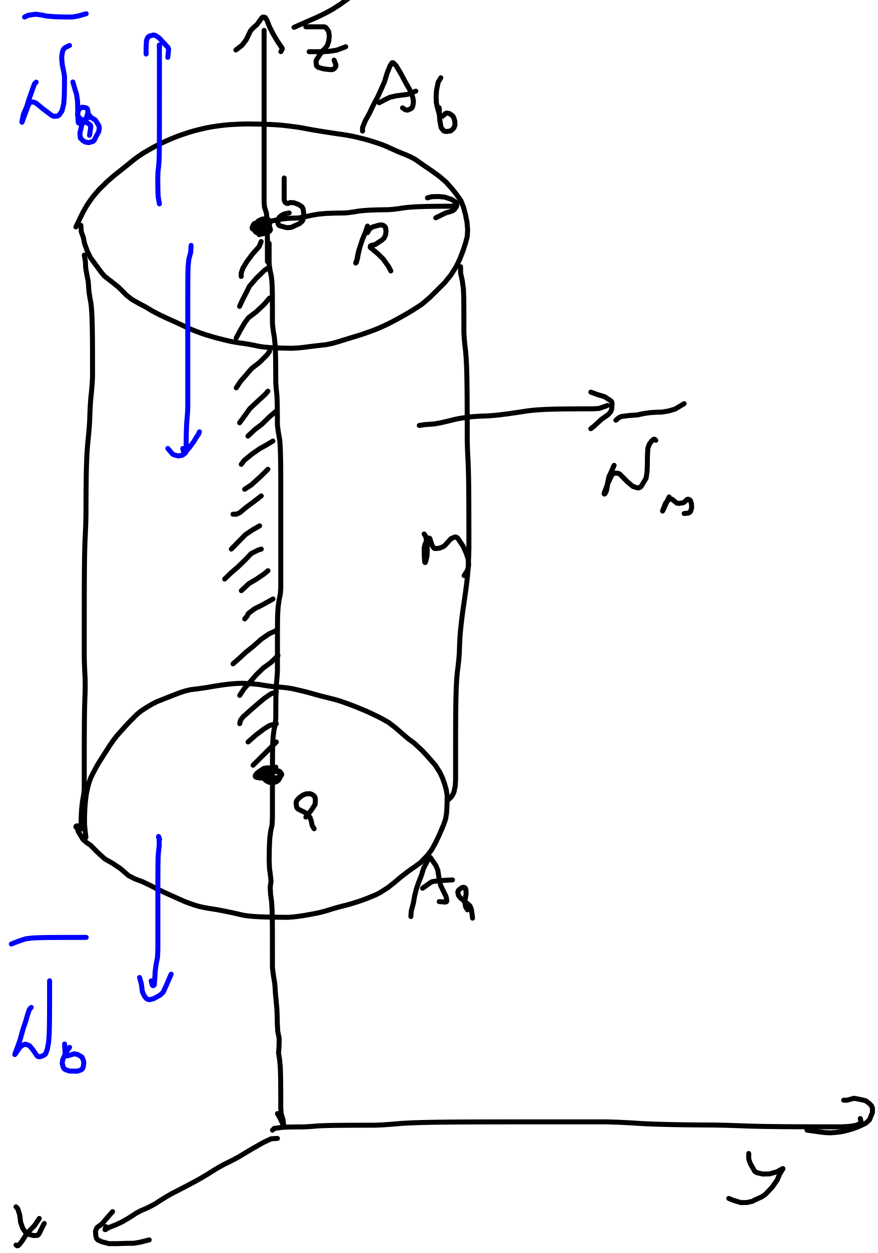
$$\bar{F} = \begin{pmatrix} x^2 \\ -y \\ z \end{pmatrix}$$



$$I = \oiint_{B} \overline{F}\, d\overline{B} =$$

$$= \iint_{M} \overline{F}\, d\overline{M} + \iint_{A_d} \overline{F}\, d\overline{A_d} +$$

$$+ \iint_{A_g} \overline{F}\, d\overline{A_g}$$

$$\overline{N} \qquad \overline{r_u} \otimes \overline{r_v}$$

(1) Mantel M :

$$M = \left\{ \bar r(\varphi, z) \;\middle|\; \bar r(\varphi, z) = \begin{pmatrix} R\cos\varphi \\ R\sin\varphi \\ z \end{pmatrix}, \quad 0 \le \varphi \le 2\pi, \quad 4 \le z \le 6 \right\}$$

Zylinderkoordinaten

$$\frac{\partial \bar r}{\partial \varphi} = \begin{pmatrix} -R\sin\varphi \\ R\cos\varphi \\ 0 \end{pmatrix}; \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\frac{\partial \bar{r}}{\partial z} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\bar{r}_\varphi \otimes \bar{r}_z = \begin{pmatrix} R\cos\varphi \\ R\sin\varphi \\ 0 \end{pmatrix}$$

$$\bar{F}(r, \varphi, z)\ !$$

$$\bar{F}(x, y, z) = \begin{pmatrix} x^2 \\ -y \\ z \end{pmatrix} =$$

$$= \begin{pmatrix} R^2 \cos^2\varphi \\ -R \sin\varphi \\ z \end{pmatrix}$$

$$\bar{F}(r,\varphi,z) \cdot \bar{r}_\varphi \otimes \bar{r}_z =$$

Zylinderkoordinaten

$$= \begin{pmatrix} R^2 \cos^2\varphi \\ -R \sin\varphi \\ z \end{pmatrix} \cdot \begin{pmatrix} R\cos\varphi \\ R\sin\varphi \\ 0 \end{pmatrix} =$$

$$= R^3 \cos^3\varphi - R^2 \sin^2\varphi;$$

$$I_M = \iint_M \bar{F}(x,y,z)\, d\bar{M} =$$

$$= \iint_{B'} \bar{F}(\bar{r}(\varphi,z)) \cdot (\bar{r}_\varphi \otimes \bar{r}_z)\, dz\, d\varphi$$

$$= \int_{\varphi=0}^{2\pi} \int_{z=a}^{b} (R^3 \cos^3\varphi - R^2 \sin^2\varphi)\, dz\, d\varphi =$$

$$= R^2(b-a)\left[R\int_0^{2\pi}\cos^3\varphi\,d\varphi - \int_0^{2\pi}\sin^2\varphi\,d\varphi\right]$$

$$= R^2(b-a)\left[R\cdot 0 - \pi\right] = -(b-a)R^2\pi$$

$$\boxed{\int \sin^2(ax)\,dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + C}$$

(2) Вода A_a

$$A_a = \left\{ \bar{r}(r,\varphi) = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ a \end{pmatrix} \;\middle|\; \begin{array}{l} 0 \le r \le R \\ 0 \le \varphi \le 2\pi \end{array} \right\}$$

$$\frac{\partial \bar{r}}{\partial \varphi} = \begin{pmatrix} -r\sin\varphi \\ r\cos\varphi \\ 0 \end{pmatrix}; \quad \frac{\partial \bar{r}}{\partial r} = \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{pmatrix}$$

$$\bar{r}_\varphi \times \bar{r}_r = \begin{pmatrix} 0 \\ 0 \\ -r \end{pmatrix}$$

$$\bar{F} = \begin{pmatrix} x^2 \\ -y \\ z \end{pmatrix} = \begin{pmatrix} r^2 \cos^2\varphi \\ -r \sin\varphi \\ a \end{pmatrix}$$

$$\bar{F} \cdot \bar{N}_b = -r \cdot a$$

$$I_{A_a} = \iint_{A_a} \bar{F} \, d\bar{A}_a = \int_{r=0}^{R} \int_{\varphi=0}^{2\pi} (-ra) \, d\varphi \, dr =$$

$$= -2\pi a \int_0^R r \, dr = -R^2 \pi a$$

Deckel A_b :

$$A_b = \left\{ \bar{r}(r,\varphi) = \begin{pmatrix} r\cos\varphi \\ r\sin\varphi \\ b \end{pmatrix}, \begin{array}{l} 0 \leq r \leq R \\ 0 \leq \varphi \leq 2\pi \end{array} \right\}$$

$$\frac{\partial \bar{r}}{\partial \varphi} = \begin{pmatrix} -r\sin\varphi \\ r\cos\varphi \\ 0 \end{pmatrix}$$

Oder $\bar{N}_b = -\bar{N}_0$

in diesem Fall

$$\bar{F} = \begin{pmatrix} r^2\cos^2\varphi \\ -r\sin\varphi \\ b \end{pmatrix}$$

$$\bar{F} \cdot \bar{n}_b = \begin{pmatrix} r^2 \cos^2\varphi \\ -r \sin\varphi \\ b \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ r \end{pmatrix} =$$

$$= br;$$

$$I_{A_b} = \iint_{A_b} \bar{F}\, d\bar{A}_b =$$

$$= \int_{r=0}^{R} \int_{\varphi=0}^{2\pi} rb \, d\varphi \, dr =$$

$$= \pi b R^2.$$

$$I = I_{A_1} + I_{A_2} + I_m =$$

$$= -9\pi R^2 + 8\pi R^2 -$$

$$-(8-9)R^2\pi =$$

$$= \pi R^2(8-9) - (8-9)R^2\pi =$$

$$= 0$$