

Lösungen zu Aufgabe 7 der Probeklausur 1

Aufg 7) Gaußsches Algorithmus

$$(A|E) = \left(\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ 2z_2 - z_1 \\ \\ \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 1 & 0 & 0 & 0 & 1 \end{array} \right) \begin{array}{l} \\ 5z_3 - z_2 \\ 5z_4 - z_2 \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 10 & 1 & 1 & -2 & 5 & 0 \\ 0 & 0 & 15 & 6 & 1 & -2 & 0 & 5 \end{array} \right) \begin{array}{l} \\ \\ z_4 \cdot 2 - 3z_3 \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 2 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 5 & 0 & -1 & -1 & 2 & 0 & 0 \\ 0 & 0 & 10 & 1 & 1 & -2 & 5 & 0 \\ 0 & 0 & 0 & 9 & -1 & 2 & -15 & 10 \end{array} \right) \begin{array}{l} 9z_1 - z_4 \\ 9z_2 + z_4 \\ 9z_3 - z_4 \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 18 & 9 & 0 & 0 & 10 & -2 & 15 & -10 \\ 0 & 45 & 0 & 0 & -10 & 20 & -15 & 10 \\ 0 & 0 & 90 & 0 & 10 & -20 & 60 & -10 \\ 0 & 0 & 0 & 9 & -1 & 2 & -15 & 10 \end{array} \right) \begin{array}{l} 5z_1 - z_2 \\ \\ \\ \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 90 & 0 & 0 & 0 & 60 & -30 & 90 & -60 \\ 0 & 45 & 0 & 0 & -10 & 20 & -15 & 10 \\ 0 & 0 & 90 & 0 & 10 & -20 & 60 & -10 \\ 0 & 0 & 0 & 9 & -1 & 2 & -15 & 10 \end{array} \right) \begin{array}{l} :90 \\ \\ :90 \\ :9 \end{array}$$

$$\Rightarrow \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2/3 & -1/3 & 1 & -2/3 \\ 0 & 1 & 0 & 0 & -2/9 & 4/9 & -3/9 & 2/9 \\ 0 & 0 & 1 & 0 & 1/9 & -2/9 & 2/3 & -1/9 \\ 0 & 0 & 0 & 1 & -1/9 & 2/9 & -5/3 & 10/9 \end{array} \right)$$

$\underbrace{\hspace{10em}}_{A^{-1}}$

$$A^{-1} = \begin{pmatrix} 2/3 & -1/3 & 1 & -2/3 \\ -2/9 & 4/9 & -3/9 & 2/9 \\ 1/9 & -2/9 & 2/3 & -1/9 \\ -1/9 & 2/9 & -5/3 & 10/9 \end{pmatrix}$$