$$\bar{r}(t) = \begin{pmatrix} e^{2t} \\ \ln t \\ 1/t \end{pmatrix}, \quad t > 0$$

$$\bar{F}(x,y,z) = \begin{pmatrix} 2x \\ z \\ -z^2 \end{pmatrix}$$

$$\dot{\bar{r}}(t) \overset{!}{=} \bar{F}(\bar{r}(t)) \quad \text{z.z.}$$

$$\dot{\bar{r}}(t) = \begin{pmatrix} \frac{\partial}{\partial t}\left(e^{2t}\right) \\ \frac{\partial}{\partial t}\left(\ln t\right) \\ \frac{\partial}{\partial t}\left(\frac{1}{t}\right) \end{pmatrix} = \begin{pmatrix} 2e^{2t} \\ \frac{1}{t} \\ -\frac{1}{t^2} \end{pmatrix} = \begin{pmatrix} 2x \\ z \\ -z^2 \end{pmatrix} = \bar{F}(\bar{r}(t)).$$

A2)

$$\bar{v}(x,y) = \begin{pmatrix} 1 \\ y \end{pmatrix}$$

$$t_0 = 0 \qquad P_0(0,2)$$

a) $$\bar{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix};$$

$$\dot{\bar{r}}(t) = \begin{pmatrix} \dot{x}(t) \\ \dot{y}(t) \end{pmatrix} = \begin{pmatrix} 1 \\ y(t) \end{pmatrix}$$

$$\begin{cases} \dot{x}(t) = 1 \\ \dot{y}(t) = y(t) \end{cases}$$

$$\int \dot{x}(t)\,dt = \int 1\,dt =$$

$$= t + c_1,$$
$$c_1 \in \mathbb{R}$$

$$\dot{y} = y;$$

$$\frac{\partial y}{\partial t} = y$$

$y \neq 0$

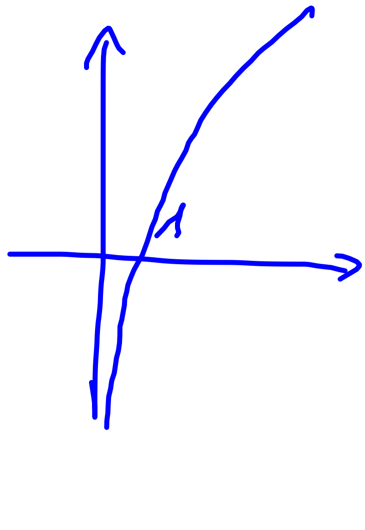
$$\int \frac{\partial y}{y} = \int dt$$

$$\ln|y| = t + c_2,$$

$c_2 \in \mathbb{R}$

$$c_2 = \ln c_3,$$

$c_3 > 0$



$$\ln|y| = t + \ln c_3$$

$$\ln \frac{|y|}{c_3} = t$$

$$e^{\ln \frac{|y|}{C_3}} = e^{t}$$

$$\frac{|y|}{C_3} = e^{t}$$

$$|y| = C_3 e^{t}, \quad C_3 > 0$$

$$|C_4| = C_3$$

$$C_4 = C_3 \cdot \operatorname{sgn}(y)$$

$$y = C_4 \cdot e^{t}, \quad C_4 \neq 0$$

$$C_4 \in \mathbb{R}$$

$$\dot{y} = y$$

$y \equiv 0$ $\qquad \dot{y} = 0 \quad \forall t$

$$\left[\begin{array}{l} y \equiv 0 \quad \text{auch eine} \\ \quad \text{Lösung!} \\ y = c_1 e^t, \quad c_1 \in \mathbb{R} \setminus \{0\} \end{array}\right.$$

$c \in \mathbb{R}$

$$y = c\, e^t, \quad c \in \mathbb{R}$$

$$\Rightarrow \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} t + c_1 \\ c e^t \end{pmatrix}$$

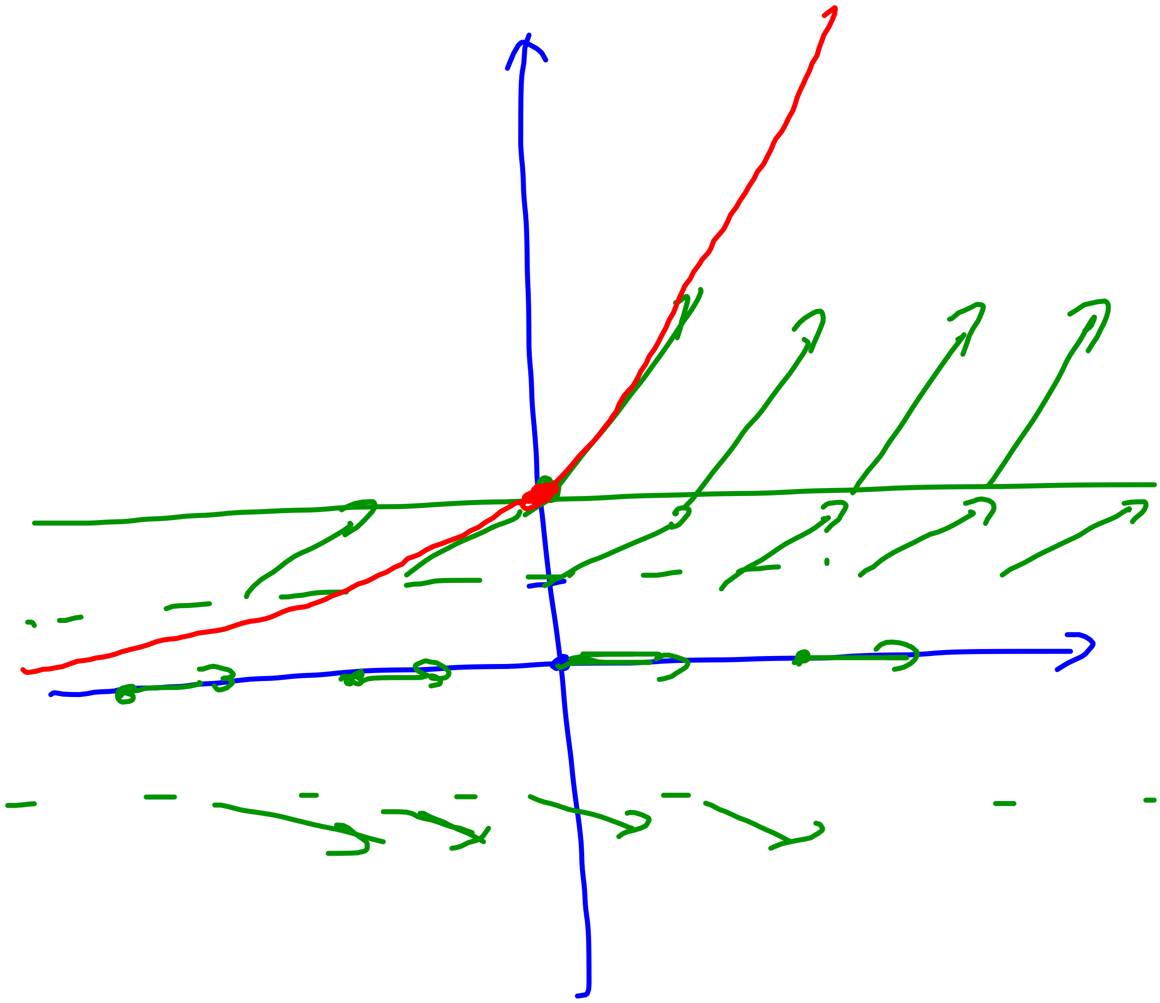
$$\begin{pmatrix} x \\ y \end{pmatrix}(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 0 + c_1 \\ c e^0 \end{pmatrix} = \begin{pmatrix} c_1 \\ c \end{pmatrix}$$

$$\Rightarrow \begin{array}{l} c_1 = 0 \\ c_2 = 2 \end{array}$$

$$\bar{r}(t) = \begin{pmatrix} t \\ 2e^{t} \end{pmatrix}$$

$$y = 2e^{x}$$



A3)

$$\bar{F}\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \begin{pmatrix} x \\ x^2 \end{pmatrix}$$

c) $x(0) = 0$

$y(0) = k$

$$\dot{\bar{r}}(t) = \bar{F}\begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\bar{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

$$\begin{cases} \dot{x} = x \\ \dot{y} = x^2 \end{cases}$$

$$\dot{x} = x \iff x = C_1 e^t, \quad C_1 \in \mathbb{R}$$

$$\int \dot{y}\, dt = x^2 = \int (C_1 e^t)^2\, dt$$

$$y = \frac{C_1^2}{2} e^{2t} + C_2 =$$

$$= \frac{(C_1 e^t)^2}{2} + C_2 = \frac{x^2}{2} + C_2$$

AB

$$x(0) = 0$$

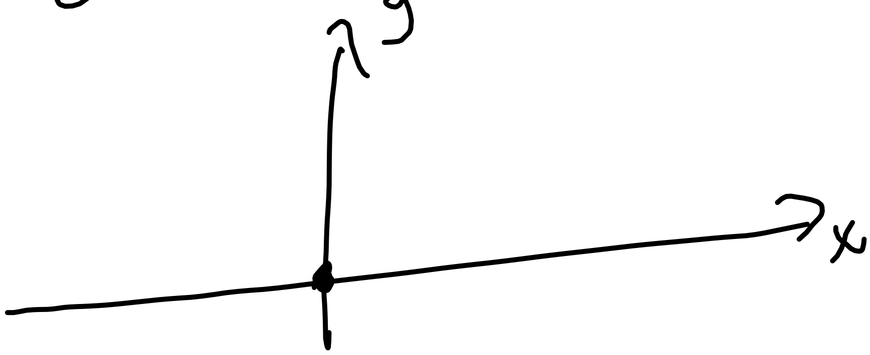
$$y(0) = k$$

$$x(0) = 0$$

$$\Rightarrow \quad C_1 = 0$$

$$y(0) = k \Rightarrow C_2 = k$$

$$y = \frac{x^2}{2} + k$$



A4)

$$F\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ -x \end{pmatrix}$$

c) $$\begin{cases} \dot{x} = y \\ \dot{y} = -x \end{cases}$$

$$x = -\dot{y}$$
$$\dot{x} = -\ddot{y}$$

$$f = -g'$$
$$f' = (-g')' = -g''$$

$$-\ddot{y} = y$$

$$y + \ddot{y} = 0$$

lineare DGl 2. Ordn.
mit konst. Koeff., homog.

$$\lambda^2 + 1 = 0$$

$$\lambda = \pm j$$

$$\Longrightarrow$$

$$y = C_1 \cos t + C_2 \sin t$$

$$x = -\dot{y} = C_1 \sin t - C_2 \cos t$$

$$\vec{r}(t) = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \sin t + \begin{pmatrix} -C_2 \\ C_1 \end{pmatrix} \cos t$$

$$x(0) = R, \quad y(0) = 0$$

$$\bar{r}(0) = \begin{pmatrix} R \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} R \\ 0 \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \sin(0) + \begin{pmatrix} -C_2 \\ C_1 \end{pmatrix} \cos 0$$

$$= \begin{pmatrix} -C_2 \\ C_1 \end{pmatrix}$$

$$C_1 = 0$$

$$C_2 = -R$$

$$\Rightarrow \bar{r}(t) = \begin{pmatrix} R \cos t \\ -R \sin t \end{pmatrix}$$

e) $\mathrm{rot}\ \bar{F} \overset{!}{=} \bar{0}$

$$\mathrm{rot}\,\bar{F} = \begin{array}{c} 3 \\ 2 \\ 1 \\ 3 \\ 2 \\ 1 \end{array} \begin{pmatrix} \dfrac{\partial F_3}{\partial y} - \dfrac{\partial F_2}{\partial z} \\[2ex] \dfrac{\partial F_1}{\partial z} - \dfrac{\partial F_3}{\partial x} \\[2ex] \dfrac{\partial F_2}{\partial x} - \dfrac{\partial F_1}{\partial y} \end{pmatrix}$$

$x = x_1$

$y = x_2$

$z = x_3$

$$F = \begin{pmatrix} F_1 \\ F_2 \\ 0 \end{pmatrix} = \begin{pmatrix} y \\ -x \\ 0 \end{pmatrix},$$

$$\operatorname{rot} \bar{F} = \begin{pmatrix} 0 - 0 \\ 0 - 0 \\ -1 - 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$

$$\neq \bar{0}$$

$\Rightarrow$ kein Potentialfeld.

f) $\text{rot } F = \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

$$\text{div } F = \frac{\partial}{\partial x} F_1 + \frac{\partial}{\partial y} F_2 + \frac{\partial}{\partial z} F_3 =$$

$$= 0 + 0 + 0 = 0$$