

Tabelle zur Substitution

Integral der Form: $\int f(u; \sqrt{a^2 - u^2}) du$

Substitution:

$$\begin{aligned}u &= a \sin(z) \\ du &= a \cos(z) dz \\ \sqrt{a^2 - u^2} &= a \cos(z)\end{aligned}$$

Rücksubstitution:

$$\begin{aligned}z &= \arcsin\left(\frac{u}{a}\right) \\ \sin(z) &= \frac{u}{a} \\ \cos(z) &= \frac{1}{a} \sqrt{a^2 - u^2} \\ \tan(z) &= \frac{x}{\sqrt{a^2 - u^2}} \\ \cot(z) &= \frac{\sqrt{a^2 - u^2}}{x}\end{aligned}$$

Integral der Form: $\int f(u; \sqrt{u^2 + a^2}) du$

Substitution:

$$\begin{aligned}u &= a \sinh(z) \\ du &= a \cosh(z) dz \\ \sqrt{u^2 + a^2} &= a \cosh(z)\end{aligned}$$

Rücksubstitution:

$$\begin{aligned}z &= \operatorname{arsinh}\left(\frac{u}{a}\right) = \ln(u + \sqrt{u^2 + a^2}) - \ln(a) \\ \sinh(z) &= \frac{u}{a} \\ \cosh(z) &= \frac{1}{a} \sqrt{u^2 + a^2} \\ \tanh(z) &= \frac{u}{\sqrt{u^2 + a^2}} \\ \coth(z) &= \frac{\sqrt{u^2 + a^2}}{u}\end{aligned}$$

Integral der Form: $\int f(u; \sqrt{u^2 - a^2}) du$

Substitution:

$$\begin{aligned}u &= a \cosh(z) \\ du &= a \sinh(z) dz \\ \sqrt{u^2 - a^2} &= a \sinh(z)\end{aligned}$$

Rücksubstitution:

$$\begin{aligned}z &= \operatorname{arcosh}\left(\frac{u}{a}\right) = \ln(u + \sqrt{u^2 - a^2}) - \ln(a) \\ \cosh(z) &= \frac{u}{a} \\ \sinh(z) &= \frac{1}{a} \sqrt{u^2 - a^2} \\ \tanh(z) &= \frac{u}{\sqrt{u^2 - a^2}} \\ \coth(z) &= \frac{\sqrt{u^2 - a^2}}{u}\end{aligned}$$