

Nr.	$x(k) = \mathcal{Z}^{-1}[X(z)](k)$	$X(z) = \mathcal{Z}[x(k)](z)$	Bemerkungen
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Definitionen:

1.	$x(k)$	$\sum_{k=0}^{\infty} x(k) z^{-k}$	Hintransformation
2.	$\frac{1}{2\pi j} \oint_{\gamma} X(z) z^{k-1} dz$	$X(z)$	Rücktransformation

Regeln:

3.	$af(k) + bg(k)$	$aF(z) + bG(z)$	Linearität
4.	$f(k - \ell)$	$z^{-\ell} F(z)$	Versch. n. re., $\ell > 0$
5.	$f(k + 1)$	$z(F(z) - f(0))$	Versch. nach links
6.	$f(k + 2)$	$z^2(F(z) - f(0) - f(1)z^{-1})$	Versch. nach links
7.	$f(k + \ell)$	$z^{\ell} \left( F(z) - \sum_{k=0}^{\ell-1} f(k)z^{-k} \right)$	Versch. n. li., $\ell > 0$
8.	$b^{-k} f(k)$	$F(bz)$	Dämpfung, $b \neq 0$
9.	$f(k) - f(k - 1)$	$(1 - z^{-1})F(z) = \frac{z-1}{z} F(z)$	$k$ -Ableitung
10.	$kf(k)$	$-zF'(z)$	$z$ -Ableitung
11.	$\sum_{i=0}^k f(i)$	$\frac{1}{1-z^{-1}} F(z) = \frac{z}{z-1} F(z)$	$k$ -Integral
12.	$\frac{f(k)}{k} = \left( 0, f(1), \frac{f(2)}{2}, \dots \right)$	$\int_z^{\infty} \frac{F(w)}{w} dw$	$z$ -Integral, $f(0) \stackrel{!}{=} 0$
13.	$(f * g)(k) = \sum_{i=0}^k f(k-i)g(i)$	$F(z)G(z)$	$k$ -Faltung
14.	$f(k)g(k)$	$\frac{1}{2\pi j} \oint_{\gamma} \frac{F(z/w)G(w)}{w} dw$	$z$ -Faltung

Potenzen:

15.	$(1, 0, 0, 0, 0, 0, 0, \dots)$	1	$\delta_{0k}$
16.	$(0, 1, 0, 0, 0, 0, 0, \dots)$	$z^{-1} = \frac{1}{z}$	$\delta_{1k}$
17.	$(0, \dots, 0, 1, 0, 0, 0, \dots)$	$z^{-i} = \frac{1}{z^i}$	$\delta_{ik}, i \geq 0$

Grenzwertsätze:

$$x(0) = \lim_{k \rightarrow 0} x(k) = \lim_{z \rightarrow \infty} X(z)$$

$$\lim_{k \rightarrow \infty} x(k) = \lim_{z \rightarrow 1} (z-1)X(z)$$

TABELLE ZUR Z-TRANSFORMATION

Nr.	$x(k) = \mathcal{Z}^{-1}[X(z)](k)$	$X(z) = \mathcal{Z}[x(k)](z)$	Bemerkungen
Nennergrad = 1:			
18.	$(1, 1, 1, 1, 1, 1, 1, \dots)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$	1
19.	$(0, 1, 1, 1, 1, 1, 1, \dots)$	$\frac{z^{-1}}{1-z^{-1}} = \frac{1}{z-1}$	$1 - \delta_{0k}$
20.	$(1, -1, 1, -1, 1, -1, \dots)$	$\frac{1}{1+z^{-1}} = \frac{z}{z+1}$	$(-1)^k$
21.	$(0, 1, -1, 1, -1, 1, -1, \dots)$	$\frac{z^{-1}}{1+z^{-1}} = \frac{1}{z+1}$	$(-1)^{k+1} + \delta_{0k}$
Nennergrad = 2:			
22.	$(1, 0, 1, 0, 1, 0, 1, 0, \dots)$	$\frac{1}{1-z^{-2}} = \frac{z^2}{z^2-1}$	$\frac{1}{2}(1 + (-1)^k)$
23.	$(0, 1, 0, 1, 0, 1, 0, 1, \dots)$	$\frac{z^{-1}}{1-z^{-2}} = \frac{z}{z^2-1}$	$\frac{1}{2}(1 + (-1)^{k+1})$
24.	$(0, 0, 1, 0, 1, 0, 1, 0, \dots)$	$\frac{z^{-2}}{1-z^{-2}} = \frac{1}{z^2-1}$	$\frac{1}{2}(1 + (-1)^k) - \delta_{0k}$
25.	$(1, 0, -1, 0, 1, 0, -1, 0, \dots)$	$\frac{1}{1+z^{-2}} = \frac{z^2}{z^2+1}$	$\cos\left(\frac{\pi}{2}k\right)$
26.	$(0, 1, 0, -1, 0, 1, 0, -1, \dots)$	$\frac{z^{-1}}{1+z^{-2}} = \frac{z}{z^2+1}$	$\sin\left(\frac{\pi}{2}k\right)$
27.	$(0, 0, 1, 0, -1, 0, 1, 0, \dots)$	$\frac{z^{-2}}{1+z^{-2}} = \frac{1}{z^2+1}$	$-\cos\left(\frac{\pi}{2}k\right) + \delta_{0k}$
28.	$(0, 0, 1, 1, 1, 1, 1, 1, \dots)$	$\frac{z^{-2}}{1-z^{-1}} = \frac{1}{z(z-1)}$	$1 - \delta_{0k} - \delta_{1k}$
29.	$(0, 0, 1, -1, 1, -1, 1, -1, \dots)$	$\frac{z^{-2}}{1+z^{-1}} = \frac{1}{z(z+1)}$	$(-1)^k - \delta_{0k} + \delta_{1k}$
30.	$(1, 2, 3, 4, 5, 6, 7, 8, \dots)$	$\frac{1}{(1-z^{-1})^2} = \frac{z^2}{(z-1)^2}$	$k+1$
31.	$(0, 1, 2, 3, 4, 5, 6, 7, \dots)$	$\frac{z^{-1}}{(1-z^{-1})^2} = \frac{z}{(z-1)^2}$	$k$
32.	$(0, 0, 1, 2, 3, 4, 5, 6, \dots)$	$\frac{z^{-2}}{(1-z^{-1})^2} = \frac{1}{(z-1)^2}$	$k-1 + \delta_{0k}$
33.	$(1, -2, 3, -4, 5, -6, \dots)$	$\frac{1}{(1+z^{-1})^2} = \frac{z^2}{(z+1)^2}$	$(-1)^k(k+1)$
34.	$(0, 1, -2, 3, -4, 5, -6, \dots)$	$\frac{z^{-1}}{(1+z^{-1})^2} = \frac{z}{(z+1)^2}$	$(-1)^{k+1}k$
35.	$(0, 0, 1, -2, 3, -4, 5, -6, \dots)$	$\frac{z^{-2}}{(1+z^{-1})^2} = \frac{1}{(z+1)^2}$	$(-1)^k(k-1) + \delta_{0k}$

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Nennergrad = 3:

36.	$k^2$	$\frac{(1+z^{-1})z^{-1}}{(1-z^{-1})^3} = \frac{z(z+1)}{(z-1)^3}$	
37.	$(k-1)^2 - \delta_{0k}$	$\frac{(1+z^{-1})z^{-2}}{(1-z^{-1})^3} = \frac{z+1}{(z-1)^3}$	
38.	$(-1)^{k+1}k^2$	$\frac{(1-z^{-1})z^{-1}}{(1+z^{-1})^3} = \frac{z(z-1)}{(z+1)^3}$	
39.	$(-1)^k(k-1)^2 - \delta_{0k}$	$\frac{(1-z^{-1})z^{-2}}{(1+z^{-1})^3} = \frac{z-1}{(z+1)^3}$	

Potenzen  $b^k$  und  $b^{-k}$ :

40.	$b^k$	$\frac{1}{1-bz^{-1}} = \frac{z}{z-b}$	
41.	$b^{-k}$	$\frac{b}{b-z^{-1}} = \frac{bz}{bz-1}$	$b \neq 0$
42.	$kb^k$	$\frac{bz^{-1}}{(1-bz^{-1})^2} = \frac{bz}{(z-b)^2}$	
43.	$kb^{-k}$	$\frac{bz^{-1}}{(b-z^{-1})^2} = \frac{bz}{(bz-1)^2}$	$b \neq 0$
44.	$k^2b^k$	$\frac{(1+bz^{-1})bz^{-1}}{(1-bz^{-1})^3} = \frac{bz(z+b)}{(z-b)^3}$	
45.	$k^2b^{-k}$	$\frac{(b+z^{-1})bz^{-1}}{(b-z^{-1})^3} = \frac{bz(bz+1)}{(bz-1)^3}$	$b \neq 0$

Trigonometrisches:

46.	$\sin(ak)$	$\frac{z^{-1} \sin a}{1-2z^{-1} \cos a + z^{-2}}$ $= \frac{z \sin a}{z^2 - 2z \cos a + 1}$	
47.	$\cos(ak)$	$\frac{1 - z^{-1} \cos a}{1 - 2z^{-1} \cos a + z^{-2}}$ $= \frac{z^2 - z \cos a}{z^2 - 2z \cos a + 1}$	
48.	$k \sin(ak)$	$\frac{(z^{-1} - z^{-3}) \sin a}{(1 - 2z^{-1} \cos a + z^{-2})^2}$ $= \frac{(z^3 - z) \sin a}{(z^2 - 2z \cos a + 1)^2}$	
49.	$k \cos(ak)$	$\frac{(z^{-1} + z^{-3}) \cos a - 2z^{-2}}{(1 - 2z^{-1} \cos a + z^{-2})^2}$ $= \frac{(z^3 + z) \cos a - 2z^2}{(z^2 - 2z \cos a + 1)^2}$	

TABELLE ZUR Z-TRANSFORMATION

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Trigonometrisches (Fortsetzung):			
50.	$b^{-k} \sin(ak)$	$\frac{bz^{-1} \sin a}{b^2 - 2bz^{-1} \cos a + z^{-2}}$ $= \frac{bz \sin a}{b^2 z^2 - 2bz \cos a + 1}$	$b \neq 0$
51.	$b^{-k} \cos(ak)$	$\frac{b^2 - bz^{-1} \cos a}{b^2 - 2bz^{-1} \cos a + z^{-2}}$ $= \frac{bz(bz - \cos a)}{b^2 z^2 - 2bz \cos a + 1}$	$b \neq 0$