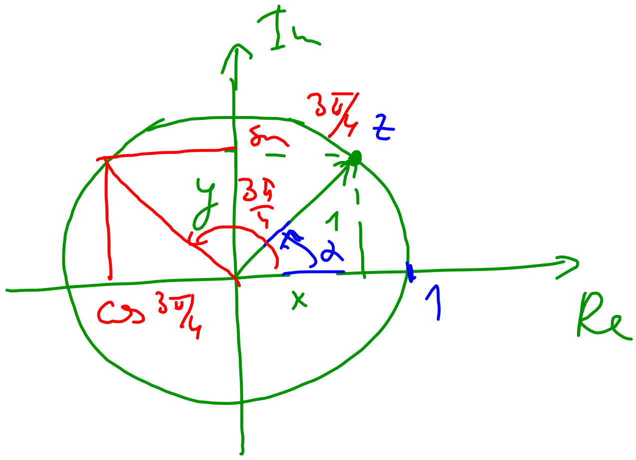
$$= -\frac{3}{\sqrt{2}} + i\frac{7}{\sqrt{2}} \quad \text{NF!}$$

$\longrightarrow$ EF

$|\cdot|$, Winkel:

$$\left|-\frac{3}{\sqrt{2}} + i\frac{7}{\sqrt{2}}\right| = \sqrt{\left(-\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{7}{\sqrt{2}}\right)^2} =$$

$$= \sqrt{\frac{9}{2} + \frac{49}{2}} = \sqrt{29}$$



$|z| = 1$

$Arg = \alpha$

$Re\, z = |z| \cos\alpha = \cos\alpha$

$Im\, z = |z| \sin\alpha = \sin\alpha$

$\Rightarrow \cos^2\alpha + \sin^2\alpha = 1$

Winkel:

$Arg\left(-\frac{3}{\sqrt{2}} + i\frac{7}{\sqrt{2}}\right)$

$\arctan \frac{Im}{Re} = \arctan \frac{7/\sqrt{2}}{-3/\sqrt{2}} =$

$= \arctan\left(-7/3\right) =$

$= -1,166$ Rad

Die Zahl liegt im II. Quadrant:

$Arg\left(-\frac{3}{\sqrt{2}} + i\frac{7}{\sqrt{2}}\right) = \pi - 1,166$

$\Rightarrow$ Komplexe Amplitude

$$\left(\sqrt{29}\cdot e^{i(\pi - 1,166)}\right)^{i3t}$$

$\Rightarrow$ Resultierende komplexe Schwingung:

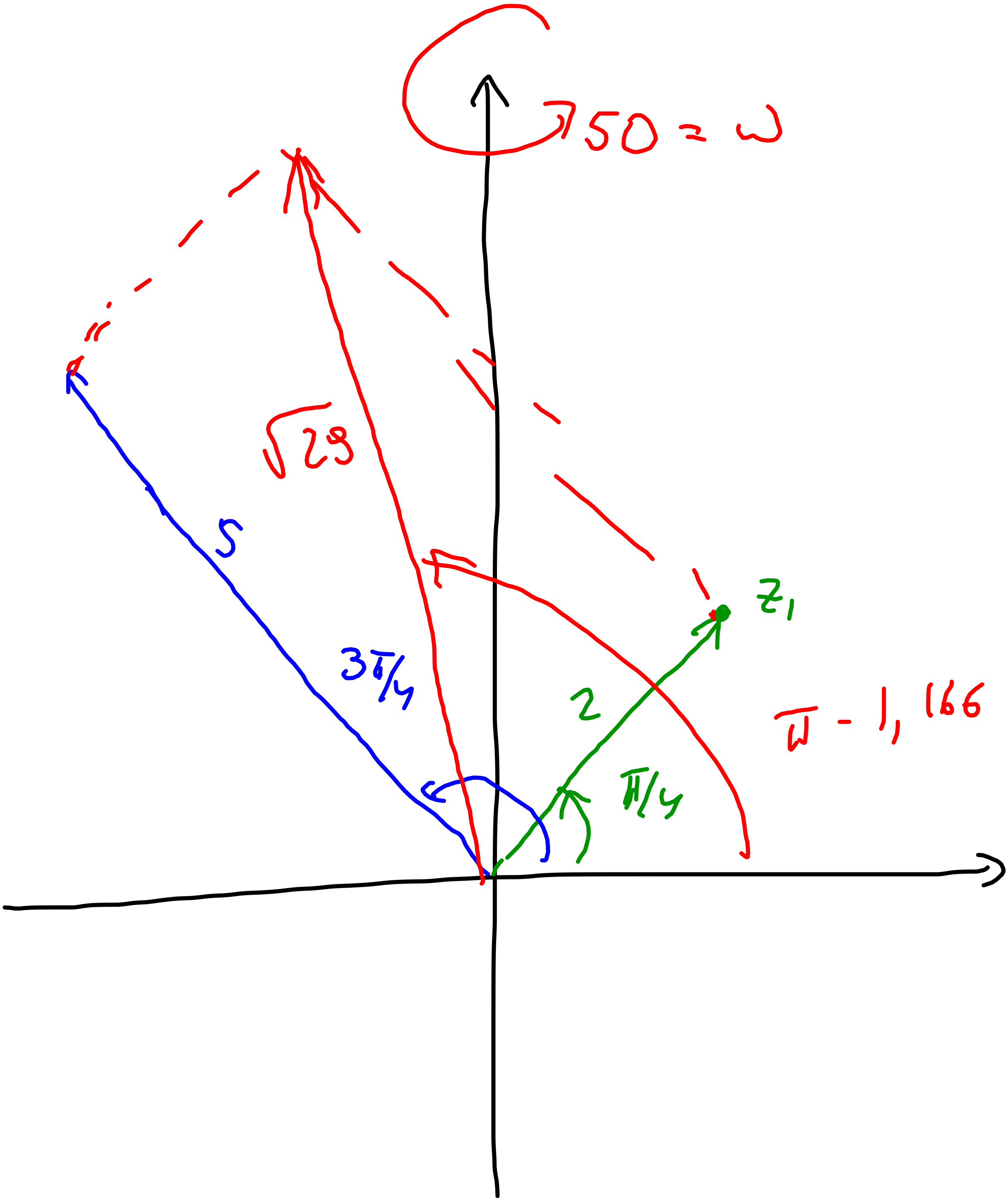
$$z(t) = \sqrt{29}\, e^{i(\pi - 1,166)} \cdot e^{i50t} =$$

$$= \sqrt{29}\cdot e^{i(50t + \pi - 1,166)}$$

$$\begin{array}{c} z(t) \\ \big\downarrow T^{-1} \\ y(t) \end{array}$$

$$y(t) = \operatorname{Im} z(t) =$$

$$\boxed{= \sqrt{29}\,\sin\left(50t + \pi - 1{,}166\right)}$$



$\sqrt{50} = w$
$\sqrt{29}$
5
$3\pi/4$
z_1
2
$\pi/4$
$\pi - 1,166$