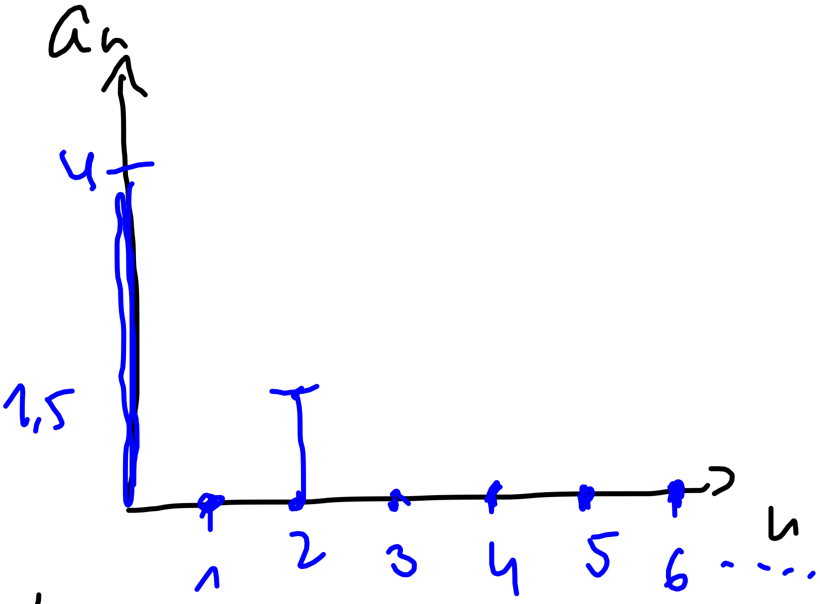
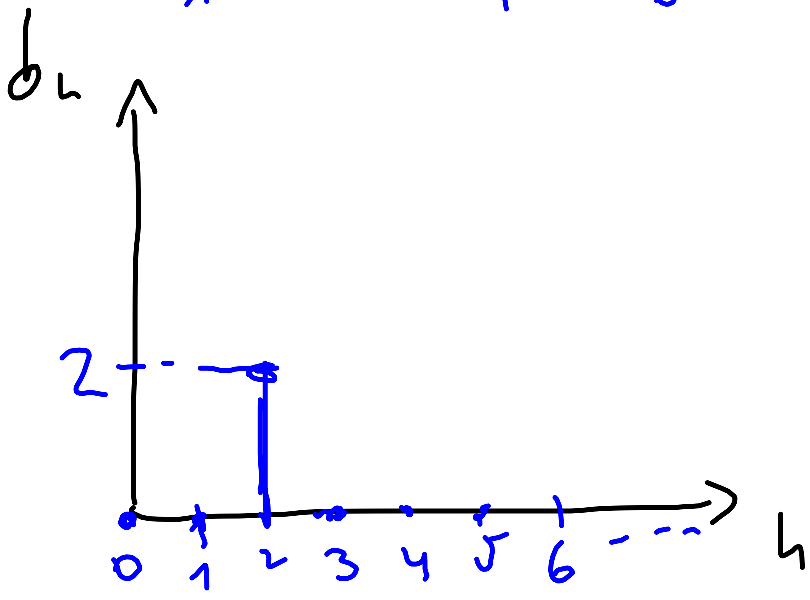




$T = 2\pi$

$T = 2\pi \Rightarrow \omega_0 = 1$

$f(t) = 2 + 1{,}5 \cos(2t) +$

$+ 2 \sin(2t).$

Bsp.

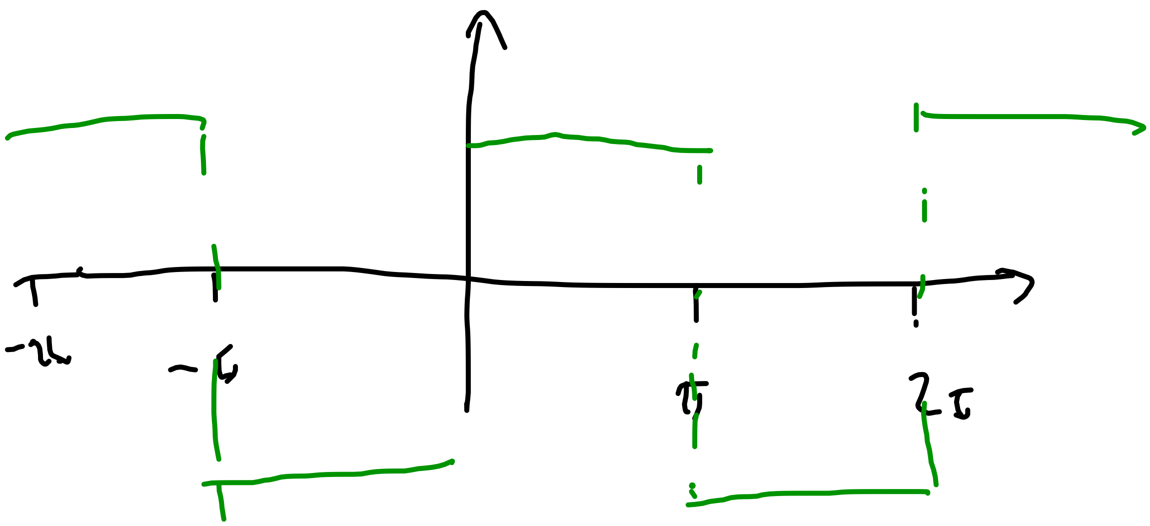
$$f(t) = \begin{cases} 1 & 0 \leq t \leq \pi \\ -1 & \pi < t < 2\pi \end{cases}$$

$+2\pi k$ $+2\pi k$ $+2\pi k$ $+2\pi k$

$k \in \mathbb{Z}$

2π- Periodisch

Symmetrie?



Punktsymmetrisch | ungerade

$$\Longrightarrow \quad a_k = 0 \quad \forall k \in \mathbb{N}_0 =$$

$$= \mathbb{N} \cup \{0\}$$

$$\Downarrow$$

$$b_n = \frac{2}{T} \int_{(T)} f(t) \, \cancel{\cos}\, \sin(n\omega_0 t) \, dt =$$

$$= \frac{2}{2\pi} \left(\int_0^{\pi} + \int_{\pi}^{2\pi} \right) =$$

$$= \frac{1}{\pi} \left(\int_0^{\pi} 1 \cdot \cancel{\cos}\, \sin(n\omega_0 t) \, dt + \right.$$

$$\left. + \int_{\pi}^{2\pi} (-1) \sin(n\omega_0 t) \, dt \right) =$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$= \frac{1}{\pi}\left[-\frac{1}{n}\cos(nt)\Big|_0^{\pi} - \right.$$

$$\left. - \left(-\frac{1}{n}\cos(nt)\Big|_{\pi}^{2\pi}\right)\right] =$$

$$= -\frac{1}{\pi n}\Big(\cos(\pi n) - \cos 0 -$$

$$- \cos(2\pi n) + \cos(\pi n)\Big) =$$

$$= -\frac{1}{\pi n}\left(\underline{2\cos(\pi n) - 2}\right) =$$

$$= \begin{cases} \dfrac{4}{\pi n}, & n \text{ ungerade} \\ 0, & \text{sonst} \end{cases}$$

$\Longrightarrow$

$$f(t) = \sum_{n=1}^{\infty} \frac{4}{(2n-1)\pi} \sin((2n-1)t);$$

Wichtige Fourier-Reihen:

1. Rechtecksignal

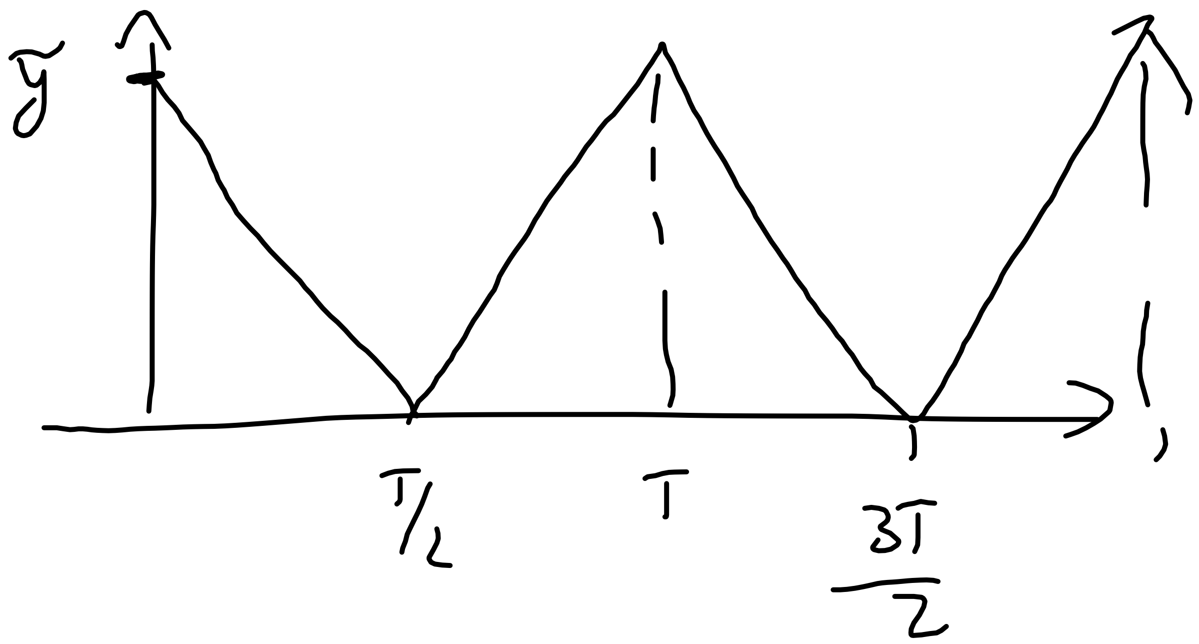
$$y(t) = \begin{cases} \tilde{y}, & 0 \le t \le T/2 \\ 0, & T/2 < t < T \end{cases}$$

T - periodisch

$$\frac{a_0}{2} = \frac{\tilde{y}}{2}, \qquad a_n = 0 \quad \forall n \in \mathbb{N}$$

$$y(t) = \frac{\tilde{y}}{2} + \frac{2\tilde{y}}{\pi} \sum_{n=1}^{\infty} \frac{1}{2n-1} \sin(n\omega_0 t)$$

2. Dreieckssignal



$$y(t) = \begin{cases} -\frac{2\tilde{y}}{T}t + \tilde{y}, & 0 \le t \le T/2 \\ \frac{2\tilde{y}}{T}t + \tilde{y}, & \frac{T}{2} \le t \le T \end{cases}$$

$$\frac{a_0}{2} = \frac{\tilde{y}}{2}, \qquad b_n = 0 \quad n \in \mathbb{N}$$

F.R:

$$y(t) = \frac{\tilde{y}}{2} + \frac{4\tilde{y}}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos(n\omega_0 t)$$

1.3. Komplexe Darstellung der Fourier-Reihe

$$\cos(nx) = \frac{1}{2}\left(e^{jnx} + e^{-jnx}\right)$$

$$\sin(nx) = \frac{1}{2j}\left(e^{jnx} - e^{-jnx}\right)$$

$\downarrow$

in Fourier-Reihe

HA: Wiederholung der Überlagerung von Schwingungen