1 RK : $\boxed{\text{AWP}}$

- Stufenzahl s
- Max. mögl. Konsistenzordnung $m \leq s$
- Err. Ordnung n

$$n \leq m \leq s$$

$$\begin{array}{c|ccc} c_2 & & & \\ \vdots & & A & \\ c_s & & & \\ \hline & b_1 & \cdots & b_s \end{array} \qquad \sum b_i = 1$$

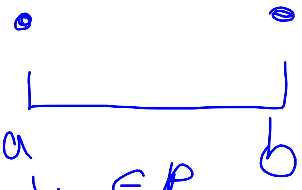
System 1. Ordn

Gl. höh. Ordn.

$\boxed{\text{RWP}}$

↳ Einfg. z. WSMS ESVt

RB'en



1. $y(a) = y_a \in \mathbb{R}$

$y(b) = y_b \in \mathbb{R}$

Ordn. der Gl. ab 2

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = A \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

$A \in \mathbb{R}^{2 \times 2}$

$\Longrightarrow EW(A) \rightarrow \lambda_1, \lambda_2$

$\det(A - \lambda E) = 0$

EVen: $\bar{v}_{1A}(\lambda_1), \bar{v}_{2A}(\lambda_2) \neq 0$

$(A - \lambda_k E)\bar{v}_k = \bar{0}$

y, y'

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = c_1 e^{\lambda_1 x} \bar{v}_{1A}(\lambda_1) + c_2 e^{\lambda_2 x} \bar{v}_{2A}(\lambda_2)$$

$y(1) = 1$

$y(2) = 3$

2. Art

$y'(1) = 0$

$y'(2) = -5$

3. Art: z.B. $y'(1) = \frac{1}{2}$

$y(2) = 3$

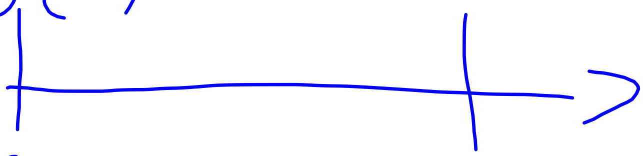
$$-y'' + p(x)\,y' + q(x)\,y = g(x)$$

$$y(a) = y_a$$

$$y(b) = y_b$$

$$[a, b]$$

$$y(a) = y_a$$



$$y'(a) = s$$

$$-y'' + py' + qy = g;$$

$$y(a) = y_a$$

$$y'(a) = s$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}(x) = \overline{y}_{all}(x) = c_1 e^{\lambda_1 x} \overline{v_1} + c_2 e^{\lambda_2 x} \overline{v_2}$$

$$y'(a) = s = y_2(a)$$

$$y_1(a) = y_1(a) = y_a$$

$$y_a = c_1 e^{\lambda_1 a} v_1^{(1)} + c_2 e^{\lambda_2 a} v_2^{(1)}$$

$$s = c_1 e^{\lambda_1 a} v_1^{(2)} + c_2 e^{\lambda_2 a} v_2^{(2)}$$

$$\Downarrow$$

$$c_1(s);\ c_2(s)$$

$$\overline{y}_{sp\,AW} = c_1(s) \cdot e^{\lambda_1 x} \overline{v_1} + c_2(s) e^{\lambda_2 x} \overline{v_2}$$

1 Parameter s !

Und 2. RB: $y(b) = \overset{\circ}{y}_b$

$$y_b = c_1(s)\, e^{\lambda_1 b} v_1^{(1)} + c_2(s)\, e^{\lambda_2 b} v_2^{(1)}$$

$$\Downarrow$$

$$s = s^* \ldots\ldots$$

$$s = s^* + \varepsilon \qquad \varepsilon \sim 10^{-8}$$

Fehler $r(\varepsilon; x)$

$$|y(x, s^*) - y(x, s^* + \varepsilon)| =$$

$$= r(\varepsilon, x)$$

Fin. Diff:

y''

y'

f_i'' → 2. Abl. von f an der Stelle x_i

f_i''

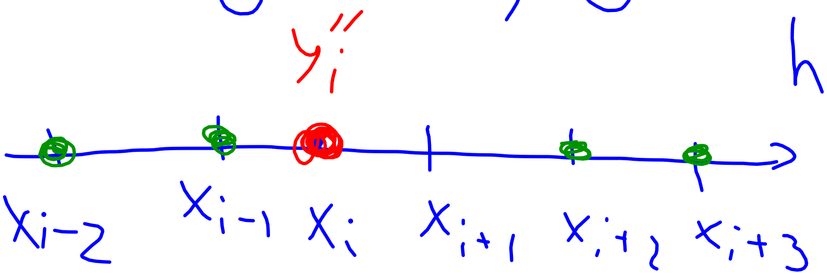
z.B.

y_i'' : mit den Werten

y_{i-2} ; y_{i-1} ;

y_{i+2} ; y_{i+3} ;

$h = x_{k+1} - x_k$;



$$y_i'' = \alpha y_{i-2} + \beta y_{i-1} + \gamma y_{i+2} + \delta y_{i+3}$$

$$y_{i\pm2} = \Big(y_i \pm 2y_i' \cdot h + \frac{y_i''}{2!}(2h)^2 \pm \frac{y_i'''}{3!}(2h)^3\Big) + \frac{y_i^{(iv)}}{4!}(2h)^4 \pm \frac{y_i^{(v)}}{5!}(2h)^5 + \frac{y_i^{(vi)}}{6!}(2h)^6 + O(h^7)$$

$$y_{i-1} = \Big(y_i - y_i' h + y_i'' \frac{h^2}{2} - \frac{y_i'''}{6} h^3\Big) + \frac{y_i^{iv}}{24} h^4 - \frac{y_i^{(v)}}{120} h^5 + \frac{y_i^{(6)}}{720} h^6 + O(h^7)$$

$$y_{i+3} = \Big(y_i + y_i'(3h) + y_i'' \frac{(3h)^2}{2} + \frac{y_i'''}{6}(3h)^3\Big) + \frac{y_i^{(4)}}{24}(3h)^4 + \frac{y_i^{(5)}}{120}(3h)^5 + \frac{y_i^{(6)}}{720}(3h)^6 + O(h^7)$$

$$y_i'' \approx \alpha y_{i-2} + \beta y_{i-1} + \gamma y_{i+2} + \delta y_{i+3}$$

$$y_i'' \approx \alpha\left(y_i - y_i' 2h + y_i'' 2h^2 - y_i''' \frac{4}{3} h^3\right)$$

$$+ \beta\left(y_i - y_i' h + y_i'' \frac{h^2}{2} - y_i''' h^3/6\right) +$$

$$+ \gamma\left(y_i + y_i' 2h + y_i'' 2h^2 + y_i''' \frac{4}{3} h^3\right)$$

$$+ \delta\left(y_i + y_i' 3h + y_i'' \frac{9}{2} h^2 + y_i''' \frac{9}{2} h^3\right) =$$

$$= 0 \cdot y_i + 0 \cdot y_i' + 1 \cdot y_i'' + 0 \cdot y_i'''$$

<u>Koeff. - Vergl.:</u>

y_i: $\quad 0 = \alpha + \beta + \gamma + \delta$

y_i': $\quad 0 = -\alpha 2\cancel{h} - \beta \cancel{h} + \gamma 2\cancel{h} + \delta 3\cancel{h}$

y_i'': $\quad 1 = \alpha 2h^2 + \beta \frac{h^2}{2} + \gamma 2h^2 + \frac{9}{2}h^2\delta;$

y_i''': $\quad 0 = \alpha\left(-\frac{4}{3}\cancel{h^3}\right) + \beta\left(-\frac{\cancel{h^3}}{6}\right) + \frac{4}{3}\cancel{h^3}\gamma +$

$\qquad + \delta\frac{9}{2}\cancel{h^3};$

$h \neq 0$

<u>LGS:</u>

$$\begin{pmatrix} 1 & 1 & 1 & 1 \\ -2 & -1 & 2 & 3 \\ 2h^2 & \frac{h^2}{2} & 2h^2 & \frac{9}{2}h^2 \\ -\frac{4}{3} & -\frac{1}{6} & \frac{4}{3} & \frac{9}{2} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ -2 & -1 & 2 & 3 & 0 \\ 4 & 1 & 4 & 9 & 2/h^2 \\ -8 & -1 & 8 & 27 & 0 \end{array}\right) \rightsquigarrow$$

$$z_2 \leftarrow z_2 + 2z_1$$
$$z_3 \leftarrow z_3 - 4z_1$$
$$z_4 \leftarrow z_4 + 8z_1$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 4 & 5 & 0 \\ 0 & -3 & 0 & 5 & 2/h^2 \\ 0 & 7 & 16 & 35 & 0 \end{array}\right)$$

$$z_3 \leftarrow z_3 + 3z_2$$
$$z_4 \leftarrow z_4 - 7z_2$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ & 1 & 4 & 5 & 0 \\ & 0 & 12 & 20 & z/h^2 \\ & 0 & -12 & 0 & 0 \end{array}\right)$$

$z_3 \longleftrightarrow z_4$; $z_3/12$

$z_3 + z_4 \longrightarrow z_4$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ & 1 & 4 & 5 & 0 \\ 0 & & 1 & 0 & 0 \\ & & & 20 & z/h^2 \end{array}\right)$$

$$\delta = \frac{1}{10h^2}$$

$$\gamma = 0$$

$$\beta = -5\delta = \frac{-5}{10h^2}$$

$$\alpha = -\beta - \overset{=0}{\gamma} - \delta = -(\beta + \delta) = \frac{4}{10h^2};$$

$$y_i'' \approx \frac{4}{10h^2} y_{i-2} - \frac{5}{10h^2} y_{i-1} + 0 \cdot y_{i+2} + \frac{1}{10h^2} y_{i+3} =$$

$$= \frac{4y_{i-2} - 5y_{i-1} + y_{i+3}}{10h^2};$$

$$y_i'' = \frac{1}{10h^2}\Big[4\Big(y_i - y_i' 2h + y_i'' 2h^2 - y_i''' \frac{(2h)^3}{6} + y_i^{(4)} \frac{(2h)^4}{24} - y_i^{(5)} \frac{(2h)^5}{120} + \frac{(2h)^6}{720} y_i^{(6)} + O(h^7)\Big) -$$

$$-5\Big(y_i - y_i' h + y_i'' \frac{h^2}{2} - y_i''' \frac{h^3}{6} + y_i^{(4)} \frac{h^4}{24} - y_i^{(5)} \frac{h^5}{120} + y^{(6)} \frac{h^6}{720} + O(h^7)\Big) + \Big(y_i + y_i' 3h + y_i'' \frac{(3h)^2}{2} + y_i''' \frac{(3h)^3}{6} + y_i'''' \frac{(3h)^4}{24} + y^{(5)} \frac{(3h)^5}{120} + \frac{(3h)^6}{720} y_i^{(6)} + O(h^7)\Big)\Big] =$$

$$= \frac{1}{10h^2}\Big[y_i \underbrace{(4-5+1)}_{=0} +$$

$$+ y_i' \big(\underbrace{-8h+5h+3h}_{=0}\big) +$$

$$+ y_i'' \Big(\underbrace{8h^2 - \frac{5h^2}{2} - \frac{9h^2}{2}}_{=1h^2}\Big) +$$

$$+ y_i''' \Big(\underbrace{\cancel{-\frac{8\cdot 4}{6}h^4} \cancel{- \frac{5}{24}h^4} \cancel{+ \frac{3^4}{24}h^4}}_{\cancel{\frac{-8\cdot 16-5+81}{24}} \neq 0}\Big)$$

$$\underbrace{\left(-\frac{4\cdot 2^3 \cdot h^3}{6} + \frac{5h^3}{6} + \frac{3^3 h^3}{6}\right)}_{h^3\left(\frac{-32}{6}+\frac{5}{6}+\frac{27}{6}\right)=0} +$$

$$+ y_i^{(4)} h^4 \underbrace{\left(\frac{4\cdot 2^4}{24} - \frac{5}{24} + \frac{3^4}{24}\right)}_{\frac{1}{24}(32-5+81)\neq 0} + O(h^5)$$

$$= y_i \cdot 0 + y_i' \cdot 0 +$$

$$+ y_i'' \cdot \cancel{h^2} + y_i''' \cdot 0 +$$

Muss! NB: RE

$$+ \underbrace{O\left(\frac{h^4}{h^2}\right)}_{= O(h^2)} .$$

Kons.-Ord.

$$y_i'' = y_i'' + O(h^{m})$$