Bsp:

$$Ax = b$$

$$A = \begin{pmatrix} 4 & 4 & 0 \\ 3 & 3 & 5 \\ 0 & 4 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} 4 \\ -7 \\ -6 \end{pmatrix}$$

QR nach Householder

$a_1 = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}, \quad \|a_1\| = \sqrt{16+9} = 5$

$Q_1 = I - 2u_1u_1^T / |u_1|^2$

$u_1 = a_1 + sgn(a_{11}) \|a_1\| e_1 =$

$= \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix} + \begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix} =$

$= \begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix}$

$e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$\dots$

$u_1 u_1^T = \begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix} (9\ 3\ 0) = \begin{pmatrix} 81 & 27 & 0 \\ 27 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$\|u_1\| \neq 1$

$v_1 = \frac{u_1}{|u_1|}, \quad |v_1| = 1$

$\langle u_1, u_1 \rangle = \|u_1\|^2 = |u_1|^2$

$u_1^T u_1 = \langle u_1, u_1 \rangle$

$a^T b = \langle a, b \rangle$

$u_1^T u_1 = (9\ 3\ 0) \begin{pmatrix} 9 \\ 3 \\ 0 \end{pmatrix} = 90$

$\Longrightarrow Q_1 = \begin{pmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{pmatrix} - \frac{2}{90} \begin{pmatrix} 81 & 27 & 0 \\ 27 & 9 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} - \frac{1}{5} \begin{pmatrix} 9 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} =$

$= \frac{1}{5} \begin{pmatrix} -4 & -3 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & \end{pmatrix}$

$$Q_1 A = \frac{1}{5}\begin{pmatrix} -4 & -3 & 0 \\ -3 & 4 & 0 \\ 0 & 0 & 5 \end{pmatrix}\begin{pmatrix} 4 & 4 & 0 \\ 3 & 3 & 5 \\ 0 & 4 & 1 \end{pmatrix} =$$

$$= \frac{1}{5}\begin{pmatrix} -25 & -25 & -15 \\ 0 & 0 & 20 \\ 0 & 20 & 5 \end{pmatrix} =$$

$$= \begin{pmatrix} -5 & -5 & -3 \\ 0 & 0 & 4 \\ 0 & 4 & 1 \end{pmatrix} = A_1$$

$$P_2 = Q_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\tilde{A}_1 = \begin{pmatrix} 0 & 4 \\ 4 & 1 \end{pmatrix}$$

$$\tilde{a}_1 = \begin{pmatrix} 0 \\ 4 \end{pmatrix}, \quad |\tilde{a}_1| = 4,$$

$$\tilde{Q}_2 = I - 2\frac{\tilde{u}_2 \tilde{u}_2^T}{\tilde{u}_2^T \tilde{u}_2} =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - 2\frac{\begin{pmatrix} 4 \\ 4 \end{pmatrix}(4, 4)}{32} =$$

$$\boxed{u_2 = \begin{pmatrix} 4 \\ 4 \end{pmatrix}}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \frac{1}{16}\begin{pmatrix} 16 & 16 \\ 16 & 16 \end{pmatrix} =$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

$$Q_2 = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & \multicolumn{2}{c}{\multirow{2}{*}{$\tilde{Q}_2$}} \\ 0 & & \end{array}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -5 & -3 \\ 0 & 0 & 4 \\ 0 & 4 & 1 \end{pmatrix} = A_1$$

$$Q_2 A_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -5 & -5 & -3 \\ 0 & 0 & 4 \\ 0 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -5 & -3 \\ 0 & -4 & -1 \\ 0 & 0 & -4 \end{pmatrix} = R$$

$$Ax = b$$

$$Q_2 Q_1 A x = Q_2 Q_1 b$$

$$Rx = Q_2 Q_1 b$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} -4/5 & -3/5 & 0 \\ -3/5 & 4/5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} 4 \\ -7 \\ -6 \end{pmatrix} = \begin{pmatrix} -4/5 & -3/5 & 0 \\ 0 & 0 & -1 \\ 3/5 & -4/5 & 0 \end{pmatrix};$$

$$= \begin{pmatrix} -\frac{16}{5} + \frac{21}{5} \\ 6 \\ \frac{12}{5} + \frac{28}{5} \end{pmatrix} = \begin{pmatrix} 1 \\ 6 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -5 & -5 & -3 \\ & -4 & -1 \\ & & -4 \end{pmatrix} x = \begin{pmatrix} 1 \\ 6 \\ 8 \end{pmatrix}$$

$x_3 = -2$

$x_2 = -2 \implies x = \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix}$

$x_1 = 3$

Givens

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix}$$

$a_{21} = 1; \quad -a_{21}s + a_{11}c = 0$

$a_{11} = 3;$

$c, s? \quad \begin{pmatrix} c & +s \\ -s & c \end{pmatrix}$

$\alpha^2 = a_{11}^2 + a_{21}^2 = 10;$

$$\boxed{c = \frac{a_{11}}{\alpha}, \quad s = \frac{a_{21}}{\alpha}}$$

$c = \frac{3}{\sqrt{10}}, \quad s = \frac{1}{\sqrt{10}}$

$G_1 \cdot A;$

$$G_1 = \begin{pmatrix} \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 \\ -\frac{1}{\sqrt{10}} & \frac{3}{\sqrt{10}} & 0 \\ 0 & 0 & 1 \end{pmatrix} =$$

$$G_1A = \begin{pmatrix} \sqrt{10} & & \\ 0 & & \\ 4 & 5 & 2 \end{pmatrix}$$

$$G_1 = \begin{pmatrix} \frac{3\sqrt{10}}{10} & \frac{\sqrt{10}}{10} & 0 \\ -\frac{\sqrt{10}}{10} & \frac{3\sqrt{10}}{10} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = G_1 A = G_1 \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 4 & 5 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{10} & \frac{3}{5}\sqrt{10} & \frac{2}{5}\sqrt{10} \\ 0 & \frac{4}{5}\sqrt{10} & \frac{1}{5}\sqrt{10} \\ 4 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} \sqrt{10} \\ 4 \end{pmatrix} = a_2, \quad |a_2| = \sqrt{26}$$

$$c = \frac{\sqrt{10}}{\sqrt{26}}, \quad s = \frac{4}{\sqrt{26}};$$

$$c = \frac{\sqrt{10}\sqrt{26}}{26}, \quad s = \frac{4\sqrt{26}}{26}$$

$$G_2 = \begin{pmatrix} \frac{\sqrt{10}\sqrt{26}}{26} & 0 & \frac{4\sqrt{26}}{26} \\ 0 & 1 & 0 \\ -\frac{4\sqrt{26}}{26} & 0 & \frac{\sqrt{10}\sqrt{26}}{26} \end{pmatrix}$$

$$A_3 = G_2 A_2 = \begin{pmatrix} \frac{\sqrt{26}\cdot\sqrt{10}}{26} & 0 & \frac{4\sqrt{26}}{26} \\ 0 & 1 & 0 \\ -\frac{4\sqrt{26}}{26} & 0 & \frac{\sqrt{26}\sqrt{10}}{26} \end{pmatrix} \cdot$$

$$\begin{pmatrix} \sqrt{10} & \frac{3}{5}\sqrt{10} & \frac{2}{5}\sqrt{10} \\ 0 & \frac{4}{5}\sqrt{10} & \frac{1}{5}\sqrt{10} \\ 4 & 5 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} \sqrt{26} & \sqrt{26} & \frac{12\sqrt{26}}{26} \\ 0 & \frac{4}{5}\sqrt{10} & \frac{1}{5}\sqrt{10} \\ 0 & \frac{1}{10}\sqrt{10}\sqrt{26} & \frac{1}{65}\sqrt{10}\sqrt{26} \end{pmatrix}$$

$$\cancel{a = \begin{pmatrix} \sqrt{26} \\ \\ \frac{\sqrt{10}}{10}\sqrt{26} \end{pmatrix}}$$

$$\cancel{\alpha = |a| = \sqrt{\frac{11}{10}\cdot 26} = \frac{\sqrt{11}\cdot\sqrt{26}}{\sqrt{10}}}$$

$$\cancel{c = \frac{\sqrt{26}\cdot\sqrt{10}}{\sqrt{11}\cdot\sqrt{26}} = \sqrt{\frac{10}{11}}}$$

$$\cancel{s = \frac{\frac{\sqrt{10}}{10}\sqrt{26}\,\sqrt{10}}{\sqrt{26}\cdot\sqrt{11}} = \frac{1}{\sqrt{11}}}$$

$\cancel{G_3 =}$ Leider gilt nicht!

$c \rightarrow$ immer <u>Diagonaleintrag</u>

$$a = \begin{pmatrix} \frac{4}{5}\sqrt{10} \\ \frac{\sqrt{10}\sqrt{26}}{10} \end{pmatrix}, \quad |a| = \left(\frac{160}{25} + \frac{260}{100}\right)^{\frac{1}{2}} = 3$$

$$c = \frac{4\sqrt{10}}{15}, \quad s = \frac{\sqrt{10}\sqrt{26}}{30}$$

$$G_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{4}{5}\sqrt{10} & \frac{\sqrt{10}\sqrt{26}}{30} \\ 0 & -\frac{\sqrt{10}\sqrt{26}}{30} & \frac{4\sqrt{10}}{15} \end{pmatrix}$$

$$G_3 A_3 = \begin{matrix} \sqrt{26} & \sqrt{26} & \frac{12\sqrt{26}}{26} \\ 0 & 3 & 2/3 \\ 0 & 0 & * \end{matrix}$$