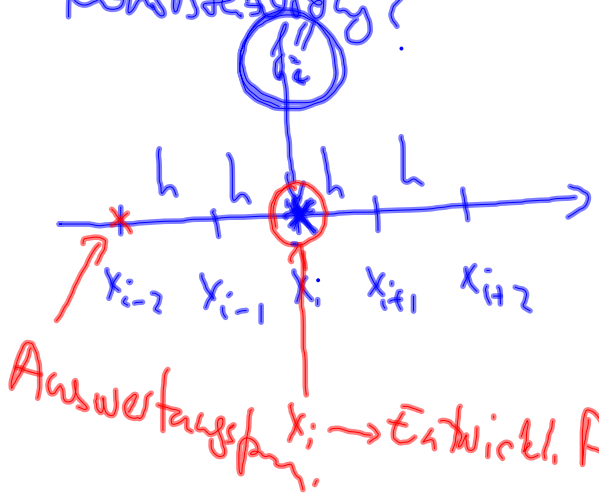
f, f_i, x_i

$i \in \mathbb{Z}$,

$x_{i+1} - x_i = h$

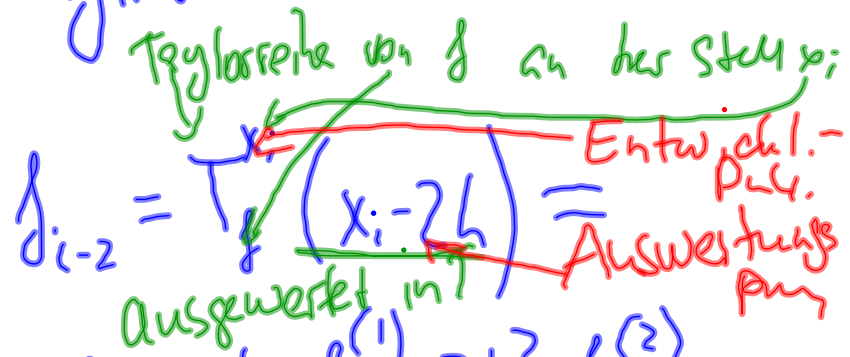
f_i'' : f_{i-2}, f_{i-1}, f_i,
f_{i+1}, f_{i+2}

Konsistenzordnung?



Auswertungspunkt x_i → Entwickl. Punkt.

Taylor:



$$= f_i - 2h f_i^{(1)} + 2h^2 f_i^{(2)} -$$

$$- \frac{4}{3} h^3 f_i^{(3)} + \frac{2}{3} h^4 f_i^{(4)} + O(h^5)$$

$$= f(x_i) + \frac{f'(x_i)}{1!}\left((x_i - 2h) - x_i\right) +$$

$$+ \frac{f''(x_i)}{2!}(-2h)^2 + \frac{f'''(x_i)}{3!}(x - x_i)^3 + \dots$$

$f(x_i) = f_i$; $f'(x_i) = f_i'$

$(x - x_i) = \left((x_i - 2h) - x_i\right) = -2h$

$$f_{i-1} = f_i - h f_i' + \frac{f_i''}{2} h^2 - \frac{f_i'''}{6} h^3 + \frac{f_i^{(4)}}{24} h^4 + O(h^5)$$

$$f_i = f_i$$

$$f_{i+1} = f_i + h f_i' + \frac{h^2}{2} f_i'' + \frac{f_i'''}{6} h^3 + \frac{f_i^{(4)}}{24} h^4 + O(h^5)$$

$$f_{i+2} = f_i + 2h f_i' + 2h^2 f_i'' + \frac{4}{3} h^3 f_i''' + \frac{2}{3} h^4 f_i^{(4)} + O(h^5)$$

$$f_i'' \approx \alpha f_{i-2} + \beta f_{i-1} + \gamma f_i + \delta f_{i+1} + \varepsilon f_{i+2}$$

$\alpha, \beta, \gamma, \delta, \varepsilon$ — ?

Taylor-Redelen

$$f_i^{(2)} \approx \alpha\left(f_i - 2hf_i' + 2h^2 f_i'' - \frac{4}{3}h^3 f_i''' + \frac{2}{3}h^4 f_i^{(4)}\right) +$$

$$+\beta\left(f_i - hf_i' + \frac{h^2}{2}f_i'' - \frac{h^3}{6}f_i''' + \frac{h^4 f_i^{(IV)}}{24}\right) +$$

$$+\gamma f_i +$$

$$+\delta\left(f_i + hf_i' + \frac{h^2}{2}f_i'' + \frac{h^3}{6}f_i''' + \frac{h^4 f_i^{(4)}}{24}\right) +$$

$$+\varepsilon\left(f_i + 2hf_i' + 2h^2 f_i'' + \frac{4}{3}h^3 f_i''' + \frac{2}{3}f_i^{(4)}h^4\right) =$$

$$= f_i(\alpha+\beta+\gamma+\delta+\varepsilon) +$$

$$+ f_i'\left(-2\alpha-\beta+\delta+2\varepsilon\right) h +$$

$$+ f_i'' h^2\left(2\alpha+\beta/2+\frac{\delta}{2}+2\varepsilon\right)+$$

$$+ f_i''' h^3\left(-\frac{4}{3}\alpha-\frac{\beta}{6}+\frac{\delta}{6}+\frac{4}{3}\varepsilon\right)$$

$$+ f_i^{(4)} h^4\left(\frac{2}{3}\alpha+\frac{1}{24}\beta+\frac{1}{24}\delta+\frac{2}{3}\varepsilon\right)$$

$$\simeq f_i^{(k)}$$

$$
\begin{cases}
\alpha+\beta+\gamma+\delta+\varepsilon = 0 \\
-2\alpha-\beta+\delta+2\varepsilon = 0 \\
2\alpha+\frac{\beta}{2}+\frac{\delta}{2}+2\varepsilon = \frac{1}{h^2} \\
-\frac{4}{3}\alpha-\frac{\beta}{6}+\frac{\delta}{6}+\frac{4}{3}\varepsilon = 0 \\
\frac{2}{3}\alpha+\frac{\beta}{24}+\frac{\delta}{24}+\frac{2}{3}\varepsilon = 0
\end{cases}
$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ -2 & -1 & 0 & 1 & 2 \\ 2 & \frac{1}{2} & 0 & \frac{1}{2} & 2 \\ -\frac{4}{3} & -\frac{1}{6} & 0 & \frac{1}{6} & \frac{4}{3} \\ \frac{2}{3} & \frac{1}{24} & 0 & \frac{1}{24} & \frac{2}{3} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \\ \varepsilon \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{h^2} \\ 0 \\ 0 \end{pmatrix}$$

$\text{Gauß} \rightsquigarrow$

$$\alpha = \varepsilon = -\frac{1}{12h^2},$$

$$\beta = \delta = \frac{4}{3h^2},$$

$$\gamma = -\frac{5}{2h^2}$$

$$f_i'' \approx -\frac{1}{12h^2} f_{i-2} + \frac{4}{3h^2} f_{i-1} - \frac{5}{2h^2} f_i$$

$$+ \frac{4}{3h^2} f_{i+1} - \frac{1}{12h^2} f_{i+2}$$

$$= \frac{1}{h^2}\left(-\frac{f_{i-2}}{12} + \frac{4f_{i-1}}{3} - \frac{5f_i}{2} + \right.$$

$$\left. + \frac{4}{3} f_{i+1} - \frac{1}{12} f_{i+2}\right)$$

$$f_{i+1}'' \approx \frac{1}{h^2}\left(-\frac{1}{12} f_{i-1} + \frac{4}{3} f_i - \frac{5}{2} f_{i+1} + \frac{4}{3} f_{i+2} - \right.$$

$$\left. - \frac{1}{12} f_{i+2}\right)$$

Zur Konsistenz:

Taylor der höheren Ordn:

$h = x - x_i$

$$f(x) = T_f^{(x_i)}(x) = f_i + \frac{f_i'}{1!}h + \frac{f_i''}{2!}h^2 + \frac{f_i'''}{6}h^3 + \frac{f_i^{(4)}}{24}h^4 + \frac{f_i^{(5)}}{120}h^5 + \frac{f_i^{(6)}}{720} \cdot h^6 + O(h^7)$$

Wieder:

$f_{i-2}, f_{i-1}, f_i, f_{i+1}, f_{i+2}$

als Taylor und

Setzen wir in unsere

berechnete Formel ein

und klammern um.

$$f_i^{(2)} \approx \frac{1}{h^2}\left[f_i \underbrace{\left(-\frac{1}{12} + \frac{4}{3} - \frac{5}{2} + \frac{4}{3} - \frac{1}{12} \right)}_{=0} + \right.$$

$$+ h f_i' \underbrace{\left(\frac{1}{6} - \frac{4}{3} + \frac{4}{3} - \frac{1}{6} \right)}_{=0} +$$

$$+ h^2 f_i^{(2)} \left(-\frac{1}{6} + \frac{4}{6} + \frac{4}{6} - \frac{1}{6} \right) +$$

$$+ h^3 f_i^{(3)} \underbrace{\left(\frac{1}{9} - \frac{2}{3} + \frac{2}{3} - \frac{1}{9} \right)}_{=0} +$$

$$+ h^4 f_i^{(4)} \underbrace{\left(-\frac{1}{18} + \frac{1}{18} + \frac{1}{18} - \frac{1}{18} \right)}_{=0} +$$

$$+ h^5 f_i^{(5)} \underbrace{\left(\frac{1}{45} - \frac{1}{90} + \frac{1}{90} - \frac{1}{45} \right)}_{=0}$$

$$+ h^6 f_i^{(6)} \left(-\frac{1}{135} + \frac{16}{135} + \frac{16}{135} - \frac{1}{135} \right)$$

$$\left. + O(h^7) \right] =$$

$y_i'' \approx \alpha y_{i-2} + \beta y_{i-1} + \gamma y_i + \delta y_{i+1} + \varepsilon y_{i+2}$

$\alpha, \beta, \gamma, \delta, \varepsilon$ sind schon bekannt.

x_i ist der Entwicklungspunkt

Was berechnet die rechte Seite der Formel?

$$= \frac{1}{h^2}\Big[0 \cdot f_i + h \cdot f_i' \cdot 0 + h^2 f_i'' \cdot 1 + h^3 \cdot f_i''' \cdot 0 + h^4 f_i^{(4)} \cdot 0 + h^5 \cdot f_i^{(5)} \cdot 0 + \frac{30}{135} h^6 f_i^{(6)} + O(h^7)\Big]$$

$$= f_i'' + \frac{30}{135} f_i^{(6)} h^4 + O(h^5)$$

A3 B2)

$$\begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix}' = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} y_1(x) \\ y_2(x) \end{pmatrix} +$$

$$+ \begin{pmatrix} e^x \\ 0 \end{pmatrix}, \quad x \geq 0$$

$y_1(0) = 0.5$, $y_2(0) = 0$.

$$y' = Ay + f(x)$$

$A \in \mathbb{R}^{2\times 2}$

- inhomogen
- linear
- Ordnung 1
- System
- konst. Koeff.

(1) homogene Gl.:

$y' = Ay$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

Eweuen A.:

$$\det(A - \lambda I) = \begin{vmatrix} 1-\lambda & 4 \\ 2 & 3-\lambda \end{vmatrix}$$

$= (1-\lambda)(3-\lambda) - 8 = \lambda^2 - 4\lambda - 5$

Char. Gl.: $\lambda^2 - 4\lambda - 5 = 0$

$\lambda_1 = -1$

$\lambda_2 = 5$

EV: $\lambda_1 = -1$:

$(A - \lambda_1 I) v_1 = 0$, $v_1 = EV(\lambda_1)$

$$\begin{pmatrix} 2 & 4 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} v_1^{(1)} \\ v_1^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

z.B. $\begin{pmatrix} -2 \\ 1 \end{pmatrix} = v_1$

$2v_1^{(1)} + 4v_1^{(2)} = 0$

$\mu = v_1^{(2)}$ - Parameter: $v_1 = \begin{pmatrix} -2\mu \\ \mu \end{pmatrix}$

z.B. für $\mu = 1$:

$v_1 = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

$\lambda_2 = 5$:

$$\begin{pmatrix} -4 & 4 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} v_2^{(1)} \\ v_2^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Fundamentale Lösung: $v_1 e^{\lambda_1 x}$, $v_2 e^{\lambda_2 x}$:

$z_1 = \begin{pmatrix} -2e^{-x} \\ e^{-x} \end{pmatrix}$, $z_2 = \begin{pmatrix} e^{5x} \\ e^{5x} \end{pmatrix}$

$\Rightarrow$ Fundamentale Matrix:

$$F = \begin{pmatrix} -2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix}$$

Allg. Lösung:

$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = F \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} =$

$= \begin{pmatrix} -2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} =$

$= \begin{pmatrix} -2c_1 e^{-x} + c_2 e^{5x} \\ c_1 e^{-x} + c_2 e^{5x} \end{pmatrix}$ –

(2) inhom. System:
Variat. der Konstanten:

$$\begin{pmatrix} -2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix} \begin{pmatrix} c_1' \\ c_2' \end{pmatrix} = f(x) = \begin{pmatrix} e^x \\ 0 \end{pmatrix}$$

$$\begin{cases} -2c_1' e^{-x} + c_2' e^{5x} = e^x \\ c_1' e^{-x} + c_2' e^{5x} = 0 \\ \downarrow \; c_1' e^{-x} = -c_2' e^{5x} \end{cases}$$

$$+2c_2' e^{5x} + c_2' e^{5x} = e^x$$

$$3c_2' e^{5x} = e^x$$

$$c_2' = \frac{1}{3} e^{-4x}$$

und integriere:

$$\int c_2'(x)\,dx = \frac{1}{3} \int e^{-4x}\,dx =$$

$$= -\frac{1}{12} \int e^{-4x}\,d(-4x)$$

Vdk:
$y' = f(x,y) + g(x)$ – inhom.
Löse hom. Gl:
$y' = f(x,y) \to y(x,c)$
$c \in \mathbb{R}$ → allg. Lösg der hom. Gl.
Vdk: ~~$c \in \mathbb{R}$~~ sondern
$c = c(x)$ solche, daß
$y(x, c(x))$ die Lösg v. inhom. Gl.

(II) $-\frac{1}{12}e^{-4x} + k_2 = c_2(x)$

$$c_1(x) = -\frac{1}{6}e^{2x} + k_1$$

$\Rightarrow$ Allg. Lösung des inhomogenen Systems:

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix} \begin{pmatrix} -\frac{1}{6}e^{2x} + k_1 \\ -\frac{1}{12}e^{-4x} + k_2 \end{pmatrix}$$

$k_1, k_2 \in \mathbb{R}$

$$= \underbrace{\begin{pmatrix} 2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix} \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}}_{\text{Allg. Lösg des hom Syst.}} + \underbrace{\begin{pmatrix} 2e^{-x} & e^{5x} \\ e^{-x} & e^{5x} \end{pmatrix} \begin{pmatrix} -\frac{1}{6}e^{2x} \\ -\frac{1}{12}e^{-4x} \end{pmatrix}}_{\text{Part. Lösg des inhom Systems}}$$

$\longrightarrow$ AW einsetzen:
$\Rightarrow$ Lösg des AWPs

$$y' = y^2 - x^2$$

$$f(x,y) = y^2 - x^2;$$

BY A2)

0			
1/2	a_{21}		
2/3	a_{31}	a_{32}	
	b_1	b_2	b_3

Orch. = 3

1) Orch.: $b_1 + b_2 + b_3 = 1$

$b_1 = 1 - b_3 - b_2$

2) Vers.: $a_{21} = \frac{1}{2}$

$\frac{2}{3} = a_{31} + a_{32} \Rightarrow$

$a_{32} = \frac{2}{3} - a_{31}$

3) Odh. 2:

$$\sum_{j,k} b_j a_{j,k} = \frac{1}{2}$$

$$\sum_j b_j \sum_k a_{jk} = \frac{1}{2}$$

$$b_2 a_{21} + b_3 a_{31} + b_3 a_{32} = \frac{1}{2}$$

$$\frac{1}{2} b_2 + b_3 \frac{2}{3} = \frac{1}{2}$$

$$b_2 = 1 - \frac{4}{3} b_3$$

4) Odh. 3:

$$\sum_{j,k,l} b_j a_{jk} a_{jl} = \frac{1}{3} \quad (*)$$

$$\sum_{j,k,l} b_j a_{jk} a_{kl} = \frac{1}{6}$$

b_3, a_{31}

$j := 2,3 \; ; \; k,l = 1,2$

$$(*): \; b_2 a_{21}^2 + b_3 a_{31}^2 + b_3 a_{31} a_{32} + b_3 a_{32}^2 = \frac{1}{3}$$

$$\left(1 - \frac{4}{3} b_3\right) \frac{1}{4} + b_3 a_{31}^2 + b_3 a_{31} \left(\frac{2}{3} - a_{31}\right) + b_3 \left(\frac{2}{3} - a_{31}\right)^2 = \frac{1}{3}$$

$$\frac{1}{4} - \frac{1}{3} = b_3 \left(\frac{1}{3} - a_{31}^2 + a_{31}^2 - \frac{2}{3} a_{31} - \left(\frac{2}{3} - a_{31}\right)^2\right)$$

$$b_3 = -\frac{1}{12} \Big/ \left(\frac{1}{3} - \frac{2}{3} a_{31} - \frac{4}{9} - a_{31}^2 + \frac{4}{3} a_{31}\right)$$

$$= -\frac{1}{12} \Big/ \left(-\frac{1}{9} - a_{31}^2 + \frac{2}{3} a_{31}\right) =$$

$$= \frac{1}{12} \Big/ \left(a_{31} - \frac{1}{3}\right)^2$$

$$b_2 a_{21} \underbrace{a_{11}}_{=0} + \dots +$$

$$+ b_3 a_{31} \underbrace{a_{11}}_{=b} \dots + $$

$$+ b_3 a_{32} a_{21} = \frac{1}{6}$$

$$\Rightarrow b_3 a_{32} \underbrace{a_{21}}_{=\frac{1}{2}} = \frac{1}{6}$$

$$\frac{1}{4\cancel{12}\left(a_{31}-\frac{1}{3}\right)^2} \cdot \cancel{\frac{1}{2}} \cdot \left(\frac{2}{3}-a_{31}\right) = \frac{1}{\cancel{6}} \cancel{\frac{1}{3}}$$

$$4\left(a_{31}-\frac{1}{3}\right)^2 = \frac{2}{3} - a_{31}$$

$$4a_{31}^2 - \frac{5}{3}a_{31} - \frac{2}{9} = 0$$

$$36a_{31}^2 - 45a_{31} - 2 = 0$$

RK: $y' = f(x, y)$

c_2	a_{21}		
c_3	a_{31}	a_{32}	
	b_1	b_2	b_3

$y_k \to y_{k+1}$

$$s_1 = f(x_k, y_k)$$

$$s_2 = f(x_k + c_2 h, y_k + a_{21} h s_1)$$

$$s_3 = f(x_k + c_3 h, y_k + a_{31} h s_1 + a_{32} h s_2)$$

$$y_{k+1} = h(b_1 s_1 + b_2 s_2 + b_3 s_3) + y_k$$

$y'' + y = r(x)$

$y'' + y = 0$ | $[a,b]$

$\lambda^2 + \lambda = 0$ | $\alpha = y(a)$

$\lambda_{1,2} = \pm i$; | $\beta = y(b)$

$y(x) = \tilde{c}_1 e^{ix} + \tilde{c}_2 e^{-xi} =$

$= c_1 \cos x + c_2 \sin x$

Allg. Lös. von hom. Gl.

$r \equiv 0 \ \forall x.$

a) $[a,b] = [0, \pi]$, $\alpha = 0$, $\beta = 0$

$\begin{cases} c_1 = 0 \\ -c_1 = 0 \end{cases} \Rightarrow c_1 = 0$

$c_2 = \{$ beliebig

$\rightsquigarrow$ viele Lös.

2) $[a,b] = [0, \pi/2]$, $\alpha = 1$, $\beta = 0$.

$\begin{cases} c_1 = 1 \\ c_2 = 0 \end{cases} \Rightarrow \exists!$ Lösung

3) $[a,b] = [0, \pi]$, $\alpha = 0$, $\beta = 1$

$\begin{cases} -c_1 = 1 \\ c_1 = 0 \end{cases}$ $\Rightarrow \nexists$ Lösung

u. s. w.

BS A3)

(*) $y'' - y' - 110y = 0$, $0 < x < 10$

(**) $y(0) = 1$, $y(10) = 1$

a) Exakte Lösung von (*)

zu AB $y(0) = s_1$, $y'(0) = s_2$

2. Ordn, linear, konst. Koeff, homogen:

Char. Gl.:

$$\lambda^2 - \lambda - 110 = 0$$

$$\lambda_1 = 11$$

$$\lambda_2 = -10$$

$$y_{allg.} = c_1 e^{11x} + c_2 e^{-10x}$$

$$c_1, c_2 \in \mathbb{R}$$

Lösg des AWPs:

$$y(0) = s_1, \quad y'(0) = s_2$$

$$\begin{cases} c_1 + c_2 = s_1 \\ 11c_1 - 10c_2 = s_2 \end{cases} \quad \begin{matrix} \\ \left| \begin{matrix} \cdot 10 \\ + \end{matrix} \right. \end{matrix}$$

$$\Rightarrow 21 c_1 = 10 s_1 + s_2$$

$$\Rightarrow \boxed{c_1 = \frac{10 s_1 + s_2}{21}}$$

$$\boxed{c_2 = s_1 - c_1 = \frac{11 s_1 - s_2}{21}}$$

Lösg des AWPs:

$$y(x, s_1, s_2) = \frac{10 s_1 + s_2}{21} \cdot e^{11x} + \frac{11 s_1 - s_2}{21} \cdot e^{-10x}$$

RB: $y(0)=1=s_1$

$y(10)=1$

$s_1=1$; $s_2=s$

$y(10,1,s_2)=y(10,1,s)=1$

$$\frac{10+s}{21}\cdot e^{110}+\frac{11-s}{21}\cdot e^{-100}=1$$

$$\Downarrow$$

$$s=s^*=\frac{-10e^{110}+21-11e^{-100}}{e^{110}-e^{-100}}$$

$\Rightarrow$ exakte Lösung des RWPs:

$$y(x)=\frac{1-e^{-100}}{e^{110}-e^{-100}}\cdot e^{11x}+\frac{e^{110}-1}{e^{110}-e^{-100}}\cdot e^{-10x}$$

$s^*\approx -10$

$y'' = \ldots$ DGL

$y(a) = \alpha$ $[a, b]$

$y(b) = \beta$

SV:

DGL → allg. Lösg (c_1, c_2)

DGL, $y(a) = \alpha$.

$y'(a)$ fehlt

→ $y'(a) = s$

Genug AB'en, um AWP zu lösen.

→ Lösg. von AWP ist von s abhängig:

$y(x, s)$

RWP: Dieselbe Gl.

$y(a) = \alpha$ – dieselbe 'AB'

$y(b) = \beta$ statt

$y'(s) = s$.

Suche s, so daß

$y(x, s)\big|_{x=b} = \beta$

$y(b, s) = \beta$

Gl bzgl. s.

b) $\tilde{y}(0) = 1 = s_1$

$\tilde{y}'(0) = 10^{-9} - 10 = s_2$;

$$\Rightarrow \tilde{y}(x) = \frac{10^{-9}}{21} e^{11x} + \frac{21 - 10^{-9}}{21} e^{-10x}$$

$$\tilde{y}(10) = \frac{10^{-9}}{21} e^{110} + \underbrace{\frac{21 - 10^{-9}}{21} e^{-100}}_{\approx 0} \approx$$

$$\approx 3 \cdot 10^{37}$$

B7A3

$$y'' - 100y = 0, \quad 0 < x < 3$$

$$y(0) = 1, \quad y(3) = e^{-30}$$

a) qual. Lösg:

Char. Gl.:

$$\lambda^2 = 100$$

$$\lambda_1 = -10, \lambda_2 = 10$$

$$y(x) = c_1 e^{-10x} + c_2 e^{10x}$$

$$\begin{cases} y(0) = c_1 + c_2 = 1; \\ y(3) = c_1 e^{-30} + c_2 e^{30} = e^{-30} \end{cases}$$

$$c_2 = 0, \quad c_1 = 1$$

$$\Rightarrow y = e^{-10x};$$

b) $y(0)=1$

$y'(0)=5$

$y(x)=c_1 e^{-10x}+c_2 e^{10x}$;

$$\begin{cases} c_1+c_2=1 \\ -10c_1+10c_2=5 \end{cases}$$

$c_1=\frac{10-5}{20}$, $c_2=\frac{10+5}{20}$;

$y(x,s)=\frac{10-s}{20}e^{-10x}+\frac{10+s}{20}e^{10x}$

c) $\left|y(3,s+\Delta s)-y(3,s)\right|=$

$$\left|\frac{10-s-\Delta s}{20}e^{-30}+\frac{10+s+\Delta s}{20}e^{30}-\right.$$

$$\left.-\frac{10-s}{20}e^{-30}-\frac{10+s}{20}e^{30}\right|=$$

$$=|\Delta s|\frac{e^{30}-e^{-30}}{20}\approx|\Delta s|\cdot 5\cdot 10^{11}$$